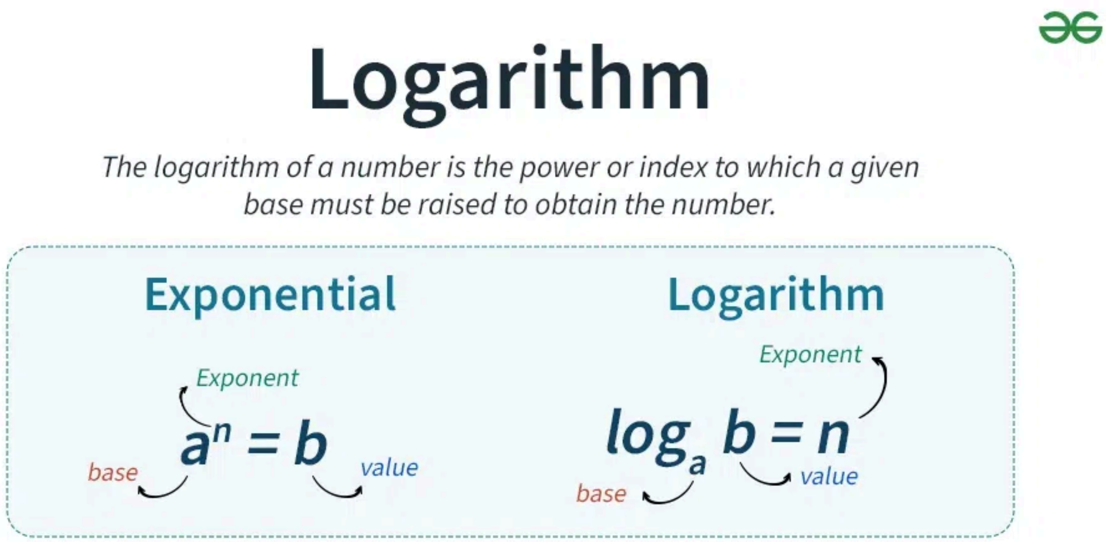
Introduction to Logarithm

Last Updated : 31 Dec, 2024

Logarithm is a mathematical function that represents the exponent to which a fixed number, known as the base, must be raised to produce a given number. In other words, it is the inverse operation of exponentiation.



Mathematical Expression for Log

If $a^n = b$ then log or logarithm is defined as the log of b at base a is equal to n. It should be noted that in both cases base is 'a' but in the log, the base is with the result and not the power.

$$a^n = b \Rightarrow \log_a b = n$$

- ***a*** *is Base*
- ***b*** *is Argument*
- ***a*** *and* ***b*** *Positive Real Numbers*

Conversion from Exponential to Log Form

If a number is expressed in the exponential form for example an = b where a is the base, n is the exponent and b is the result of the exponent then to convert it into the logarithmic form the base 'a' remains base in logarithm, the result 'b' becomes an argument and the exponent 'n' becomes the result here.

$$a^n = b \Rightarrow \log_a b = n$$

Conversion from Log to Exponential Form

If the expression is in logarithmic form then we can convert it into exponential form by making the argument as a result and the result of logarithm becomes the exponent while the base remains the same. It can be better understood from the expression mentioned below:

$$\log_a b = n \Rightarrow a^n = b$$

Types of Logarithm

Depending upon the base, there are two types of logarithm, which are,

Common Logarithm

The logarithm with base 10 is known as **Common Logarithm**. It is written as $\log_{10}X$. The common logarithm is generally written as **log** only instead of **$\log_{10}$**.

Let's see some examples.

- $\log 10 = \log_{10} 10 = 1$
- $\log 1 = \log_{10} 1 = 0$
- $\log 1000 = \log_{10} 1000 = 3$

Natural Logarithm

The logarithm with base e, where e is a mathematical constant is called **Natural Logarithm**. It is written as $\log_e X$. The natural logarithm is also written in the abbreviated form as **ln** i.e. $\log_e X = \ln X$.

Let's see some example:

- $\log_e 2 = X \Rightarrow e^X = 2$
- $\log_e 5 = y \Rightarrow e^y = 5$

Difference between Common Logarithm (Log) and Natural Log (ln)

The basic difference between log and ln is tabulated below:

Log	Ln

Log	Ln
It is given as $\log_{10}X$	It is given as $\log_e x$
It is called a common log and is represented as log x	It is called natural log and is represented as ln x
It is mostly used for solving large numbers and simplifying calculations.	It is less commonly used

Learn More: Difference Between Log and Ln

Rules of Logarithm | Log Rules

The common properties or rules of Log are :

- Product Rule
- Division Rule
- Power Rule
- Change of Base Rule
- Base Switch Rule
- Equality of Log
- Number raised to Log Power
- Negative Log Rule

Read in Detail: Logarithm Rules | List of all the Log Rules with Examples

Apart from the above-mentioned properties, there are some other properties of Log. Using these properties we can directly put their values in any equation. These properties are mentioned below:

- Log 1 =0 This property of log states that the value of Log 1 is always zero, no matter what the base is. This is because any number raised to because any number raised to power 1 results in the number itself. Hence, ln e = $\log_e e$ = 1 r.

- The value of log of zero is not defined as there is no such number when raised to any power that results in zero. Hence, log 0 = Not defined.

Log and Antilog Table

Log Table

Log Table is **used to find the value of the log** without the use of a calculator. The log table provides the logarithmic value of a number at a particular base.

A **log table has mainly three columns.** The first column contains two-digit numbers from 10 to 99, the second column contains differences for digits 0 to 9 and hence called the difference column and the third column contains mean difference from 1 to 9 and hence called **mean difference column**.

The **log table for base e is called the natural logarithm table and that for base 2 is called the binary log table**.

The logarithmic value of a number contains two parts named **characteristics** and **mantissa** both separated by a decimal. Characteristic is the integral part written o the left side of the table and can be positive or negative while the mantissa is the fraction or decimal that is always positive.

Read More : Log Table

Anti Log Table

Antilog is the process of finding the inverse of the log of the number. This is used when the number is already given in log value and we need to

Antilog table is helpful in finding the Antilog value without using the calculator. The antilog table also consists of 3 columns among which the first column contains numbers from .00 to .99, the second block which is the difference column contains digits from 0 to 9, and the third block which is the mean difference column contains digits from 1 to 9.

Read More: Antilog Table

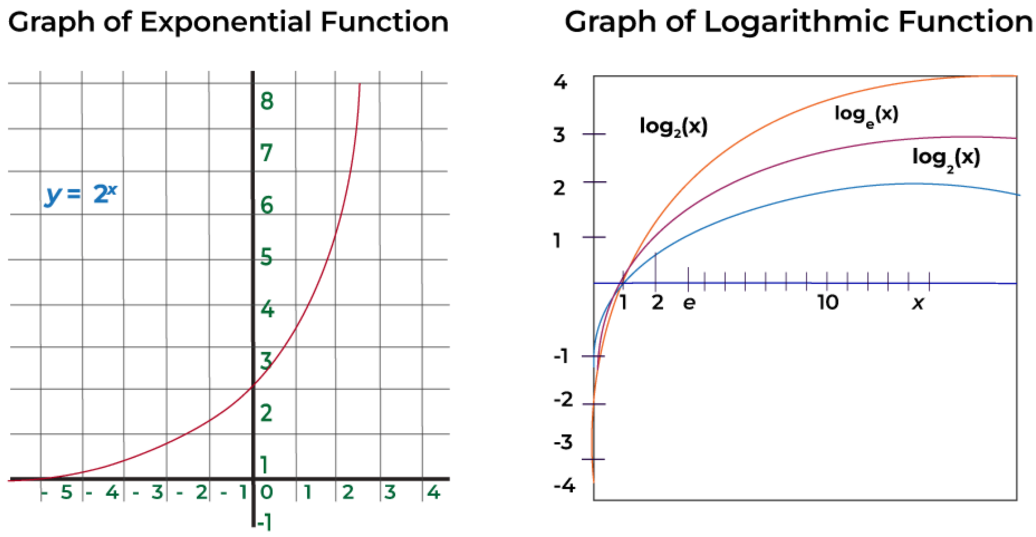
Logarithmic Function

A logarithmic function is the inverse of an exponential function and is defined for positive real numbers with a positive base (not equal to 1). The logarithmic function to the base *b* is represented as $f(x) = \log_b(x)$, where $x>0$ and $b>0$. In this function, X is the argument of the logarithm, and b is the base.

Graph of Logarithmic Function

A logarithmic function is the inverse of an exponential function and is defined for positive real numbers with a positive base (not equal to 1). The logarithmic function to the base *b* is represented as $f(x) = \log_b(x)$, where $x>0$ and $b>0$. In this function, X is the argument of the logarithm, and b is the base.

We know that the domain of Logarithmic Function is (0, ∞) and its range is a set of all real numbers. If we plot the graph using the set of domain and range we find that the graph of the logarithmic function is just the inverse of the graph obtained for the exponential function.



Logarithmic Graph

This indicates the inverse relationship between exponential and logarithmic functions. Also, the logarithmic graph is symmetric around the line y = x. We know that the value of log 1 is zero at any base value. Hence it has an intercept (1,0) on the x-axis and no intercept on the y-axis as log 0 is not defined.

Articles Related to Logarithms

- [Laws of Logarithm]
- [Change of Base Rule]

Solved Examples on Logarithms

Example 1: Find $\log_a 16 + 1/2 \log_a 225 - 2\log_a 2$

Solution:

$$\log_a 16 + 1/2 \times 2\log_a 15 - \log_a 2^2$$

$$\Rightarrow \log_a 16 + \log_a 15 - \log_a 4$$
$$\Rightarrow \log_a (16 \times 15) - \log_a 4$$

$$\Rightarrow \log_a (16 \times 15/4) = \log_a 60$$

Example 2: Solve $\log_b 3 - \log_b 27$

⇒ $log_2(3/48)$
⇒ $log_2(1/16)$
⇒ $log_2(-16)$
⇒ $-log_2 2^4$
⇒ $-4log_2 2 = -4$

Example 3: Find x in $\log_b x + \log_b(x - 3) = \log_b 10$

Solution:

Given $log_b x + log_b(x - 3) = log_b 10$

⇒ $log_b(x)(x - 3) = log_b 10$
⇒ $(x)(x - 3) = 10$
⇒ $x^2 - 3x - 10 = 0$
⇒ $x^2 - 5x + 2x - 10 = 0$
⇒ $x(x - 5) + 2(x - 5)$
⇒ $(x - 5)(x + 2) = 0$
⇒ $x = 5, -2$

People Also Read,

- Logarithmic Differentiation
- Continuity and Differentiability of Logarithm
- Derivative of Exponential and Logarithmic Functions

Next Article

Introduction to Logarithm

Similar Reads

1. Logarithmic Function
2. Logarithmic Differentiation
3. Solving Logarithmic Equations
4. Logarithmic Functions Practice Problems