

Random Variable

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Random variable is a fundamental concept in statistics that bridges the gap between theoretical probability and real-world data. A **Random variable** in statistics is a function that assigns a real value to an outcome in the sample space of a random experiment. **For example:** if you roll a die, you can assign a number to each possible outcome.

There are two basic types of random variables:

- Discrete Random Variables (which take on specific values).
- Continuous Random Variables (assume any value within a given range).

We define a random variable as a **function that maps from the sample space of an experiment to the real numbers**. Mathematically, Random Variable is expressed as,

$$X: S \rightarrow R$$

where,

- X is Random Variable (It is usually denoted using capital letter)
- S is Sample Space
- R is Set of [Real Numbers](#)

Table of Content

- [Random Variable Examples](#)
- [Variate](#)

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Got It !

- [Continuous Random Variable](#)
- [Random Variable Formulas](#)
- [Random Variable Functions](#)
- [Probability Distribution and Random Variable](#)

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Random variables are generally represented by capital letters like X and Y. This is explained by the example below:

Random Variable Examples

Example 1

If two unbiased coins are tossed then find the random variable associated with that event.

Solution:

*Suppose Two (unbiased) coins are tossed
X = number of heads. [X is a random variable or function]
Here, the sample space $S = \{HH, HT, TH, TT\}$*

Example 2

Suppose a random variable X takes m different values, $X = \{x_1, x_2, x_3, \dots, x_m\}$, with corresponding probabilities $P(X = x_i) = p_i$, where $1 \leq i \leq m$. The probabilities must satisfy the following conditions :

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- $0 \leq p_i \leq 1$; where $1 \leq i \leq m$
- $p_1 + p_2 + p_3 + \dots + p_m = 1$ or we can say $0 \leq p_i \leq 1$ and $\sum p_i = 1$

***For example,** Suppose a die is thrown (X = outcome of the dice).*

Here, the sample space $S = \{1, 2, 3, 4, 5, 6\}$.

The output of the function will be:

- $P(X = 1) = 1/6$
- $P(X = 2) = 1/6$
- $P(X = 3) = 1/6$
- $P(X = 4) = 1/6$
- $P(X = 5) = 1/6$
- $P(X = 6) = 1/6$

This also satisfies the condition $\sum_{i=1}^6 P(X = i) = 1$, since:

$$P(X = 1) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) = 6 \times 1/6 = 1$$

Variate

A **variate** is a general term often used interchangeably with a **random variable**, particularly in contexts where the random variable is not yet fully specified by a particular probabilistic experiment. A variate is an abstract

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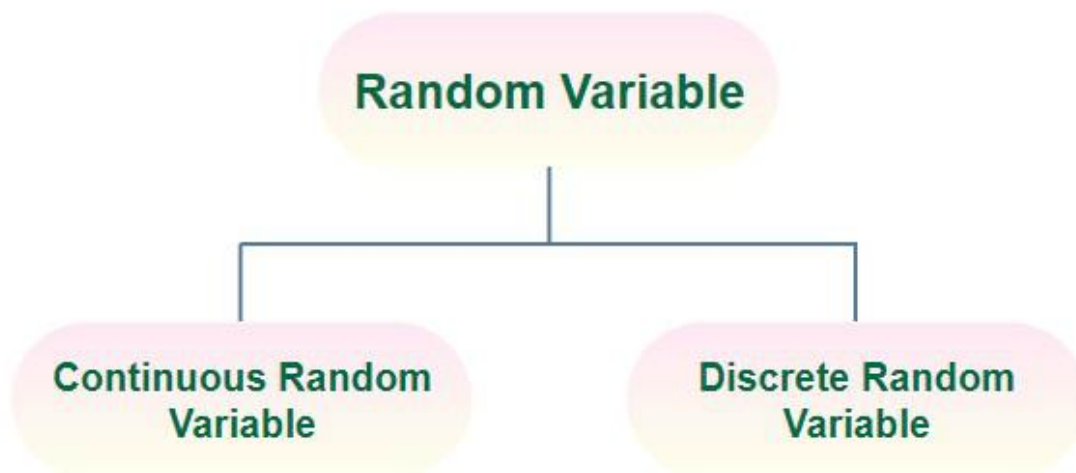
distribution.

It has the same properties as a random variable, such as a defined range of possible values. The range of values that a random variable X can take is denoted as R_X , and individual values within this range are called **quantiles**. The probability of the random variable X taking a specific value x is written as $P(X = x)$.

Types of Random Variables

Random variables are of two types that are,

- Discrete Random Variable
- Continuous Random Variable



Discrete Random Variable

A Discrete Random Variable takes on a finite number of values. The probability function associated with it is said to be **PMF**.

PMF(Probability Mass Function)

If X is a discrete random variable and the PMF of X is $P(x_i)$, then

- $0 \leq p_i \leq 1$

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Discrete Random Variables Example

Example: Let $S = \{0, 1, 2\}$

x_i	0	1	2
$P_i(X = x_i)$	P_1	0.3	0.5

Find the value of $P(X = 0)$

Solution:

We know that the sum of all probabilities is equal to 1. And $P(X = 0)$ be P_1

$$P_1 + 0.3 + 0.5 = 1$$

$$P_1 = 0.2$$

Then, $P(X = 0)$ is 0.2

Continuous Random Variable

Continuous Random Variable takes on an infinite number of values. The probability function associated with it is said to be [PDF \(Probability Density Function\)](#).

PDF (Probability Density Function)

If X is a continuous random variable. $P(x < X < x + dx) = f(x)dx$ then,

- $0 \leq f(x) \leq 1$; for all x
- $\int f(x) dx = 1$ over all values of x

Then $P(X)$ is said to be a PDF of the distribution.

Find the value of $P(1 < X < 2)$

Such that,

- $f(x) = kx^3; 0 \leq x \leq 3 = 0$

Otherwise $f(x)$ is a density function.

Solution:

If a function f is said to be a density function, then the sum of all probabilities is equal to 1.

Since it is a continuous random variable Integral value is 1 overall sample space s .

$$\int f(x) dx = 1$$

$$\int kx^3 dx = 1$$

$$K[x^4]/4 = 1$$

Given interval, $0 \leq x \leq 3 = 0$

$$K[3^4 - 0^4]/4 = 1$$

$$K(81/4) = 1$$

$$K = 4/81$$

Thus,

$$P(1 < X < 2) = k \times [X^4]/4$$

$$P = 4/81 \times [16-1]/4$$

$$P = 15/81$$

Random Variable Formulas

There are two main random variable formulas,

- Mean of Random Variable
- Variance of Random Variable

Let's learn about the same in detail,

Mean of Random Variable

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$$\text{Mean}(\mu) = \sum X.P$$

where,

- X is the random variable that consist of all possible values.
- P is the probability of respective variables

Variance of Random Variable

The variance of a random variable tells us how the random variable is spread about the mean value of the random variable. The variance of the Random Variable is calculated using the formula,

$$\text{Var}(x) = \sigma^2 = E(X^2) - \{E(X)\}^2$$

where,

- $E(X^2) = \sum X^2 P$
- $E(X) = \sum X P$

Random Variable Functions

For any random variable X if it assume the values x_1, x_2, \dots, x_n where the probability corresponding to each random variable is $P(x_1), P(x_2), \dots, P(x_n)$, then the expected value of the variable is,

Expectation of X , $E(x) = \sum x.P(x)$

Now for any new random variable Y in which the random variable X is its input, i.e. $Y = f(X)$, then the cumulative distribution function of Y is,

$$F_Y(Y) = P(g(X) \leq y)$$

Probability Distribution and Random Variable

For a random variable, its probability distribution is calculated using three

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- Experimental listing of outcomes followed with their observed relative frequencies.
- Subjective listing of outcomes followed with their subjective probabilities.

The probability of a random variable X that takes values x is defined using a probability function of X that is denoted by $f(x) = P(X = x)$.

Various probability distributions are,

- [Binomial Distribution](#)
- [Poisson Distribution](#)
- [Bernoulli's Distribution](#)
- [Exponential Distribution](#)
- [Normal Distribution](#)

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Solved Questions on Random Variable

Here are some of the solved examples on Random variable. Learn random variables by practicing these solved examples.

Question 1: Find the mean value for the continuous random variable, $f(x) = x^2, 1 \leq x \leq 3$

Solution:

Given,

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$$E(x) = \int_1^3 x^3 \cdot dx$$

$$E(x) = [x^4/4]_1^3$$

$$E(x) = 1/4 \times \{3^4 - 1^4\} = 1/4 \times \{81 - 1\}$$

$$E(x) = 1/4 \times \{80\} = 20$$

Question 2: Find the mean value for the continuous random variable,

$$f(x) = e^x, 1 \leq x \leq 3$$

Solution:

Given,

$$f(x) = e^x$$

$$1 \leq x \leq 3$$

$$E(x) = \int_1^3 x \cdot f(x) dx$$

$$E(x) = \int_1^3 x \cdot e^x \cdot dx$$

$$E(x) = [x \cdot e^x - e^x]_1^3$$

$$E(x) = [e^x(x - 1)]_1^3$$

$$E(x) = e^3(2) - e(0)$$

Question 3: Given the discrete random variable X with the following probability distribution:

X	1	2	3	4
P(X)	0.1	0.2	0.4	0.3

Find the mean value (or expected value) of the random variable X.

Solution:

To find the mean value (expected value) of a discrete random variable X, we use the formula:

Using the relation: $E(X) = \mu_X = x_1P(x_1) + x_2P(x_2) + \dots + x_nP(x_n)$

$$E(X) = \sum_i X_i \cdot P(X_i)$$

The expected value $E(X)$, or mean μ_X of a discrete random variable

$$E(X) = 0.100 + 0.400 + 1.200 + 1.200 = 2.900$$

$$E(X) = 2.900$$

Question 4: Given the discrete random variable X with the following probability distribution:

Suppose a discrete random variable X represents the number of defective items in a sample of 10 items from a batch of 100 items. The possible values of X are 0, 3, 5, and 7 defective items, with the following probability distribution:

X	0	3	5	7
P(X)	0.2	0.5	0.2	0.1

Find the mean value (or expected value) of the random variable X.

Solution:

*The formula for the **mean (or expected value)** of a discrete random variable X is:*

$$E(X) = \sum_i X_i \cdot P(X_i)$$

The expected value $E(X)$, or mean μ_X of a discrete random variable X

$$E(X) = \mu_X = \sum [x_i * P(x_i)]$$

$$E(X) = 0 * 0.2 + 3 * 0.5 + 5 * 0.2 + 7 * 0.1$$

$$E(X) = 0.000 + 1.500 + 1.000 + 0.700 = 3.200$$

$$E(X) = 3.200$$

Practice Problems on Random Variables

Question 1: Find the mean value for the continuous random variable, $f(x) = x^3$, $1 \leq x \leq 5$

Question 2: Find the mean value for the continuous random variable, $f(x) =$

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Question 3: Given the discrete random variable X with the following probability distribution:

X	1	2	3	4
$P(X)$	0.2	0.3	0.4	0.1

Find the mean value (or expected value) of the random variable X .

Question 4: Given the discrete random variable X with the following probability distribution:

X	0	1	3	5
$P(X)$	0.3	0.3	0.3	0.1

Find the mean value (or expected value) of the random variable X .

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9. JavaScript Random

10. Python Random - random() Function



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