Random Variable

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Random variable is a fundamental concept in statistics that bridges the gap between theoretical probability and real-world data. A R**andom variable** in statistics is a function that assigns a real value to an outcome in the sample space of a random experiment**. For example**: if you roll a die, you can assign a number to each possible outcome.

There are two basic types of random variables:

- Discrete Random Variables (which take on specific values).
- Continuous Random Variables (assume any value within a given range).

We define a random variable as a **function that maps from the sample space of an experiment to the real numbers**. Mathematically, Random Variable is expressed as,

$$X: S \rightarrow R$$

where,

- ***X*** *is Random Variable (It is usually denoted using capital letter)*
- ***S*** *is Sample Space*
- ***R*** *is Set of Real Numbers*

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Read: Probability in Maths

Random variables are generally represented by capital letters like X and Y. This is explained by the example below:

Random Variable Examples

Example 1

If two unbiased coins are tossed then find the random variable associated with that event.

Solution:

> *Suppose Two (unbiased) coins are tossed*
> *X = number of heads. [X is a random variable or function]*
>
> *Here, the sample space S = {HH, HT, TH, TT}*

Example 2

Suppose a random variable X takes m different values, X = {x_1, x_2, x_3………x_m}, with corresponding probabilities $\mathbf{P(X = x_i) = p_i}$, where $1 \le i \le m$. The probabilities must satisfy the following conditions :

- $0 \leq p_i \leq 1$; where $1 \leq i \leq m$
- $p_1 + p_2 + p_3 + \ldots\ldots + p_m = 1$ or we can say $0 \leq p_i \leq 1$ and $\sum p_i = 1$

For example, *Suppose a die is thrown (X = outcome of the dice).*

Here, the sample space S = {1, 2, 3, 4, 5, 6}.
The output of the function will be:

- *P(X = 1) = 1/6*
- *P(X = 2) = 1/6*
- *P(X = 3) = 1/6*
- *P(X = 4) = 1/6*
- *P(X = 5) = 1/6*
- *P(X = 6) = 1/6*

This also satisfies the condition $\sum^{6}_{i=1} P(X = i) = 1$*, since:*
P(X = 1) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) = 6 × 1/6 = 1

Variate

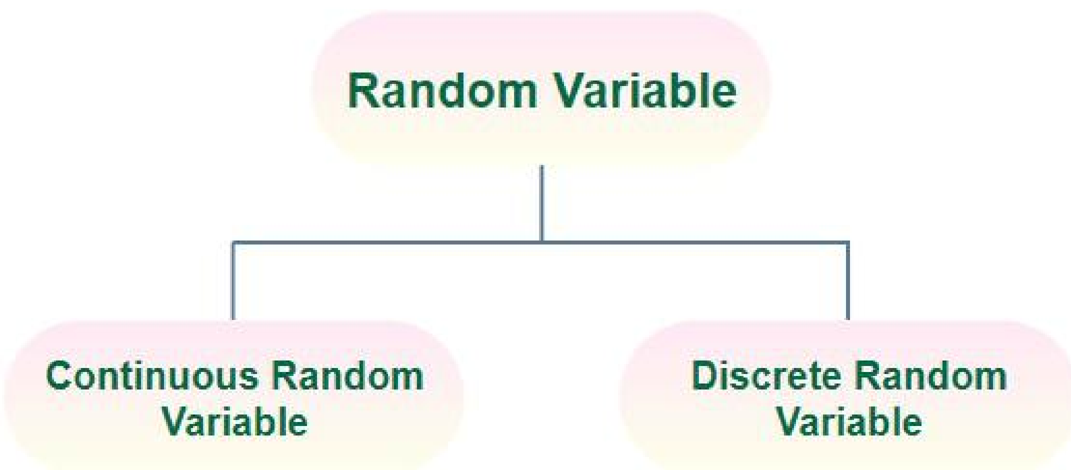
A **variate** is a general term often used interchangeably with a **random variable**, particularly in contexts where the random variable is not yet fully specified by a particular probabilistic experiment. A variate is an abstract distribution.

It has the same properties as a random variable, such as a defined range of possible values. The range of values that a random variable X can take is denoted as R_X, and individual values within this range are called **quantiles**. The probability of the random variable X taking a specific value x is written as P(X = x).

Types of Random Variables

Random variables are of two types that are,

- Discrete Random Variable
- Continuous Random Variable



Discrete Random Variable

A **Discrete Random Variable** takes on a finite number of values. The probability function associated with it is said to be **PMF**.

PMF(Probability Mass Function)

If X is a discrete random variable and the PMF of X is P(xi), then

- $0 \le p_i \le 1$

Discrete Random Variables Example

Example: Let S = {0, 1, 2}

x_i	0	1	2
Pi(X = xi)	P_1	0.3	0.5

Find the value of P (X = 0)

Solution:

> *We know that the sum of all probabilities is equal to 1. And P (X = 0) be P_1*
> *$P_1 + 0.3 + 0.5 = 1$*
> *$P_1 = 0.2$*
>
> *Then, P (X = 0) is 0.2*

Continuous Random Variable

Continuous Random Variable takes on an infinite number of values. The probability function associated with it is said to be PDF (Probability Density Function).

PDF (Probability Density Function)

If X is a continuous random variable. P (x < X < x + dx) = f(x)dx then,

- $0 \le f(x) \le 1$; for all x
- $\int f(x)\, dx = 1$ over all values of x

Then P (X) is said to be a PDF of the distribution.

Find the value of P (1 < X < 2)

Such that,

- **$f(x) = kx^3$; 0 ≤ x ≤ 3 = 0**

Otherwise f(x) is a density function.

Solution:

> *If a function f is said to be a density function, then the sum of all probabilities is equal to 1.*
>
> *Since it is a continuous random variable Integral value is 1 overall sample space s.*
>
> $\int f(x)\,dx = 1$
> $\int kx^3\,dx = 1$
> $K[x^4]/4 = 1$
>
> *Given interval, 0 ≤ x ≤ 3 = 0*
>
> $K[3^4 - 0^4]/4 = 1$
> $K(81/4) = 1$
> $K = 4/81$
>
> *Thus,*
>
> $P\ (1 < X < 2) = k \times [X^4]/4$
> $P = 4/81 \times [16-1]/4$
> $P = 15/81$

Random Variable Formulas

There are two main random variable formulas,

- Mean of Random Variable
- Variance of Random Variable

Let's learn about the same in detail,

Mean of Random Variable

$$\boldsymbol{Mean(\mu) = \sum X.P}$$

where,

- ***X** is the random variable that consist of all possible values.*
- ***P** is the probability of respective variables*

Variance of Random Variable

The variance of a random variable tells us how the random variable is spread about the mean value of the random variable. The variance of the Random Variable is calculated using the formula,

$$\boldsymbol{Var(x) = \sigma^2 = E(X^2) - \{E(X)\}^2}$$

where,

- $E(X^2) = \sum X^2 P$
- $E(X) = \sum XP$

Random Variable Functions

For any random variable X if it assume the values x_1, x_2,...x_n where the probability corresponding to each random variable is $P(x_1)$, $P(x_2)$,...$P(x_n)$, then the expected value of the variable is,

Expectation of X, $E(x) = \sum x.P(x)$

Now for any new random variable Y in which the random variable X is its input, i.e. Y = f(X), then the cumulative distribution function of Y is,

$$\boldsymbol{F_y(Y) = P(g(X) \leq y)}$$

Probability Distribution and Random Variable

For a random variable, its probability distribution is calculated using three

- Experimental listing of outcomes followed with their observed relative frequencies.
- Subjective listing of outcomes followed with their subjective probabilities.

The probability of a random variable X that takes values x is defined using a probability function of X that is denoted by f (x) = f (X = x).

Various probability distributions are,

- Binomial Distribution
- Poisson Distribution
- Bernoulli's Distribution
- Exponential Distribution
- Normal Distribution

People Also Read:

- *Probability Distribution Function*
- *Expected Value*
- *Variance and Standard Deviation*
- *Continous and Discrete Uniform Distribution Formula*
- *Random Variables Practice Problems*

Solved Questions on Random Variable

Here are some of the solved examples on Random variable. Learn random variables by practicing these solved examples.

Question 1: Find the mean value for the continuous random variable, $f(x) = x^2$, $1 \le x \le 3$
Solution:

> *Given,*
> $f(x) = x^2$

$E(x) = \int^3_1 x^3.dx$
$E(x) = [x^4/4]^3_1$
$E(x) = 1/4 \times \{3^4 - 1^4\} = 1/4 \times \{81 - 1\}$
$E(x) = 1/4 \times \{80\} = 20$

Question 2: Find the mean value for the continuous random variable, $f(x) = e^x$, $1 \leq x \leq 3$

Solution:

Given,
$f(x) = e^x$
$1 \leq x \leq 3$

$E(x) = \int^3_1 x.f(x)dx$
$E(x) = \int^3_1 x.e^x.dx$
$E(x) = [x.e^x - e^x]^3_1$
$E(x) = [e^x(x - 1)]^3_1$
$E(x) = e^3(2) - e(0)$

Question 3: Given the discrete random variable X with the following probability distribution:

X	1	2	3	4
P(X)	0.1	0.2	0.4	0.3

Find the mean value (or expected value) of the random variable X.

Solution:

To find the mean value (expected value) of a discrete random variable X, we use the formula:
Using the relation: $E(X) = \mu_X = x_1P(x_1) + x_2P(x_2) + \ldots + x_nP(x_n)$
$E(X) = \sum_i X_i \, . P(X_i)$

The expected value E(X), or mean μ_X of a discrete random variable X

$E(X) = 0.100 + 0.400 + 1.200 + 1.200 = 2.900$
$E(X) = 2.900$

Question 4: Given the discrete random variable X with the following probability distribution:

Suppose a discrete random variable X represents the number of defective items in a sample of 10 items from a batch of 100 items. The possible values of X are 0, 3, 5, and 7 defective items, with the following probability distribution:

X	**0**	**3**	**5**	**7**
P(X)	0.2	0.5	0.2	0.1

Find the mean value (or expected value) of the random variable X.
Solution:

*The formula for the **mean** (or **expected value**) of a discrete random variable X is:*
$E(X) = \sum_i Xi \cdot P(X_i)$

The expected value E(X), or mean μ_X of a discrete random variable X

$E(X) = \mu_X = \sum [\, x_i * P(x_i) \,]$
$E(X) = 0 * 0.2 + 3 * 0.5 + 5 * 0.2 + 7 * 0.1$
$E(X) = 0.000 + 1.500 + 1.000 + 0.700 = 3.200$
$E(X) = 3.200$

Practice Problems on Random Variables

Question 1: Find the mean value for the continuous random variable, $f(x) = x^3$, $1 \leq x \leq 5$

Question 2: Find the mean value for the continuous random variable, f(x) =

Question 3: Given the discrete random variable X with the following probability distribution:

X	1	2	3	4
P(X)	0.2	0.3	0.4	0.1

Find the mean value (or expected value) of the random variable X.

Question 4: Given the discrete random variable X with the following probability distribution:

X	0	1	3	5
P(X)	0.3	0.3	0.3	0.1

Find the mean value (or expected value) of the random variable X.

Next Article

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