Binomial Distribution in Probability

Last Updated : 04 Dec, 2024

Binomial Distribution is a probability distribution used to model the number of successes in a fixed number of independent trials, where each trial has only two possible outcomes: success or failure. This distribution is useful for calculating the probability of a specific number of successes in scenarios like flipping coins, quality control, or survey predictions.

Binomial Distribution is based on **Bernoulli trials**, where each trial has an independent and identical chance of success. The probability distribution for a Bernoulli trial is called the **Bernoulli Distribution**.

> *A **Binomial Distribution** for a random variable X = 0, 1, 2,..., n is defined as the probability of success or failure in a series of independent trials. Each trial is independent of the others, and the distribution helps calculate the probability of various outcomes in these trials.*

Conditions for Binomial Distribution

The Binomial distribution can be used in scenarios where the following conditions are satisfied:

1. **Fixed Number of Trials**: There are a set number of trials or experiments (denoted by n), such as flipping a coin 10 times.
2. **Two Possible Outcomes**: Each trial has only two possible outcomes, often labeled as "success" and "failure." For example, getting heads or tails in a coin flip.
3. **Independent Trials**: The outcome of each trial is independent of the

4. **Constant Probability**: The probability of success (denoted by p) remains the same for each trial. For example, if you're flipping a fair coin, the probability of getting heads is always 0.5.

The Binomial distribution is an appropriate model to use for calculating the probabilities of obtaining a certain number of successes in the given trials.

Read More: Bernoulli trails

Negative Binomial Distribution

The Negative Binomial Distribution is used to model the number of trials needed to achieve a certain number of successes in a sequence of independent trials, where the probability of success in each trial is constant.

For example, consider a situation where getting 6 is the success of throwing a die. Now if we throw the die and not get 6 then it is a failure. Now we throw again and do not get 6. Let's say we don't get 6 for three successive attempts and 6 is obtained in the fourth attempt and onwards then the binomial distribution of the number of getting 6 is called the Negative Binomial Distribution.

Negative Binomial Distribution Formula

The formula for Negative Binomial Distribution is given as

$$P(x) = {}^{n+r-1}C_{r-1}p^r q^n$$

Where,

- **n** = Total Number of Trials.
- **r** = Number of Trials in which we get the first success.
- **p** = Probability of Success in Each Trial.
- **q** = Probability of Failure in Each Trial.

Binomial Distribution Formula

The Binomial Distribution Formula which is used to calculate the probability, for a random variable X = 0, 1, 2, 3,....,n is given as

$$P(X = r) = {}^{n}C_r p^r q^{n-r},\ r = 0, 1, 2, 3....$$

Where,

- **n** = Total number of trials
- **r** = Number of successes
- **p** = Probability of success
- **q** = Probability of failure (q = 1 - p)

Bernoulli Trial is a trial that gives results of dichotomous nature i.e. result in yes or no, head or tail, even or odd. It means it gives two types of outcomes out of which one favors the event while the other doesn't. A random experiment is called Bernoulli Trial if it satisfies the following conditions:

- Trials are finite in number
- Trials are independent of each other
- Each trial has only two possible outcomes
- The probability of success and failure in each trial is the same.

Binomial Random Variable

A Binomial Random Variable can be defined by two possible outcomes such as “success” and **binomial** “failure”. For instance, consider rolling a fair six-sided die and recording the value of the face. The binomial distribution formula can be put into use to calculate the probability of success for binomial distributions. Often it states “plugin” the numbers to the formula and calculates the requisite values.

The binomial distribution is based on the following characteristics:

- Experiment contains n identical trials.
- Each trial results in one of the two outcomes either success or failure.
- The probability of success, denoted p, remains the same from trial to trial.
- All the n trials are independent.

> ***Example: A fair coin is flipped 20 times;***
> *X represents the number of heads*
> *X is a binomial random variable with n = 20 which is the total number of trials and p = 1/2 is the probability of getting head in each trial.*
> *The value of X represents the number of trials in which you succeed in getting head.*

Binomial Distribution in statistics is used to compute the probability of likelihood of an event using the above formula. To calculate the probability using binomial distribution we need to follow the following steps:

- ***Step 1:*** *Find the number of trials and assign it as 'n'*
- ***Step 2:*** *Find the probability of success in each trial and assign it as 'p'*
- ***Step 3:*** *Find the probability of failure and assign it as q where q = 1-p*
- ***Step 4:*** *Find the random variable X = r for which we have to calculate the binomial distribution*
- ***Step 5:*** *Calculate the probability of Binomial Distribution for X = r using the Binomial Distribution Formula.*

The use of the above steps has been illustrated using an example below:

Binomial Distribution Examples

- Finding the probability of getting exactly 6 heads when a fair coin is flipped 10 times.
- Finding the probability of exactly 3 bulbs being defective when a batch of 100 bulbs is tested and each bulb has a 2% chance of being defective.
- To find the Probability of exactly 7 patients responding positively to the treatment when the drug is tested on 8 patients and has a 90% success rate.

Let's say we toss a coin twice, and getting head is a success we have to calculate the probability of success and failure then, in this case, we will calculate the probability distribution as follows:

In each trial getting a head that is a success, its probability is given as:

- p = 1/2

- r = 0 for no success, r = 1 for getting head once and r = 2 for getting head twice

Probability of failure q = 1 - p = 1 - 1/2 = 1/2.
P(Getting 1 head) = P(X = 1) = $^{n}c_{r}p^{r}q^{n-r} = {}^{2}c_{1}\,(1/2)^{1}(1/2)^{1} = 2 \times 1/2 \times 1/2 = 1/2$
P(Getting 2 heads) = P(X = 2) = $^{2}c_{2}(1/2)^{2}(1/2)^{0} = 1/4$
P(Getting 0 heads) = P(X = 0) = $^{2}c_{0}(1/2)^{0}(1/2)^{2} = 1/4$

Random Variable (X = r)	P(X = r)
X = 0 (Getting 0 Head)	1/4
X = 1 (Getting 1 Head)	1/2
X = 2 (Getting 2 Head)	1/4

As of now, we know that Binomial Distribution is calculated for the Random Variables obtained in Bernoulli Trials. Hence, we should understand these terms.

Binomial Distribution Table

The binomial distribution for a situation when getting 6 is a success on throwing two dies is discussed in this section. First of all, we see that it is a Bernoulli Trial as getting 6 is the only success, and getting any different is a failure. Now we can get six on both die in a trial or six on only one of the die in a trial and getting no six on both die. Hence, the random variable for which we have to find the probability takes the value X = r = 0, 1, 2.

The Binomial Distribution Table for getting 6 as success is plotted below:

Random Variable (X = r)	P(X = r)
X = 0 (Getting no 6)	25/36
X = 1 (Getting one 6)	10/36
X = 2 (Getting two 6)	1/36

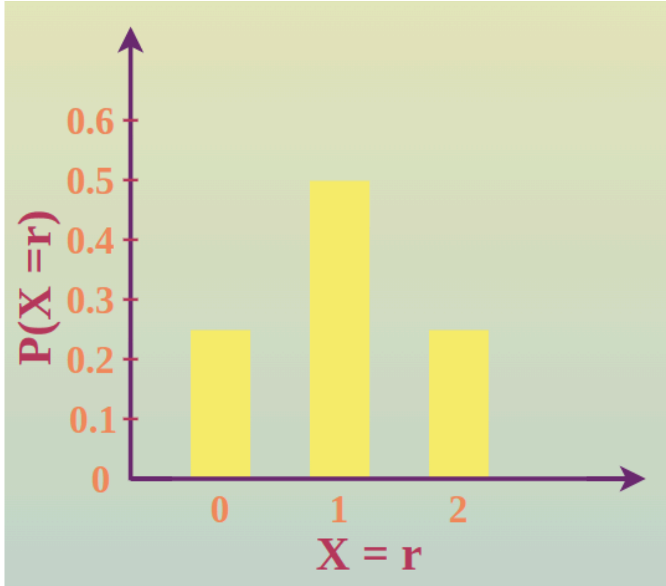
We see that sum of all the probabilities 25/36 + 10/36 + 1/36 = 1.

Binomial Distribution Graph

Binomial Distribution Graph is plotted for X and P(X). We will plot a Binomial Distribution Graph for tossing a coin twice where getting the head is a success. If we toss a coin twice, the possible outcomes are {HH, HT, TH, TT}. The binomial distribution Table for this is given below:

X (Random Variable)	P(X)
X = 0 (Getting no head)	P(X = 0) = 1/4 = 0.25
X = 1 (Getting 1 head)	P(X = 1) = 2/4 = 1/2 = 0.5
X = 2 (Getting two heads)	P(X = 2) = 1/4 = 0.25

Binomial Distribution Graph for the above table is given below:



Binomial Distribution in Statistics

Measures of central tendency, specifically the mean, provide insights into the distribution's central or typical value for the number of successes in a series of independent trials. For a binomial distribution defined by parameters n (number of trials) and p (probability of success on each trial), the measures of central tendency are characterized as follows:

- Binomial Distribution Mean
- Binomial Distribution Variance
- Binomial Distribution Standard Deviation

Measure of Central Tendency for Binomial Distribution

The formulas for Mean, Variance, and Standard Deviation of Binomial Distribution are listed below:

Binomial Distribution Mean

The Mean of Binomial Distribution is the measurement of average success that would be obtained in the 'n' number of trials. The Mean of Binomial Distribution is also called Binomial Distribution Expectation. The formula for Binomial Distribution Expectation is given as where,

- **μ** is the Mean or Expectation
- **n** is the Total Number of Trials
- **p** is the Probability of Success in Each Trial

Read more about, Expected Value or Expectation

Example: If we toss a coin 20 times and getting head is the success then what is the mean of success?

Solution:

> *Total Number of Trials n = 20*
> *Probability of getting head in each trial, p = 1/2 = 0.5*
> *Mean = n.p = 20 × 0.5*
>
> *It means on average we would head 10 times on tossing a coin 20 times.*

Binomial Distribution Variance

Variance of Binomial Distribution tells about the dispersion or spread of the distribution. It is given by the product of the number of trials, probability of success, and probability of failure. The formula for Variance is given as follows:

$$\sigma^2 = n.p.q$$

where

- σ^2 is Variance
- **n** is the Total Number of Trials
- **p** is the Probability of Success in Each Trial
- **q** is the Probability of Failure in Each Trial

Solution:

We have, $n = 20$

Probability of Success in each trial (p) = 0.5
Probability of Failure in each trial (q) = 0.5
Variance of the Binomial Distribution, $\sigma = n.p.q = (20 \times 0.5 \times 0.5) = 5$

Binomial Distribution Standard Deviation

Standard Deviation of Binomial Distribution tells about the deviation of the data from the mean. Mathematically, Standard Deviation is the square root of the variance. The formula for the Standard Deviation of Binomial Distribution is given as

$$\sigma = \sqrt{n.p.q}$$

where,

- **σ** is the Standard Deviation
- **n** is the Total Number of Trials
- **p** is the Probability of Success in Each Trial
- **q** is the Probability of Failure in Each Trial

Example: If we toss a coin 20 times and getting head is the success then what is the standard deviation?

Solution:

We have, $n = 20$

Probability of Success in each trial (p) = 0.5
Probability of Failure in each trial (q) = 0.5

Standard Deviation of the Binomial Distribution, $\sigma = \sqrt{n.p.q}$
$\Rightarrow \sigma = \sqrt{(20 \times 0.5 \times 0.5)}$
$\Rightarrow \sigma = \sqrt{5} = 2.23$

- There are only two possible outcomes: success or failure, yes or no, true or false.
- There is a finite number of trials given as 'n'.
- The probability of success and failure in each trial is the same.
- Only Success is calculated out of all trials.
- Each trial is independent of any other trial.

Binomial Distribution Applications

Binomial Distribution is used where we have only two possible outcomes. Let's see some of the areas where Binomial Distribution can be used.

- To find the number of male and female students in an institute.
- To find the likeability of something in Yes or No.
- To find defective or good products manufactured in a factor.
- To find positive and negative reviews on a product.
- Votes are collected in the form of 0 or 1.

Binomial Distribution vs Normal Distribution

Binomial Distribution differs from the Normal Distribution in many aspects. The key differences and characteristics of the Binomial and Normal distributions are highlighted in the following table:

Aspect	Binomial Distribution	Normal Distribution
Type	Discrete probability distribution	Continuous probability distribution
Outcomes	Two possible outcomes per trial (success or failure)	Infinite possible outcomes within a continuous range
Parameters	n (number of trials), p (probability of success)	μ (mean), σ (standard deviation)

Aspect	Binomial Distribution	Normal Distribution
Shape	Varies depending on n and p; typically skewed unless p=0.5 and n is large	Bell-shaped curve (symmetric)
Support	x can take integer values from 0 to n	x can take any real number (from $-\infty$ to $+\infty$)
Mean	μ = np	μ
Variance	σ^2 = np(1 -p)	σ^2
Applicability	Used for modeling the number of successes in a fixed number of independent trials	Used for modeling continuous data that cluster around a mean
Examples	Flipping coins, quality control (defective items)	Heights of people, test scores, measurement errors
Approximation	Approximates Normal distribution for large n and p not too close to 0 or 1	Considered the limit of the Binomial Distribution as n becomes large and p is near 0.5

People Also Read:

- *Probability Theory*
- *Probability Distribution Function*
- *Baye's Theorem*

Binomial Distribution in Probability Examples

Example 1: A die is thrown 6 times and if getting an even number is a success what is the probability of getting

(i) 4 Successes

(ii) No success

Solution:

Given: $n = 6$, $p = 3/6 = 1/2$, and $q = 1 - 1/2 = 1/2$

$P(X = r) = {}^nC_r p^r q^{n-r}$

(i) $P(X = 4) = {}^6C_4(1/2)^4(1/2)^2 = 15/64$
(ii) $P(X = 0) = {}^6C_0(1/2)^0(1/2)^6 = 1/64$

Example 2: A coin is tossed 4 times what is the probability of getting at least 2 heads?

Solution:

Given: $n = 4$
Probability of getting head in each trial, $p = 1/2 \Rightarrow q = 1 - 1/2 = 1/2$

$P(X = r) = {}^4C_r(1/2)^r(1/2)^{4-r}$
$\Rightarrow P(X = r) = {}^4C_r(1/2)^4$ *{Using the laws of Exaponents}*

And we know, Probability of getting at least 2 heads = $P(X \geq 2)$

$\Rightarrow$ Probability of getting at least 2 heads = $P(X = 2) + P(X = 3) + P(X = 4)$
$\Rightarrow$ Probability of getting at least 2 heads = ${}^4C_2(1/2)^4 + {}^4C_3(1/2)^4 + {}^4C_4(1/2)^4$
$\Rightarrow$ Probability of getting at least 2 heads = $({}^4C_2 + {}^4C_3 + {}^4C_4)(1/2)^4$
$\Rightarrow$ Probability of getting at least 2 heads = $11(1/2)^4 = 11/16$

Example 3: A pair of dice is thrown 6 times and getting sum 5 is a success then what is the probability of getting (i) no success (ii) two success (iii) at most two success

Solution:

Probability of getting the sum 5 in each trial, p = 4/36 = 1/9
Probability of not getting sum 5 = 1 - 1/9 = 8/9

(i) Probability of getting no success, $P(X = 0) = {}^6C_0(1/9)^0(8/9)^6 = (8/9)^6$
(ii) Probability of getting two success, $P(X = 2) = {}^6C_2(1/9)^2(8/9)^4 = 15(8^4/9^6)$
(iii) Probability of getting at most two successes, $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$\Rightarrow P(X \leq 2) = (8/9)^6 + 6(8^5/9^6) + 15(8^4/9^6)$

Practice Problems on Binomial Distribution in Probability

1. A box has 5 red, 7 black,? and 8 white balls. If three balls are drawn one by one with replacement what is the probability that all,

i) all are white
ii) all are red
iii) all are black

2. What is the probability distribution of the number of tails when three coins are tossed together?

3. A die is thrown three times what is the probability distribution of getting six?

4. A coin is tossed 4 times then what is the probability distribution of getting head.

Next Article

Binomial Mean and Standard Deviation - Probability | Class 12 Maths

2. What is Binomial Probability Distribution with Example?
3. Probability Distribution Function
4. Binomial Distribution Practice Problems
5. Python - Binomial Distribution
6. Discrete Probability Distribution
7. Variance of Binomial Distribution
8. Mathematics | Probability Distributions Set 4 (Binomial Distribution)
9. Bernoulli Distribution in R
10. How to use Binomial Distribution in Excel?