Probability Theory

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Probability theory is an advanced branch of mathematics that deals with **measuring the likelihood** of events occurring. It provides tools to analyze situations involving **uncertainty** and helps in determining how likely certain outcomes are. This theory uses the concepts of random variables, sample space, probability distributions, and more to determine the outcome of any situation.

> ***For Example: Flipping a Coin***
>
> *Flipping a coin is a random event with two possible outcomes: heads or tails. Each time you flip a fair coin, there are exactly two possible outcomes, each with an equal chance of occurring. Therefore, the probability of landing on heads is 1/2, and similarly, the probability of landing on tails is also 1/2.*

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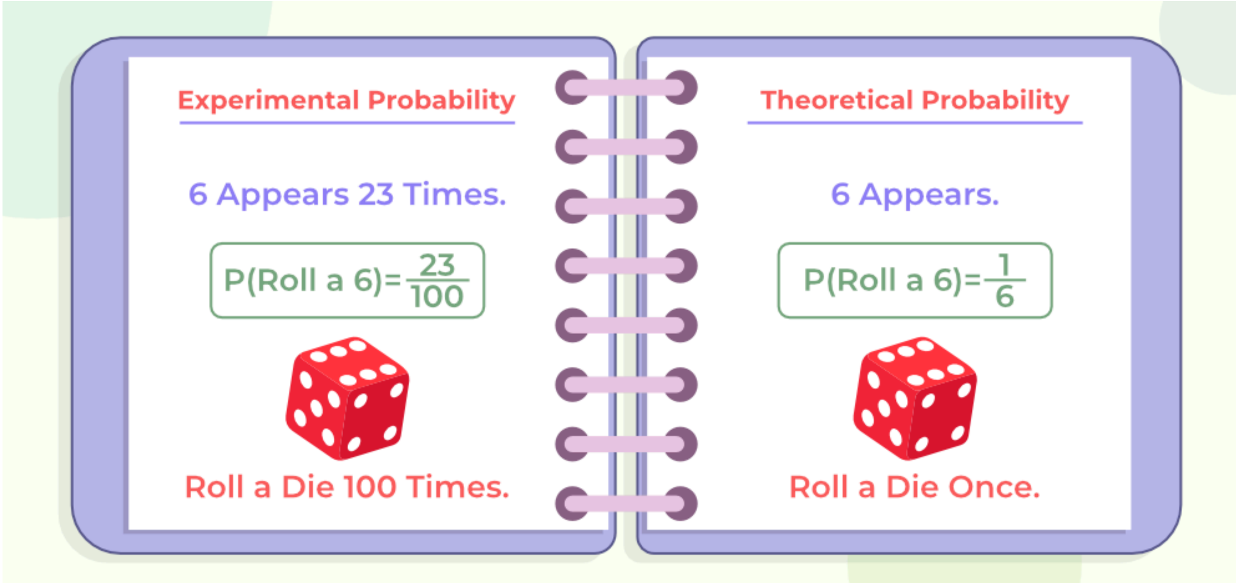
Different Approaches In Probability Theory

Probability theory studies random events and tells us about their occurrence. The four main approaches for studying probability theory are:

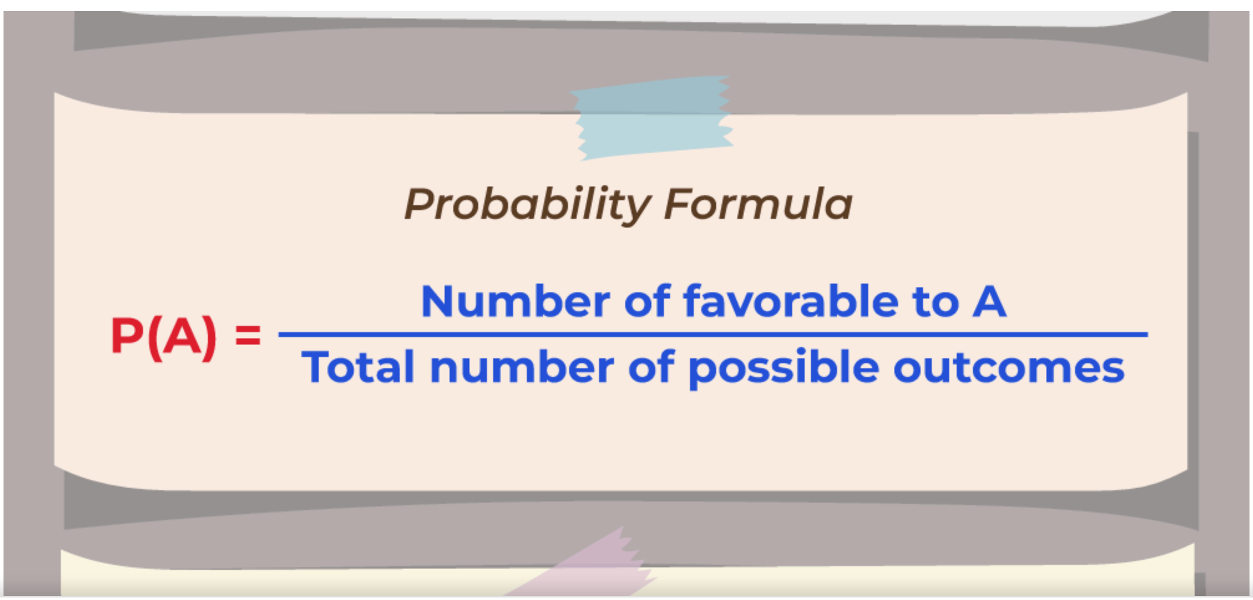


Theoretical Probability

Theoretical probability deals with assumptions to avoid unfeasible or costly repetition of experiments. The theoretical Probability for an event **A** can be calculated as follows:

> ***P(A) = (Number of outcomes favorable to Event A) / (Number of all possible outcomes)***

The image shown below shows the theoretical probability formula.



Now, as we learn the formula, let's put this formula in our coin-tossing case. In tossing a coin, there are two outcomes: Head or Tail. Hence, The Probability of the occurrence of a Head on tossing a coin is P(H) = 1/2

Similarly, The Probability of the occurrence of a Tail on tossing a coin is P(T) = 1/2

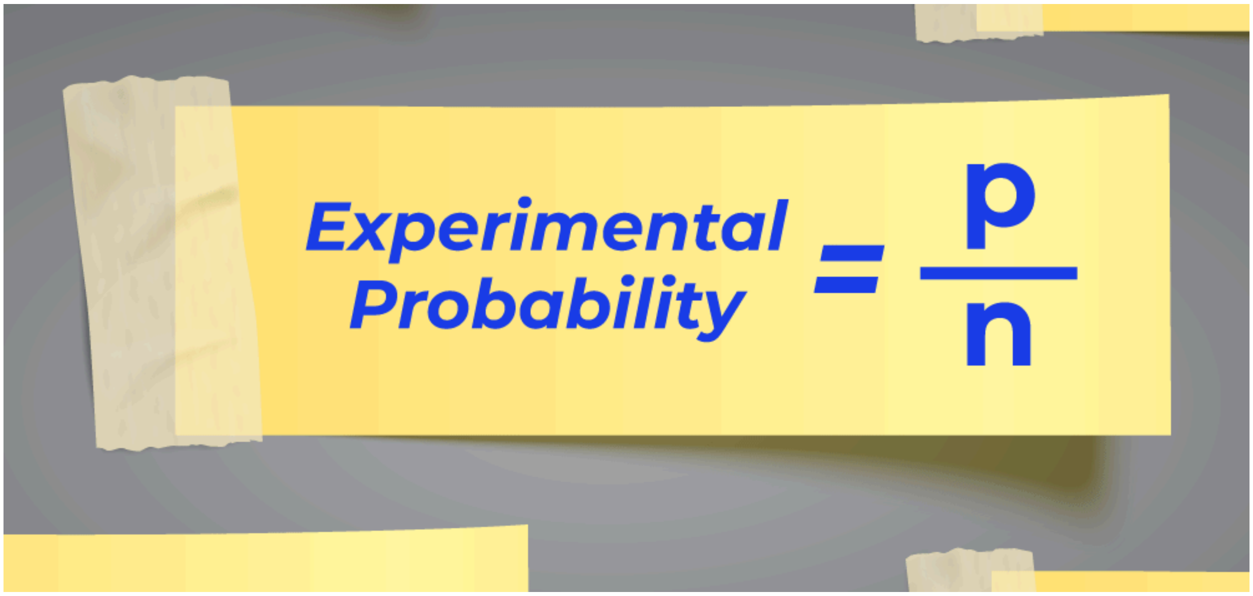
➢ **Read more about-** Theoretical probability

Experimental Probability

Experimental probability is found by performing a series of experiments and observing their outcomes. These random experiments are also known as trials. The experimental probability for Event A can be calculated as follows:

P(E) = (Number of times event A happened) / (Total number of trials)

The following image shows the Experimental Probability Formula,



Now, as we learn the formula, let's put this formula in our coin-tossing case. If we tossed a coin 10 times and recorded heads 4 times and tails 6 times, then the Probability of occurrence of heads on tossing a coin: P(H) = 4/10

Similarly, the Probability of Occurrence of Tails on tossing a coin: P(T) = 6/10

➢ **Read more about-** Experimental Probability

Subjective Probability

Subjective probability refers to the likelihood of an event occurring, as estimated by an individual based on their personal beliefs, experiences, intuition, or knowledge, rather than on objective statistical data or formal mathematical models.

> ***Example***: *A cricket enthusiast might assign a 70% probability to a team's victory based on their understanding of the team's recent form, the opponent's strengths and weaknesses, and other relevant factors.*

Axiomatic Probability

defined on a σ-algebra of events, satisfying a set of axioms (rules). This approach is highly abstract and provides a rigorous framework for probability theory.

> ***Example***: *For a fair coin, there are two possible outcomes: Heads (H) or Tails (T). So, the sample space S is:*
> *S = {Heads, Tails}*
> *P(Heads) + P(Tails) = 1*
> *So, using the axiomatic framework, we've established that:*
> *P(Heads) = 0.5, P(Tails) = 0.5*

➢ **Read more about-** Axiomatic Probability

Basics of Probability Theory

Some important concepts of probability theory are:

Random Experiment

In probability theory, any event that can be repeated multiple times and whose outcome is not hampered by its repetition is called a Random Experiment.
For example, tossing a coin, rolling the dice, etc., are random experiments.

Sample Space

The set of all possible outcomes for any random experiment is called the sample space.
For example, throwing dice results in six outcomes, which are 1, 2, 3, 4, 5, and 6. Thus, its sample space is (1, 2, 3, 4, 5, 6)

Event

The outcome of any experiment is called an event. Various types of events

events.
For example, the output of tossing a coin in repetition is not affected by its previous outcome.

- **Dependent Events:** The events whose outcomes are affected by the outcome of other events are called dependent events.
  For example, picking oranges from a bag that contains 100 oranges without replacement.

- **Mutually Exclusive Events:** The events that can not occur simultaneously are called mutually exclusive events.
  For example, obtaining a head or a tail in tossing a coin, because both (head and a tail can not be obtained together.

- **Equally likely Events:** The events that have an equal chance or probability of happening are known as equally likely events.
  For example, observing any face in rolling the dice has an equal probability of 1/6.

Random Variable

A variable that can assume the value of all possible outcomes of an experiment is called a random variable in Probability Theory. Random variables in probability theory are of two types, which are discussed below,

Discrete Random Variable

Variables that can take countable values, such as 0, 1, 2, ..., are called discrete random variables.
Examples: The number of heads when flipping 3 coins, the number of cars in a parking lot, or the number of correct answers on a test.

Variables that can take an infinite number of values in a given range are called continuous random variables.

Examples: The height of a person, the time it takes for a chemical reaction to occur, or the temperature of a substance.

Probability Theory Formulas

Various formulas are used in probability theory, and some of them are discussed below,

- **Theoretical Probability Formula:** (Number of Favourable Outcomes) / (Number of Total Outcomes)
- **Empirical Probability Formula:** (Number of times event A happened) / (Total number of trials)
- **Addition Rule of Probability:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Complementary Rule of Probability:** $P(A') = 1 - P(A)$
- **Independent Events:** $P(A \cap B) = P(A) \cdot P(B)$
- **Conditional Probability:** $P(A \mid B) = P(A \cap B) / P(B)$
- **Bayes' Theorem:** $P(A \mid B) = P(B \mid A) \cdot P(A) / P(B)$

Read in Detail: Bayes' Theorem

Probability Theory in Statistics

Probability has various applications in Statistics. These are mentioned below:

- **Descriptive Statistics**: Probability theory helps in understanding and interpreting data summaries and distributions.
- **Inferential Statistics**: This forms the basis for making inferences about populations from samples, including hypothesis testing and the construction of confidence intervals.
- **Regression Analysis**: Probability distributions of errors are used to estimate the relationships between variables.
- **Bayesian Statistics**: Uses probability to represent uncertainty about the

Applications of Probability Theory

Probability theory is widely used in our lives, It is used to find answers to various types of questions, such as Will it rain tomorrow? What is the chance of landing on the Moon? What is the chance of the evolution of humans? And others. Some of the important uses of probability theory are,

- Probability theory is used to predict the performance of stocks and bonds.
- In casinos and gambling, probability theory is used to find the chances of winning.
- Probability theory is used in weather forecasting.
- Probability theory is used in Risk mitigation.
- In consumer industries, the risk of product failure is mitigated by using Probability theory.

Solved Questions on Probability Theory

We can study the concept of probability with the help of the example discussed below.

Question 1: Let's take two random dice and roll them randomly. Now now the probability of getting a total of 10 is calculated.

Solution:

> *Total Possible events that can occur (sample space) {(1,1), (1,2),..., (1,6),..., (6,6)}. The total spaces are 36.*
> *Now the required events, {(4,6), (5,5), (6,4)} are all which adds up to 10.*
>
> *So the probability of getting a total of 10 is = 3/36 = 1/12*

Question 2: A fair coin is tossed three times. What is the probability of getting exactly two heads?

Solution:

Outcomes with exactly two heads: HHT, HTH, THH (3 outcomes).

Probability of getting exactly two heads:
P(exactly 2 heads)=Number of favorable outcomes/ Total outcomes.

P(exactly 2 heads)=3/ 8.

Question 3: A standard deck of cards contains 52 cards. What is the probability of drawing an Ace or a King from the deck?

Solution:

Total number of cards = 52.
Number of Aces = 4.
Number of Kings = 4.

Total number of favorable outcomes (Aces or Kings) = 4 + 4 = 8.

Probability of drawing an Ace or a King:

P(Ace or King)=Number of favorable outcomes/Total outcomes
P(Ace or King)=Number of Aces or Kings/Total number of cards.

P(Ace or King) = 8/52 = 2/13.

Question 4: Consider a jar with 7 red marbles, 3 green marbles, and 4 blue marbles. What is the probability of randomly selecting a non-blue marble from the jar?

Solution:

Given,
Number of Red Marbles = 7, Number of Green Marbles = 3, Number of Blue Marbles = 4

So, Total number of possible outcomes in this case: 7 + 3 + 4 = 14
Now, Number of non-blue marbles are: 7 + 3 = 10
According to the formula of theoretical Probability we can find,
P(Non-Blue) = 10/14 = 5/7

Hence, theoretical probability of selecting a non-blue marble is 5/7.

Solution:

Let N and M represent the events that Naveena wins the match and Isha wins the match, respectively.

The probability of Naveena's winning P(N) = 0.62 (given)
The probability of Isha's winning P(I) = ?
Winning of the match is an mutually exclusive event, since only one of them can win the match.
Therefore,

P(N) + P(I) =1
P(I) = 1 – P(N)
P(I) = 1 – 0.62 = 0.38

Thus, the Probability of Isha winning the match is 0.38.

Question 6: If someone takes out one card from a 52-card deck, what is the probability of the card being a heart? What is the probability of obtaining a 7-number card?

Solution:

Total number of cards in a deck = 52
Total Number of heart cards in a deck = 13

So, the probability of obtaining a heart,

P(heart) = 13/52 = 1/4
Total number of 7-number cards in a deck = 4

So, the probability of obtaining a 7-number card,
P(7-number) = 4/52 = 1/13

Question 7: Find the probability of rolling an even number when you roll a die containing the numbers 1-6. Express the probability as a fraction, decimal, ratio, or percent.

Solution:

Out of 1 to 6 number, even numbers are 2, 4, and 6.

Probability of obtaining an even number P(Even)= 1/2 = 0.5 = 1 : 2 = 50%

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