Indian Institute of Technology
Hyderabad

FREQUENCY MODULATION

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Contents

Chapter 1

FREQUENCY MODULATION

1 Message

The message signal is available in

fm/msg/codes/Sound_Noise.wav

1. Plot the spectrum of the message signal.

   Solution: The spectrum of input audio signal is plotted in Fig. 1.1 using below code

   /codes/input.py

   To plot the spectrum of the message signal, we need to compute the Fourier Transform of the message signal, which will give us the frequency domain representation of the signal.

$$M_k = \sum_{n=0}^{N-1} m(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1.1.1}$$

In code for computing the FFT of m(n)

$$M_k = \texttt{fft}(x(n), N) \tag{1.1.2}$$

Where M_k is the frequency representation of the signal, m(n) is the input signal.

2. Find the bandwidth of the message signal.

   Solution: :

   (a) By calculating the Power Spectral Density:

   $$PSD(M) = |M(k)|^2$$

   Where PSD(M) is the power spectral density at frequency f, M(K) is the Fourier Transform of the input signal

   (b) Finding the Frequency Range with Significant Power:

   $$mask(f) = \begin{cases} 1 & PSD(f) > T \\ 0 & otherwise \end{cases}$$

   Bandwidth is calculate as the difference between the maximum and minimum frequencies in the range with significant power.

   $$B = f_u - f_l$$

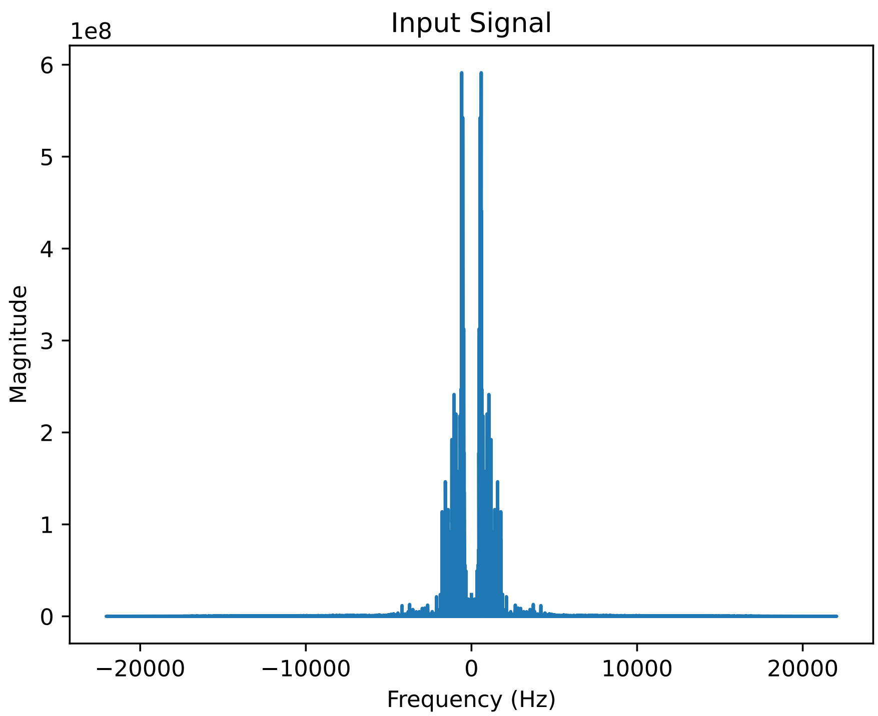

Figure 1.1: spectrum analysis of input signal

where f_l and $f_u$arethe lower and upper frequency bounds of the range respectively.

2 Transmitter

1. The modulated signal is given by

$$s(t) = \cos\left(2\pi f_c t + \phi(t)\right) \tag{1.1}$$

where

$$\phi(t) = 2\pi k_f \int_0^t m(\tau)\, d\tau \tag{1.2}$$

List the various parameters in a table.

Parameter	**Value**	**Description**
T	0.1	Threshold
K_f	20 Hz/volt	Frequency sensitivity
A_c	1	amplitude of the carrier signal
F_c	100 MHz	Frequency of the carrier signal
F_s	44100 Hz	Sampling rate
t	22 μ	Sampling time

2. Obtain a difference equation for computing $\phi(t)$. Suggest a sampling rate.

Solution: To obtain the difference equation for computing $\phi(t)$, we need to discretize the integral in the given equation. We can use the rectangular rule for numerical integration.

Let us divide the interval $[t_0, t]$ into N equal subintervals of width $\Delta t = (t - t_0)/N$. Then, we can approximate $m(\tau)$ by its value at the midpoint of each subinterval, $\tau_n = t_0 + (n + 1/2)\Delta t$, where $n = 0, 1, 2, ..., N - 1$. This gives:

$$m(\tau_n) \approx m(n\Delta t)$$

we can approximate the integral in the given equation 1.2

$$\phi(t) \approx 2\pi k f_c \Delta t \sum_{n=0}^{N-1} m(n\Delta t)$$

ϕ at the nth time step $t_n = t_0 + n\Delta t$. Then, we can write:

$$\phi_n = 2\pi k f_c \Delta t \sum_{k=0}^{n-1} m(k\Delta t) \tag{2.1}$$

$$\phi_{n-1} = 2\pi k f_c \Delta t \sum_{k=0}^{n-2} m(k\Delta t) \tag{2.2}$$

Subtracting the 2.2 from 2.1, we get:

$$\phi_n - \phi_{n-1} = 2\pi k_f f_c \Delta t, m((n - 1)\Delta t) \tag{2.3}$$

3. Plot the spectrum of the transmitted signal.

   Solution: The spectrum of FM signal is plotted in Fig. 1.2 using below

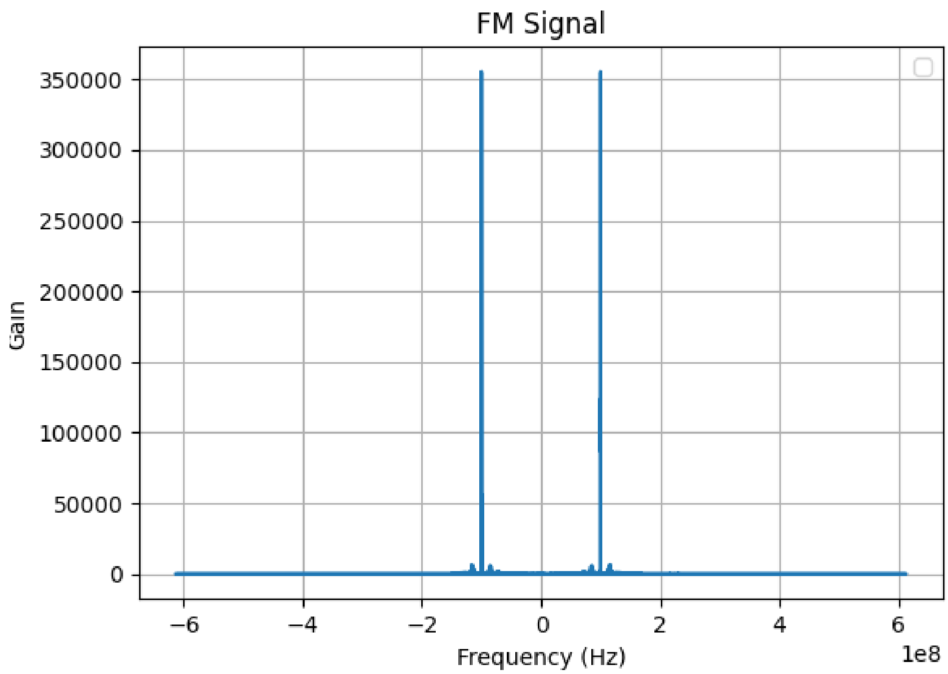

Figure 1.2: spectrum analysis of fm signal

code

/codes/mod.py

4. Compute the bandwidth of the transmitted signal.

   Solution:

   By computing the Fourier Transform:

$$S_k = \sum_{n=0}^{N-1} s(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{4.1}$$

In code for computing the FFT of s(n)

$$S_k = \texttt{fft}(s(n), N) \tag{4.2}$$

Where S_k is the frequency representation of the signal, s(n) is the transmitted signal.

5. Find the bandwidth of the message signal.

   solution: we need to calculate its power spectral density. This can be done using the equation (5.1)

   Calculating the Power Spectral Density:

$$PSD(s) = |S_k|^2 \tag{5.1}$$

we can identify the frequency range with significant power using a mask function.

$$mask(s) = \begin{cases} 1 & PSD(s) > T \\ 0 & otherwise \end{cases}$$

Bandwidth can then be calculated as

$$B = f_u - f_l$$

where f_l and $f_u$arethe lower and upper frequency bounds of the range respectively.