Bayes' Theorem

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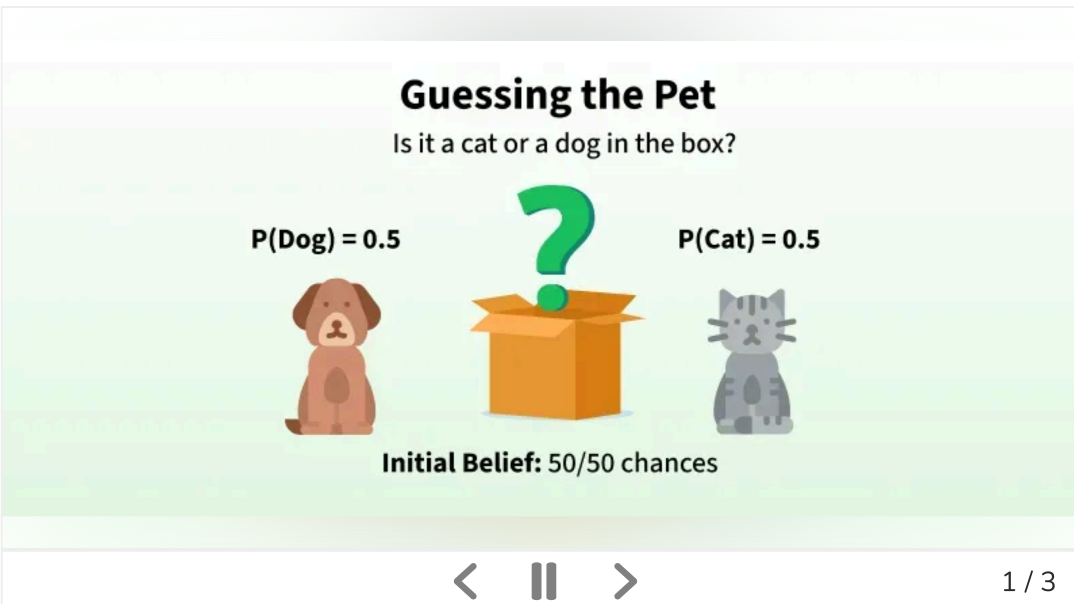
Bayes' Theorem is a mathematical formula that helps determine the **conditional probability** of an event based on prior knowledge and new evidence.

It adjusts probabilities when new information comes in and helps make better decisions in uncertain situations.



Bayes' Theorem helps us update probabilities based on prior knowledge and new evidence. In this case, knowing that the pet is quiet (new information), we can use Bayes' Theorem to calculate the updated probability of the pet being a cat or a dog, based on how likely each animal is to be quiet.

Bayes Theorem and Conditional Probability

occurred.

The general statement of Bayes' theorem is "The conditional probability of an event A, given the occurrence of another event B, is equal to the product of the event of B, given A, and the probability of A divided by the probability of event B." i.e.

> ***For example,*** *if we want to find the probability that a white marble drawn at random came from the first bag, given that a white marble has already been drawn, and there are three bags each containing some white and black marbles, then we can use Bayes' Theorem.*

Check: Bayes's Theorem for Conditional Probability

Bayes Theorem Formula

For any two events A and B, **Bayes's** formula for the Bayes theorem is given by:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Formula for the Bayes theorem

Where,

- **P(A)** and **P(B)** are the probabilities of events A and B, also, P(B) is never equal to zero.
- **P(A|B)** is the probability of event A when event B happens,
- **P(B|A)** is the probability of event B when A happens.

Bayes Theorem Statement

Bayes's Theorem for n sets of events is defined as,

Let E_1, E_2,..., E_n be a set of events associated with the sample space S, in which all the events E_1, E_2,..., E_n have a non-zero probability of occurrence. All the events E_1, E_2,..., E form a partition of S. Let A be an event from space S for which we have to find the probability, then according to Bayes theorem,

$$P(E_i \mid A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{k=1}^{n} P(E_k) \cdot P(A|E_k)}$$

for k = 1, 2, 3, ..., n

Bayes Theorem Derivation

The proof of Bayes's, Theorem is given as, according to the conditional probability formula,

$P(E_i \mid A) = \frac{P(E_i \cap A)}{P(A)}$ **.....(i)**

Then, by using the multiplication rule of probability, we get

$P(E_i \cap A) = P(E_i) \cdot P(A \mid E_i)$**......(ii)**

Now, by the total probability theorem,

$P(A) = \sum_{k=1}^{n} P(E_k) \cdot P(A \mid E_k)$**.....(iii)**

Substituting the value of $P(E_i \cap A)$ and P(A) from eq (ii) and eq(iii) in eq(i) we get,

$$P(E_i \mid A) = \frac{P(E_i) \cdot P(A|E_i)}{\sum_{k=1}^{n} P(E_k) \cdot P(A|E_k)}$$

Bayes’ theorem is also known as the formula for the probability of “causes”. As we know, the E_i‘s are a partition of the sample space S, and at any given time, only one of the events E_i occurs. Thus, we conclude that the Bayes theorem formula gives the probability of a particular E_i, given that event A has occurred.

Terms Related to Bayes Theorem

After learning about **Bayes** theorem in detail, let us understand some important terms related to the concepts we covered in the formula and derivation.

Hypotheses

- Hypotheses refer to possible events or outcomes in the sample space, they are denoted as $\mathbf{E_1, E_2, \ldots, E_n}$.
- Each hypothesis represents a distinct scenario that could explain an observed event.

Priori Probability

- Priori Probability $P(E_i)$ is the initial probability of an event occurring before any new data is taken into account.
- It reflects existing knowledge or assumptions about the event.
- **Example:** The probability of a person having a disease before taking a test.

Posterior Probability

- Posterior probability (P(Ei|A) is the updated probability of an event after considering new information.
- It is derived using the Bayes Theorem.
- **Example:** The probability of having a disease given a positive test result.

- The probability of an event A based on the occurrence of another event B is termed conditional Probability.
- It is denoted as **P(A|B)** and represents the probability of A when event B has already happened.

Joint Probability

- When the probability of two or more events occurring together and at the same time is measured, it is marked as Joint Probability.
- For two events A and B, it is denoted by joint probability is denoted as **P(A∩B).**

Random Variables

- Real-valued variables whose possible values are determined by random experiments are called random variables.
- The probability of finding such variables is the experimental probability.

Bayes Theorem Applications

Bayesian inference is very important and has found application in various activities, including medicine, science, philosophy, engineering, sports, law, etc., and Bayesian inference is directly derived from Bayes theorem.

Some of the Key Applications are:

- **Medical Testing** → Finding the real probability of having a disease after a positive test.
- **Spam Filters** → Checking if an email is spam based on keywords.
- **Weather Prediction** → Updating the chance of rain based on new data.
- **AI & Machine Learning** → Used in **Naïve Bayes classifiers** to predict outcomes.

Check, Bayes' Life Applications of Bayes theorem

Difference Between Conditional Probability and Bayes Theorem

The difference between Conditional Probability and Bayes's. The theorem can be understood with the help of the table given below.

Bayes Theorem	Conditional Probability
Bayes's Theorem is derived using the definition of conditional probability. It is used to find the reverse probability.	Conditional Probability is the probability of event A when event B has already occurred.
Formula: P(A|B) = [P(B|A)P(A)] / P(B)	**Formula:** P(A|B) = P(A∩B) / P(B)
Purpose: To update the probability of an event based on new evidence.	**Purpose**: To find the probability of one event based on the occurrence of another.
Focus: Uses prior knowledge and evidence to compute a revised probability.	**Focus**: Direct relationship between two events.

Theorem of Total Probability

Let E1, E2,…., En be **mutually exclusive and exhaustive events** of a sample space S, and let E be any event that occurs with some E_i. Then, prove that :

$$P(E) = {}^{n}\sum_{i=1} P(E/E_i) \,.\, P(E_i)$$

Proof:

Let S be the sample space

$S = E_1 \cup E_2 \cup E_3 \cup \ldots \cup En$ and $E_i \cap E_j = \varnothing$ for $i \neq j$.
Now, consider the event E: $E = E \cap S$
Substituting S with the union of Ei's:
$\Rightarrow E = E \cap (E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n)$
Using distributive law:
$\Rightarrow E = (E \cap E_1) \cup (E \cap E_2) \cup \ldots \cup (E \cap E_n)$
*Since the events Ei are mutually exclusive, the intersections E∩Ei are also **mutually exclusive**. Therefore:*

$P(E) = P\{(E \cap E_1) \cup (E \cap E_2) \cup \ldots \cup (E \cap E_n)\}$
$\Rightarrow P(E) = P(E \cap E_1) + P(E \cap E_2) + \ldots + P(E \cap E_n)$
{Therefore, $(E \cap E_1), (E \cap E_2), \ldots, (E \cap E_n)$} are pairwise disjoint}

$\Rightarrow P(E) = P(E/E_1) . P(E_1) + P(E/E_2) . P(E_2) + \ldots + P(E/E_n) . P(E_n)$ [by multiplication theorem]
$\Rightarrow P(E) = {}^n\sum_{i=1} P(E/E_i) . P(E_i)$

Articles Related to Bayes' Theorem-

- Conditional Probability
- Probability Distribution

Bayes Theorem for Programmers-

- *Bayes Theorem in Artificial Intelligence*
- *Bayes Theorem in Machine Learning*
- *Bayes' Theorem in Data Mining*

Solved Examples of Bayes's Theorem

Example 1: A person has undertaken a job. The probabilities of completion of the job on time with and without rain are 0.44 and 0.9, **and** 5, respectively. If the probability that it will rain is 0.45, then determine the probability that the job will be completed on time**.**

Solution:

Let E_1 be the event that the mining job will be completed on time

By multiplication law of probability,
$P(E_1) = 0.44$, and $P(E_2) = 0.95$

Since, events A and B form partitions of the sample space S, by total probability theorem, we have

$P(E) = P(A)\ P(E_1) + P(B)\ P(E_2)$
$\Rightarrow P(E) = 0.45 \times 0.44 + 0.55 \times 0.95$
$\Rightarrow P(E) = 0.198 + 0.5225 = 0.7205$

So, the probability that the job will be completed on time is 0.7205

Example 2: There are three urns containing 3 white and 2 black balls, 2 white and 3 black balls, and 1 black and 4 white balls, respectively. There is an equal probability of each urn being chosen. One ball is equal probability chosen at random. What is the probability that a white ball will be drawn?

Solution:

Let E_1, E_2, and E_3 be the events of choosing the first, second, and third urn respectively. Then,
$P(E_1) = P(E_2) = P(E_3) = 1/3$

Let E be the event that a white ball is drawn. Then,
$P(E/E_1) = 3/5$, $P(E/E_2) = 2/5$, $P(E/E_3) = 4/5$

By theorem of total probability, we have
$P(E) = P(E/E_1) . P(E_1) + P(E/E_2) . P(E_2) + P(E/E_3) . P(E_3)$
$\Rightarrow P(E) = (3/5 \times 1/3) + (2/5 \times 1/3) + (4/5 \times 1/3)$
$\Rightarrow P(E) = 9/15 = 3/5$

Example 3: A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts. Find the probability of the lost card being a heart.

Solution:

Let E_1, E_2, E_3, and E_4 be the events of losing a card of hearts, clubs, spades, and diamonds respectively.

$P(E|E_1)$ = probability of drawing 2 hearts, given that a card of hearts is missing
⇒ $P(E|E_1) = {}^{12}C_2 / {}^{51}C_2 = (12 \times 11)/2! \times 2!/(51 \times 50) = 22/425$

$P(E|E_2)$ = probability of drawing 2 clubs ,given that a card of clubs is missing
⇒ $P(E|E_2) = {}^{13}C_2 / {}^{51}C_2 = (13 \times 12)/2! \times 2!/(51 \times 50) = 26/425$

$P(E|E_3)$ = probability of drawing 2 spades ,given that a card of hearts is missing
⇒ $P(E|E_3) = {}^{13}C_2 / {}^{51}C_2 = 26/425$

$P(E|E_4)$ = probability of drawing 2 diamonds ,given that a card of diamonds is missing
⇒ $P(E|E_4) = {}^{13}C_2 / {}^{51}C_2 = 26/425$

Therefore,

$P(E_1|E)$ = probability of the lost card is being a heart, given the 2 hearts are drawn from the remaining 51 cards

⇒ $P(E_1|E) = P(E_1) . P(E|E_1)/P(E_1) . P(E|E_1) + P(E_2) . P(E|E_2) + P(E_3) . P(E|E_3) + P(E_4) . P(E|E_4)$
⇒ $P(E_1|E) = (1/4 \times 22/425) / \{(1/4 \times 22/425) + (1/4 \times 26/425) + (1/4 \times 26/425) + (1/4 \times 26/425)\}$
⇒ $P(E_1|E) = 22/100 = 0.22$

Hence, The required probability is 0.22.

Example 4: Suppose 15 men out of 300 men and 25 women out of 1000 are good orators. An orator is chosen at random. Find the probability that a male person is selected.

Solution:

Given,

- *Total Men = 300*
- *Total Women = 1000*
- *Good Orators among Men = 15*
- *Good Orators among Women = 25*

P(Male Orator) = Numbers of male orators / total no of orators = 15/40 = 3/8

Example 5: A man is known to speak the lies 1 out of 4 times. He throws a die and reports that it is a six. Find the probability that it **Bayes'** actually a six.

Solution:

In a throw of a die, let
E_1 *= event of getting a six,*
E_2 *= event of not getting a six and*
E = event that the man reports that it is a six.

Then, $P(E_1) = 1/6$*, and* $P(E_2) = (1 - 1/6) = 5/6$

$P(E|E_1)$ *= probability that the man reports that six occurs when six has actually occurred*
$\Rightarrow P(E|E_1)$ *= probability that the man speaks the truth*
$\Rightarrow P(E|E_1) = 3/4$

$P(E|E_2)$ *= probability that the man reports that six occurs when six has not actually occurred*

$\Rightarrow P(E|E_2)$ *= probability that the man does not speak the truth*
$\Rightarrow P(E|E_2) = (1 - 3/4) = 1/4$

Probability of getting a six ,given that the man reports it to be six

$P(E_1|E) = P(E|E_1) \times P(E_1)/P(E|E_1) \times P(E_1) + P(E|E_2) \times P(E_2)$ *[by Bayes theorem]*
$\Rightarrow P(E_1|E) = (3/4 \times 1/6)/\{(3/4 \times 1/6) + (1/4 \times 5/6)\}$
$\Rightarrow P(E_1|E) = (1/8 \times 3) = 3/8$

Hence the probability required is 3/8.

Next Article

Probability Distribution - Function, Formula, Table

1. Bayes' Theorem in Data Mining
2. Probability Theory
3. Bayes Theorem in Machine learning
4. Theoretical Probability
5. Nyquist Sampling Theorem
6. Bernoulli Naive Bayes
7. Mid Point Theorem
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9. Real Life Applications of Bayes' Theorem
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