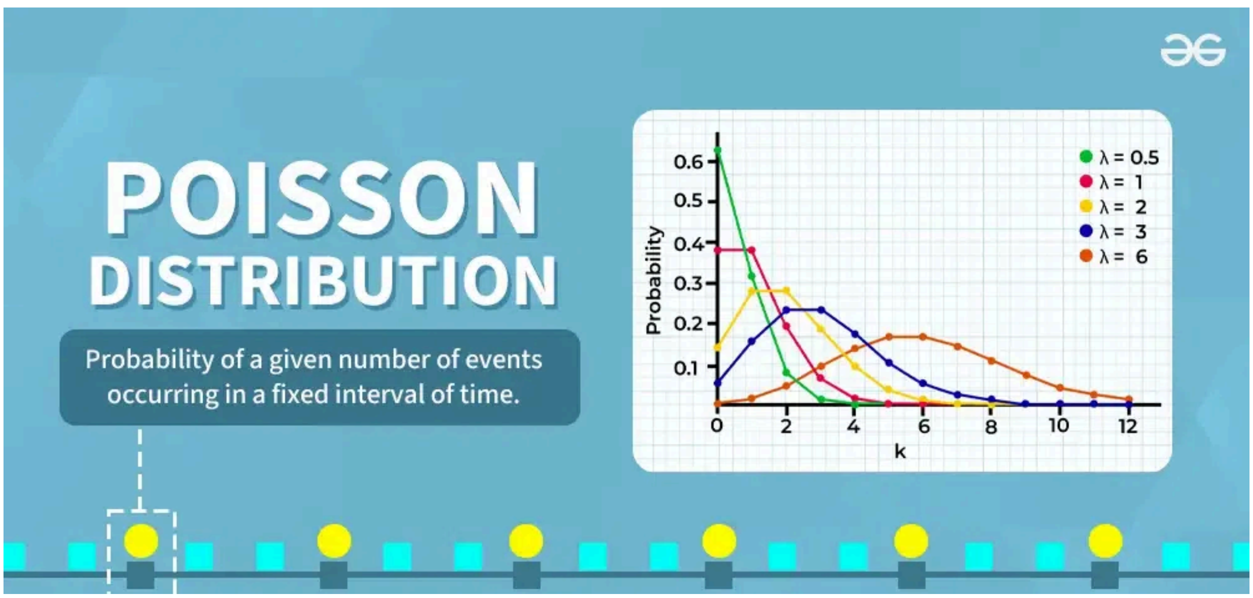
# Poisson Distribution | Definition, Formula, Table and Examples

Last Updated : 11 Apr, 2025

The **Poisson distribution** is a **discrete probability distribution** that calculates the likelihood of a certain number of events happening in a fixed time or space, assuming the events occur independently and at a constant rate.

It is characterized by a single parameter, **λ (lambda)**, which represents the event's average occurrence rate. The distribution is used when the events are rare, the number of occurrences is non-negative, and can take on integer values (0, 1, 2, 3,...).




The key assumptions of the Poisson distribution are:

1. *Events occur independently of each other.*
2. *The average rate of occurrence (λ) is constant over the given*

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

Got It !

In summary, the Poisson distribution is used to model the likelihood of events happening at a certain rate within a fixed time or space, under the assumptions of independence and constant occurrence.

**Table of Content**



## Poisson Distribution Formula

Poisson distribution is characterized by a single parameter, lambda (λ), which represents the average rate of occurrence of the events. The probability mass function of the Poisson distribution is given by:

$$P\ (X = k) = e^{-\lambda}\lambda^{k}\ /\ k!$$

Where,

- **P(X = k)** is the Probability of observing k Events
- **e** is the Base of the Natural Logarithm (approximately 2.71828)
- **λ** is the Average Rate of Occurrence of Events
- **k** is the Number of Events that Occur

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

This table is a tabulation of probabilities for a Poisson distribution and probabilities here can be calculated using the Probability Mass Function of Poisson Distribution which is given by PMF $= \frac{\lambda^k e^{-\lambda}}{k!}$

## Related searches

Binomial and Poisson Distribution Questions and Answers Pdf

Po

The following table is one such example of the Poisson Distribution Table.

| k (Number of Events) | P(X = k) |
|---|---|
| 0 | 0.0498 |
| 1 | 0.1494 |
| 2 | .2241 |
| 3 | 0.2241 |
| 4 | 0.1681 |
| 5 | 0.1009 |

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

| k (Number of Events) | P(X = k) |
|---|---|
| 7 | 0.0214 |
| 8 | 0.0080 |
| 9 | 0.0027 |
| 10 | 0.0008 |

## Poisson Distribution Characteristics

- **Probability Mass Function (PMF)**: PMF describes the likelihood of observing a specific number of events in a fixed interval. It is given by:
  $P(X = k) = (e^{-\lambda} \times \lambda^k) / k!$ , k=0,1,2,...

- **Cumulative Distribution Function (CDF)**: CDF gives the probability that the random variable is less than or equal to a certain value. It is expressed as:
  $F(x) = \sum(\text{from } k=0 \text{ to } \lfloor x \rfloor)\ (e^{-\lambda} \times \lambda^k) / k!$
  where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.

- **Moment Generating Function (MGF)**: MGF provides a way to derive moments of the distribution. It is represented by:
  $M(t) = e^{(\lambda(e^\wedge t - 1))}$
  The MGF helps derive moments like mean and variance by differentiating them.

- **Characteristic Function (CF)**: CF is an alternative way to describe the distribution and is given by:
  $\phi(t) = e^{(\lambda(e^\wedge(it) - 1))}$

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

- **Probability Generating Function (PGF)**: PGF generates the probabilities of the distribution and is expressed as:
  $G(z) = e^{(\lambda(z - 1))}$

- Median: The Median, which represents the central value, is approximately $\lambda + (1/3) - 0.02/\lambda$.

- Mode: Mode, or the most probable value, is simply the integer part of λ, denoted as $\lfloor\lambda\rfloor$.
  If λ is an **integer**, then both λ and $\lambda-1$ are modes.
- If λ is **not an integer**, the mode is simply $\lfloor\lambda\rfloor$ (integer part of λ).

- Mean and Variance: The mean (λ) and variance (λ) of a Poisson distribution are equal. This means that both the average number of events and the spread or variability around this average are characterized by the same parameter.

- **Non-negative and Discrete**: The Poisson distribution describes the probability of non-negative integer values only, as it models counts of events. It is a discrete probability distribution.

- **Memorylessness**: Events in a Poisson process are memoryless, meaning the probability of an event occurring in the future is independent of the past, given the current state. For example, if you're waiting for a bus, the probability of the bus arriving in the next minute doesn't depend on how long you've already been waiting.

- **Independent Increments**: The number of events occurring in non-overlapping intervals is independent. For instance, if you're counting the number of cars passing through an intersection in one minute, the number of cars in the next minute is independent of the number in the previous minute.

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

distribution can approximate the binomial distribution. This is known as the "rare events" approximation, where the binomial distribution with a large number of trials and a small probability of success converges to a Poisson distribution.

- **Skewness and Kurtosis**: The Poisson distribution is positively skewed (skewness > 0) and leptokurtic (kurtosis > 0), meaning it has a longer tail on the right side and heavier tails than the normal distribution. However, for large values of λ, it becomes increasingly symmetric and bell-shaped, resembling a normal distribution.

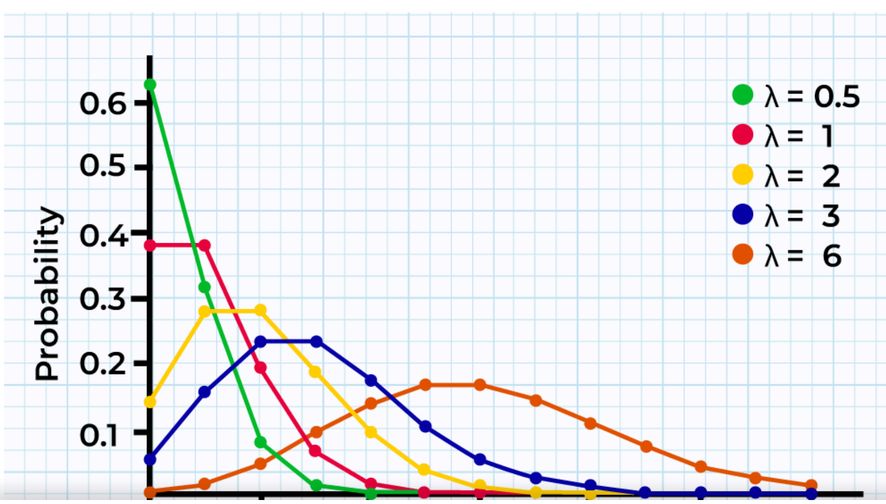
Some other properties are:

- Poisson distribution has only one parameter "λ" where λ = np.
- Poisson distribution is positively skewed and leptokurtic.

> ***Note:*** *Here* ***leptokurtic*** *means values greater* ***kurtosis*** *than the normal distribution, and* ***kurtosis*** *is the nothing but the sharpness of the peak of the* ***frequency distribution curve****.*

## Poisson Distribution Graph

The following illustration shows the Graph of the Poisson Distribution or the Poisson Distribution Curve.




We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

## Poisson Distribution Mean and Variance

In the Poisson distribution, both the mean (average) and variance are equal and are denoted by the parameter λ (lambda). This property of equal mean and variance is a distinctive characteristic of the Poisson distribution and simplifies its statistical analysis.

### Poisson Distribution Mean

The mean for poison distribution is equal to the parameter i.e., λ. Mathematically, this equation is represented as follows:

$$E[X] = \lambda$$

where,

- **E[X]** is the Mean of Poisson's Distribution
- **λ** is the Parameter of the Distribution
- **X** is a Random Variable following a Poisson distribution

Other than this, we have one more formula for the mean of expectation of the distribution,n, that is:

$$Mean = \lambda = np$$

where,

- **n** is the Number of Trails
- **p** is the Probability of Success

### Poisson Distribution Variance

Variance is the measure of the spread or dispersion of the random variable around its mean. For Poisson Distribution, variance is equal to the

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

***Var(X) = λ***

where,

- Var(X) is the variance of the Poisson-distributed random variable X
- λ is the parameter of the Poisson distribution

## Standard Deviation of Poisson Distribution

Standard Deviation of a Poisson distribution is a measure of the amount of variability or dispersion in the distribution. Mathematically, it is given by:

***σ = √λ***

Where,

- **λ** (lambda) is the Average Rate of Occurrence of Events
- **σ** (sigma) is the Standard Deviation of the Distribution

## Probability Mass Function of Poisson Distribution

The **Probability Mass Function (PMF)** of a Poisson-distributed random variable X represents the probability that X takes on a specific value k.

$$\text{PMF} = \frac{\lambda^{\mathbf{k}}\mathbf{e}^{-\lambda}}{\mathbf{k}!}$$

where,

- **λ** is the Parameter, which is also equal to the Mean, and Variance
- **k** is the Number of times an event occurs
- **e** is Euler's Number (≈2.718)

## Properties of PMF in Poisson Distribution

- **Non-Negativity**: P ( X = k ) ≥ 0 for all k.

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

$\sum_{k=0}^{\infty} P(X=k)=1$

**Example:** Suppose a hospital receives an average of λ = 4 emergency cases per hour. What is the probability that exactly **2** cases occur in an hour?

**Solution:**

Using the Poisson formula:

***$P(X = 2) = e^{-4}\, 4^2 / 2! = e^{-4} \times 16/2 = 0.0183 \times 16/2 = 0.1465$***

## Probability mass function graphs

A probability mass function graph is a visual representation of a Poisson distribution that can be represented visually as a graph of the. A probability mass function is a function that describes a discrete probability distribution.

The event with the highest probability is represented by the peak of the distribution—the mode.

- When λ is a non-integer, the mode is the closest integer smaller than λ.
- When λ is an integer, there are two modes: λ and λ−1.

When λ is low, the distribution is much more distributed on the right side of its peak than its left (right-skewed).

As λ increases, the distribution starts to appear more and more similar to a normal distribution. When λ is 10 or greater, a normal distribution is a good approximation of the Poisson distribution.

## Difference between Binomial and Poisson Distribution

The key differences between Poisson Distribution and Binomial Distribution are listed in the following table:

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

| Difference between Binomial and Poisson Distribution | | |
|---|---|---|
| **Aspect** | **Binomial Distribution** | **Poisson Distribution** |
| **Nature** | Discrete | Discrete |
| **Number of Trials** | Fixed (n) | Unlimited |
| **Outcome** | Success or Failure | Rare Events |
| **Parameter** | Probability of Success (p) | Average Event Rate (λ) |
| **Possible Values** | 0 to n | 0, 1, 2, . . . |
| **Mean** | $\mu = n \times p$ | $\mu = \lambda$ |
| **Variance** | $\sigma^2 = n \times p \times (1 - p)$ | $\sigma^2 = \lambda$ |
| **Applicability** | Limited to a fixed number of trials | Rare events over a large population |
| **Example** | Flipping a coin multiple times | Counting occurrences of an event |
| **Assumptions** | Independent trials, constant p | Rare events, low probability of success |

## Poisson Distribution Solved Examples

### Example 1:If 4% of the total items made by a factory are defective. Find

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

**Solution:**

*Here we have, n = 50, p = (4/100) = 0.04, q = (1-p) = 0.96, λ = 2*

*Using Poisson's Distribution,*
*P(X = 0) =* $\frac{2^0 e^{-2}}{0!}$ *= 1/e*$^2$ *= 0.13534*
*P(X = 1) =* $\frac{2^1 e^{-2}}{1!}$ *= 2/e*$^2$ *= 0.27068*

*Hence the probability that less than 2 items are defective in sample of 50 items is given by:*
*P( X > 2 ) = P( X = 0 ) + P( X = 1 ) = 0.13534 + 0.27068 = 0.40602*

**Example 2: If the probability of a bad reaction from medicine is 0.002, determine the chance that out of 1000 persons more than 3 will suffer a bad reaction from medicine.**

**Solution:**

*Here we have, n = 1000, p = 0.002, λ = np = 2*
*X = Number of person suffer a bad reaction*

***Using Poisson's Distribution***

*P(X > 3) = 1 - {P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)}*
*P(X = 0) =* $\frac{2^0 e^{-2}}{0!}$ *= 1/e*$^2$
*P(X = 1) =* $\frac{2^1 e^{-2}}{1!}$ *= 2/e*$^2$
*P(X = 2) =* $\frac{2^2 e^{-2}}{2!}$ *= 2/e*$^2$
*P(X = 3) =* $\frac{2^3 e^{-2}}{3!}$ *= 4/3e*$^2$
*P(X > 3) = 1 - [19/3e*$^2$*] = 1 - 0.85712 = 0.1428*

**Example 3:If 1% of the total screws made by a factory are defective. Find the probability that less than 3 screws are defective in a sample of 100 screws.**

**Solution:**

*Here we have, n = 100, p = 0.01, λ = np = 1*

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

*P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)*
*P(X = 0) =* $\frac{1^0 e^{-1}}{0!}$ *= 1/e*
*P(X = 1) =* $\frac{1^1 e^{-1}}{1!}$ *=1/e*
*P(X = 2) =* $\frac{1^2 e^{-1}}{2!}$ *=1/2e*

*Thus, P(X < 3) = 1/e + 1/e +1/2e = 2.5/e = 0.919698*

**Example 4: If in an industry there is a chance that 5% of the employees will suffer from corona. What is the probability that in a group of 20 employees, more than 3 employees will suffer from coronavirususthe?**

**Solution:**

*Here we have, n = 20, p = 0.05, λ = np = 1*
*X = Number of employees who will suffer corona*

***Using Poisson's Distribution***

*P(X > 3) = 1-[P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)]*
*P(X = 0) =* $\frac{1^0 e^{-1}}{0!}$ *= 1/e*
*P(X = 1) =* $\frac{1^1 e^{-1}}{1!}$ *= 1/e*
*P(X = 2) =* $\frac{1^2 e^{-1}}{2!}$ *=1/2e*
*P(X = 3) =* $\frac{1^3 e^{-1}}{3!}$ *=1/6e*
*P(X > 3) = 1 - [1/e + 1/e + 1/2e + 1/6e]*

*⇒ P(X > 3) = 1 - [ 8/3e] = 0.018988*

| People Also Read | |
|---|---|
| Poisson Distribution Meaning | Types of Frequency Distribution |
| Probability Distribution | Binomial Distribution |

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

P-Value: Comprehensive Guide to Understand, Apply, and Interpret

## Similar Reads

1. Uniform Distribution | Formula, Definition and Examples
2. Normal Distribution | Definition, Uses & Examples
3. Binomial Distribution in Business Statistics - Definition, Formula & Examples
4. Geometric Distribution | Formula, Mean and Examples
5. Compound Probability: Definition, Formulas, Examples
6. Measures of Dispersion | Types, Formula and Examples
7. Mathematics | Probability Distributions Set 5 (Poisson Distribution)
8. Negative Binomial Distribution : Properties, Applications and Examples
9. Mathematics | Probability Distributions Set 3 (Normal Distribution)
10. Poisson Distribution : Meaning, Characteristics, Shape, Mean and Variance

GeeksforGeeks
Sanchhaya Education Private Limited

**Corporate & Communications Address:**
A-143, 7th Floor, Sovereign Corporate Tower, Sector- 136, Noida, Uttar Pradesh (201305)

**Registered Address:**
K 061, Tower K, Gulshan Vivante Apartment, Sector 137, Noida, Gautam Buddh Nagar, Uttar Pradesh, 201305

GET IT ON Google Play
Download on the App Store

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

About Us
Legal
Privacy Policy
Careers
In Media
Contact Us
Corporate Solution
Campus Training Program

Job-A-Thon
Offline Classroom Program
DSA in JAVA/C++
Master System Design
Master CP
Videos

## Tutorials

Python
Java
C++
PHP
GoLang
SQL
R Language
Android

## DSA

Data Structures
Algorithms
DSA for Beginners
Basic DSA Problems
DSA Roadmap
DSA Interview Questions
Competitive Programming

## Data Science & ML

Data Science With Python
Machine Learning
ML Maths
Data Visualisation
Pandas
NumPy
NLP
Deep Learning

## Web Technologies

HTML
CSS
JavaScript
TypeScript
ReactJS
NextJS
NodeJs
Bootstrap
Tailwind CSS

## Python Tutorial

Python Examples
Django Tutorial
Python Projects
Python Tkinter
Web Scraping
OpenCV Tutorial
Python Interview Question

## Computer Science

GATE CS Notes
Operating Systems
Computer Network
Database Management System
Software Engineering
Digital Logic Design
Engineering Maths

## DevOps

Git
AWS

## System Design

High Level Design
Low Level Design


We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy

DevOps Roadmap

System Design Bootcamp

Interview Questions

### School Subjects

Mathematics

Physics

Chemistry

Biology

Social Science

English Grammar

### Databases

SQL

MYSQL

PostgreSQL

PL/SQL

MongoDB

### Preparation Corner

Company-Wise Recruitment Process

Aptitude Preparation

Puzzles

Company-Wise Preparation

### More Tutorials

Software Development

Software Testing

Product Management

Project Management

Linux

Excel

All Cheat Sheets

### Courses

IBM Certification Courses

DSA and Placements

Web Development

Data Science

Programming Languages

DevOps & Cloud

### Programming Languages

C Programming with Data Structures

C++ Programming Course

Java Programming Course

Python Full Course

### Clouds/Devops

DevOps Engineering

AWS Solutions Architect Certification

Salesforce Certified Administrator Course

### GATE 2026

GATE CS Rank Booster

GATE DA Rank Booster

GATE CS & IT Course - 2026

GATE DA Course 2026

GATE Rank Predictor

@GeeksforGeeks, Sanchhaya Education Private Limited, All rights reserved

We use cookies to ensure you have the best browsing experience on our website. By using our site, you acknowledge that you have read and understood our Cookie Policy & Privacy Policy