Probability Distribution Function

Last Updated : 04 Dec, 2024

Probability Distribution refers to the function that gives the probability of all possible values of a random variable.It shows how the probabilities are assigned to the different possible values of the random variable.

Common types of probability distributions Include:

- Binomial Distribution.
- Bernoulli Distribution.
- Normal Distribution.
- Geometric Distribution.

> *Note: A Probability Distribution can also be seen as the set of all possible outcomes of a random experiment, showing how probabilities are distributed across the values of the random variable.*

A **Probability Distribution Function (PDF)** is a mathematical function that describes the likelihood of different outcomes in a random experiment. For any random variable **X**, where its value is evaluated at the points 'x', then the probability distribution function gives the probability that X takes the value less than equal to x.

We represent the probability distribution as, F(x) = P (X ≤ x)

Probability Distribution Function is also called Cumulative Distribution Function(CDF), The CDF represents the cumulative probability up to a certain value of the random variable.

The cumulative probability for a closed interval(a, b] is given by:

Note: For probability distribution function the value of the variable lies between 0 and 1: 0 ≤ F(x) ≤ 1

Related Reads: Probability Distribution

Probability Distribution Function (PDF) Formula

The probability distribution function formula gives the probability of all the possible outcomes of any random variable. Depending on the type of random variable (discrete or continuous), the formulas for the probability distribution function can differ.

Different formulas for the probability distribution function are listed beolw:

Probability Distribution of a Discrete Random Variable

Discrete Random Variable is the variable that takes distinct countable values are 0, 1, 2, 3 ... The formula for the probability distribution of a discrete random variable is,

Probability Distribution Function:

$$F(x) = P\ (X \leq x)$$

A continuous Random Variable is a variable that takes infinitely many values. The formula for the probability distribution of a continuous random variable is,
Probability Distribution Function: **F(x) = P (X ≤ x)**

Additionally, the **Probability Density Function (PDF)** is the derivative of the CDF:

f(x) = d/dx (F(x))

where,

- ***F(x) =*** $\int_{-\infty}^{x}$ ***f(u)du***

Normal Probability Distribution Formula

The **Normal Distribution**, also known as the **Gaussian distribution**, is commonly represented by a bell-shaped curve. The formula for a normal probability distribution is:

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where,

- ***μ*** *is the* ***Mean***
- ***σ*** *is the* ***Standard Distribution***
- ***x*** *is the* ***Normal random variable***

Note: If mean(μ) = 0 and standard deviation(σ) = 1, then this distribution is called the Normal Distribution.

Binomial Probability Distribution Formula

The **Binomial Distribution** describes the probability of exactly r successes in n independent trials, where each trial has a probability p of success and 1 − p of failure. The formula for the binomial probability distribution is:

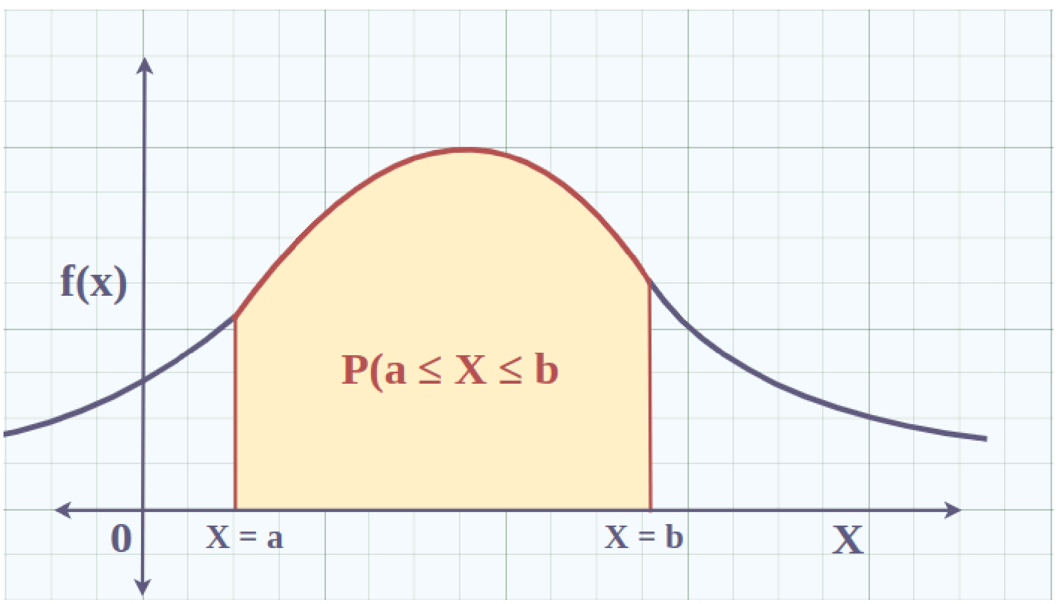
- *n is the Total number of events*
- *r is the Total number of successful events*
- *p is the Probability of success on a single trial*
- *1 – p is the Probability of failure*

Note: The binomial coefficient($^n{}_r$) is given by: ^nC_r **= n!/r!(n – r)!**

Probability Distribution Graph

The graph that plot the Probability Distribution Functions are called the Probability Distribution graphs. These graphs help us to visualize the probability distribution around a random variable and help us to easily find the required solution.

The sum of all the probabilities in any discrete distribution is one and for a continuous distribution of random variables the area under the graph is equal to 1. The distribution graph of the continuous distribution function is added below, where X (the random variable) lies between **a** and **b.** It is made using the Probability Density Function



For discrete random variables, the probability distribution is given using the Bernoulli distribution.

Probability Distribution Function and Probability

We easily describe the Probability distribution using the Probability Distribution Function and Probability Density Function. Using a probability distribution function is very useful for both continuous probability distribution and discrete probability distribution, while the probability density function(pdf) is only used for continuous probability distribution.

Articles related to Probability Distribution Function:

- *Poisson Distribution*
- *Bernoulli Trials*
- *Binomial Distribution*

Uses of Probability Distribution Function

- **Statistical Inference**: PDFs are fundamental in statistical inference, allowing for the estimation of population parameters and hypothesis testing.
- **Modeling and Simulations**: PDFs are used to model real-world phenomena and to simulate random processes in fields like engineering, finance, and the natural sciences.
- **Risk Assessment**: In finance and insurance, PDFs help assess risks and determine the likelihood of various financial outcomes.

Probability Distribution Function Examples

Example 1: Suppose we toss two dice. Make a table of the probabilities for the sum of the dice. The possibilities are: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Solution:

Probability Distribution Table

3	*2/36*
4	*3/36*
5	*4/36*
6	*5/36*
7	*6/36*
8	*5/36*
9	*4/36*
10	*3/36*
11	*2/36*
12	*1/36*

Example 2: The number of old people living in houses on a randomly selected city block is described by the following probability distribution.

Number of adults	**Probability**
(x)	**P(x)**
3	0.50
4	0.25
5	0.10

What is the probability that 6 or more old people live in a randomly selected house?

Solution:

> *Sum of all the p(probability) is equal to 1*
>
> *Probability that six or more old peoples live in a house,*
>
> *= 1 - (0.50 + 0.25 + 0.10)*
> *= 0.15*
>
> *Thus, probability that six or more old peoples live in a house is equal to 0.15*

Example 3: When a fair coin is tossed 8 times, then the **Probability of:**

- **Exactly Four Heads**
- **At least Four Heads**

Solution:

> *Every coin tossed can be considered as the Bernoulli trial. Suppose X be the number of heads in this experiment,*
>
> $n = 8$
> $p = 1/2$
>
> *So,*
> $P(X = x) = {}^nC_x\, p^{n-x}\, (1 - p)^x$, $x = 0, 1, 2, 3,\ldots n$
> $P(X = x) = {}^8C_x p^{8-x}(1 - p)^x$
>
> ***P(Exactly 4 Heads)***
> $= P(x = 4)$
> $= {}^8C_4\, p^4\, (1 - p)^4$
> $= 8!/4!4!(1/2)^4(1/2)^4$
> $= (8 \times 7 \times 6 \times 5/2 \times 3 \times 4) \times (1/16) \times (1/16)$
> $= 420/1536$
> $= 35/128$
>
> *Thus, the probability of Exactly Four Heads in a Eight Coin Toss experiment is 35/128*

= $^8C_4\, p^4\, (1 - p)^4 + {}^8C_5\, p^3\, (1 - p)^5 + {}^8C_6\, p^2\, (1 - p)^6 + {}^8C_7\, p^1 (1 - p)^7 + {}^8C_8 (1 - p)^8$
= $8!/4!4!(1/2)^8 + 8!/5!3!(1/2)^8 + 8!/6!2!(1/2)^8 + 8!/7!1!(1/2)^8 + 8!/8!(1/2)^8$
= 8 × 7 × 6 × 5/4 × 3 × 2 × 256 + 8 × 7 × 6/3 × 2 × 256 + 8/256 + 1/256
= 1680/6144 + 336/1536 + 9/256
= 70/256 + 56/256 + 9/256
= 135/256

Thus, the probability of Atleast Four Heads in a Eight Coin Toss experiment is 135/256

Example 4: Calculate the probability of getting 10 heads, if a coin is tossed 12 times.

Solution:

Given,

- *Number of Trials(n) = 12*
- *Number of Success(r) = 10 (getting 10 heads)*
- *Probability of Single Head(p) = 1/2 = 0.5*

To find ^nC_r = n!/r!(n – r)!

= 12!/10!(12 – 10)!
= (12 × 11 × 10!)/10!2!
= 66

To find $p^r = (0.5)^{10} = 0.00097665625$
So, the probability of getting 10 heads is:

$P(x) = {}^nC_r\, p^r\, (1 - p)^{n - r}$
$= 66 × 0.00097665625 × (1 – 0.5)^{(12-10)}$
$= 0.0644593125 × (0.5)^2$
= 0.016114828125

The probability of getting 10 heads = 0.0161...

Example 5: Suppose that each time you take a free throw shot, you have

Solution:

Given,

- $n = 25$
- $r = 15$
- $p = 0.35$
- $q = 0.65$

Compute
$C_{25,15} (0.35)^{15} (0.65)^{10} = 0.165$

There is a 16.5% chance of making exactly 15 shots.

Example 6: There is a total of 5 people in the room, what is the possibility that someone in the room shares His / Her birthday with at least someone else?

Solution:

P(s) = p(At least someone shares with someone else)

P(d) = p(No one share their birthday everyone has a different birthday)

$p(s) + p(d) = 1$ or 100%
$p(s) = 100\% - p(d)$

There are 5 people in the room, the possibility that no one shares his/her birthday

$= (365 \times 364 \times 363 \times 362 \times 361) / (365)^5$
$= (365!/(365 - 5)!)/365^5$
$= (365!/360!)/365^5$
$= 0.9728$

$p(d) = 0{,}9728$ or 97.28%

$p(s) = 100\% - p(d)$

$= 100\% - 97.28\%$ or $1 - 0.9728$
$= 2.72\% \approx 0.0272$

Practice Questions on the Probability Distribution Function

Q1: Find the Probability Distribution of the the **Number of Heads when two coins are tossed Simultaneously.**

Q2: What is the Probability Distribution of the of **number of Kings when three cards are drawn at random.**

Q3: A die is thrown twice. Find the probability of getting a number of sixes.

Q4: A coin is thrown until a tail appears or the the **head appears three times continuously. Find the probability distribution of tosses**.

Similar Reads

1. Plot Probability Distribution Function in R
2. Probability Density Function
3. Probability Distribution - Function, Formula, Table
4. Discrete Probability Distribution
5. Probability Mass Function
6. Discrete Probability Distributions for Machine Learning
7. Plot Cumulative Distribution Function in R
8. Cumulative Distribution Function
9. Poisson Distribution Practice Problems
10. Applications of Probability

Corporate & Communications Address:

A-143, 7th Floor, Sovereign Corporate Tower, Sector- 136, Noida, Uttar Pradesh (201305)

Registered Address:

K 061, Tower K, Gulshan Vivante Apartment, Sector 137, Noida, Gautam Buddh Nagar, Uttar Pradesh, 201305

Deep Learning

Bootstrap
Tailwind CSS

Python Tutorial

Python Examples
Django Tutorial
Python Projects
Python Tkinter
Web Scraping
OpenCV Tutorial
Python Interview Question

Computer Science

GATE CS Notes
Operating Systems
Computer Network
Database Management System
Software Engineering
Digital Logic Design
Engineering Maths

DevOps

Git
AWS
Docker
Kubernetes
Azure
GCP
DevOps Roadmap

System Design

High Level Design
Low Level Design
UML Diagrams
Interview Guide
Design Patterns
OOAD
System Design Bootcamp
Interview Questions

School Subjects

Mathematics
Physics
Chemistry
Biology
Social Science
English Grammar

Databases

SQL
MYSQL
PostgreSQL
PL/SQL
MongoDB

Preparation Corner

Company-Wise Recruitment Process
Aptitude Preparation
Puzzles
Company-Wise Preparation

More Tutorials

Software Development
Software Testing
Product Management
Project Management
Linux
Excel
All Cheat Sheets

Courses

IBM Certification Courses
DSA and Placements

Programming Languages

C Programming with Data Structures
C++ Programming Course