NONNEGATIVE AND LOCAL LINEAR REGRESSION FOR CLASSIFICATION IN SAR IMAGERY

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ABSTRACT

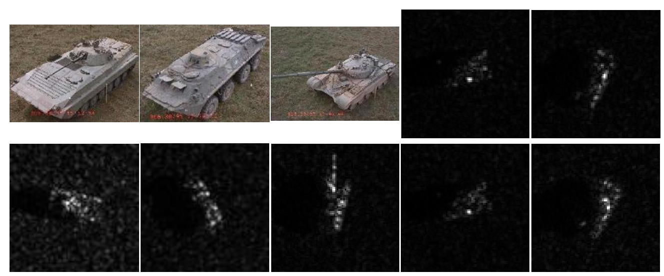
In this paper, the classification via nonnegative and local linear regression model is proposed for SAR image-based target recognition. Recently, a simple yet effective method, linear regression for pattern recognition has been presented. By assuming that images from a single-object class lie on a linear subspace, it represents the test image as a linear combination of class-specific galleries. The representation is obtained by solving a typical inverse problem with least-square strategy. Since the negative weights play a counteractive role in reconstruction, it may be unreasonable to generate the negative weights. In addition, those elements close to the test sample should contribute much more than the ones far from the test. Thus this paper limits the feasible set of the representation by nonnegative and locality constraint. The decision is ruled in favor of the class with the minimum reconstruction error. Extensive experiments on MSTAR database demonstrate that the proposed methods significantly improve the accuracy than the standard one.

***Index Terms*—** Linear regression, classification, SAR, target recognition, nonnegative

1. INTRODUCTION

Synthetic aperture radar (**SAR**) has been extensively studied in many fields, *i.e.*, environment monitoring, earth-resource mapping, target reconnaissance and surveillance. Automatic target recognition is an important topic in SAR imagery interpretation. It is an extension of computer vision and machine learning to classify unknown targets in SAR imagery.

In recently years, many algorithms have been presented to SAR image-based target recognition. The representative includes feature-based method [1], model-based method [2], correlation filter-based method [3], graph embedded based method [4], sparse representation based method [5], machine learning based method [6], etc. Despite great efforts, it remains a challenging community especially in extended operating condition, in which a single operational parameter is significantly different between the images used for training and those used for testing. This is due in part to the complexity of SAR image formation, *i.e.*, the specular reflections of a coherent source, which results in the abrupt and quick variation of the characteristics with small changes in pose, depression, and configuration. A simple example of SAR images is given as follows. SAR images of BMP2, BTR70 and T72 with the corresponding optical images are displayed in Fig. 1 (a), (b), and (c); SAR images (T72) with 2° change of aspect view and with 2° change of depression angle are presented in Fig.1 (d) and (e). As can be esaily seen, significant difference exists between each pair of images.



(a) (b) (c) (d) (e)

Fig. 1. Samples of SAR images.

Linear regression is one of the most basic research for statistical signal processing. Though simplicity, it could achieve great performance especially for the small sample size problem. Ref [7] introduces a novel pose-invariant face recognition method, where the linear regression is applied to small local image patches obtained using dense sampling strategy. In Ref [8], the pattern recognition problem is formulated as one of classifying among multiple linear regression models. Since patterns from a single-object class is assumed to span a linear subspace, the unknown can be linearly represented using the base elements in the subspace. The representation can be obtained by solving a typical inverse problem, thus any preprocessing or training procedure is not needed. To estimate the subspace more precisely, Ref [9] also suggest a robust linear regression classification method, where the reconstruction error is modeled by M-estimators, and the well-conditioned inverse problem is solved using an application of the robust linear Huber estimation.

Inspired by the former works [7, 8, 9], this paper advocates two modified version of linear regression classification, *i.e.*, nonnegative linear regression and local linear regression classification. They are applied to target recognition in

SAR imagery. Actually, linear regression based classification (**LRC**) [8] falls in the category of nearest subspace classification (**NS**). Since the unknown is represented by a linear combination of the base elements in the specified subspace, all elements in the subspace contribute to the recovery of the test. The weight coefficients provide the contribution to the reconstruction of the test. The bigger the weights, the more the element contributes. Therefore, it may be unreasonable to generate the negative weight coefficients because the negative weights usually play the counteractive role during the reconstruction of the test, and result in biased estimate accordingly.

To improve the estimate accuracy of regression model, this paper proposes to generate nonnegative regression pattern by an additional constraint on the representation. The decision is then made by evaluating which class of samples could recover the test sample as accurately as possible with the generated pattern.

On the other hand, those elements close to the test sample should contribute to the recovery much more than the ones far from the test sample. This conclusion motivates us to represent the test sample only by the nearest neighbor elements rather than all elements of the subspace. The main advantage of the proposed methods lies in improving the reconstruction accuracy with acceptable computational cost. Extensive experiments on MSTAR database prove that the proposed methods improve the recognition accuracy than the standard one.

2. THE PROPOSED METHOD

In the typical recognition problem, the goal is to predict the class membership of the test by knowledge learned by the present samples. Suppose there are n_k training samples from kth class, $\mathbf{X}_k = [\mathbf{x}_{k,1}, \mathbf{x}_{k,2}, ..., \mathbf{x}_{k,n_k}] \in \mathbb{R}^{m \times n_k}$, where each sample stacks by its columns, we aim at inferring the identity of the test $\mathbf{y} \in \mathbb{R}^m$ according to n training samples from K distinct classes ($n = \sum_{j=1}^{K} n_j$).

2.1. Linear Regression Classification (LRC)

It is known that samples from a specific object class approximately lie on a linear subspace [10]. The samples are also called the regressors (or predictors) in statistics. For any test sample from kth class, it could be well represented by a linear combination of the samples from the same class,

$$\mathbf{y} = \mathbf{X}_k \beta_k = \mathbf{x}_{k,1}\beta_{k,1} + \mathbf{x}_{k,2}\beta_{k,2} + \cdots + \mathbf{x}_{k,n_k}\beta_{k,n_k} \quad (1)$$

where $\beta_k = [\beta_{k,1}, \beta_{k,2}, ..., \beta_{k,n_k}]^T \in \mathbb{R}^{n_k}$ is the regression coefficients. Considering the well condition problem ($m > n_k$), an intuitive idea to obtain the representation β_k is to solve the inverse problem,

$$\min_k \{\|\mathbf{y} - \mathbf{X}_k \beta_k\|_2\} \quad (2)$$

It is a least square fit problem, and hence the solution of (2) is

$$\hat{\beta}_k = (\mathbf{X}_k^T \mathbf{X}_k)^{-1} \mathbf{X}_k^T \beta_k. \quad (3)$$

With the representation $\hat{\beta}_k$, the reconstruction of $\mathbf{y}$ in terms of kth class is $\hat{\mathbf{y}}_k = \mathbf{X}_k \hat{\beta}_k$, $i = 1, 2, ..., C$. $\hat{\mathbf{y}}_k$ can be viewed as the projection of signal $\mathbf{y}$ on the subspace spanned using the samples from kth class. The inference is reached by evaluating which class of samples could reconstruct the test as accurately as possible,

$$\min_{k=1,...K} \left\{ \|\mathbf{y} - \hat{\mathbf{y}}_k\|_2^2 \right\} \quad (4)$$

2.2. Nonnegative Linear Regression Classification (NNLRC)

In LRC, the test sample is recovered using the base elements of each class-specific subspace, and the decision is ruled in favor of the class with the minimum reconstruction error. In other words, LRC mainly focuses on the reconstruction accuracy, thus it is natural to produce some negative coefficient weights. Usually, the negative weights play a counteractive role in signal reconstruction, thus it is meaningful to avoid the nonnegative weight coefficients. In light of this analysis, this paper pays attention to the sign of the regression coefficients besides the reconstruction accuracy. On the basis of LRC (2), we limit the feasible set of the representation via nonnegative constraint,

$$\min_k \{\|\mathbf{y} - \mathbf{X}_k \beta_k\|_2\}, s.t., \beta_k > 0 \quad (5)$$

Then the weight coefficients committed to the base elements of class-specific subspace are either positive or zeroth. The system shown in (5) is a fundamental optimization problem that arises naturally in applications where in addition to satisfying a least squares model, the variables must also satisfy the nonnegativity constraints. The decision is made as similar to LRC shown in (4).

2.3. Local Linear Regression Classification (LLRC)

LRC recovers the test sample using all atoms of the subspace, despite what relations between the predictors and the test sample. This strategy may results in biased estimation. Theoretically, elements that are close to the test sample should play much more important role than the ones far from the test sample in signal reconstruction. Thus it is advantageous to employ different atoms from sample to sample. Based on the analysis, this paper suggests local linear regression model, in which only the elements close to the test sample rather than all elements of subspace are used during the reconstruction,

$$\mathbf{y} = \mathbf{X}_{\mathcal{N}_k} \beta_k \quad (6)$$

where $\mathbf{X}_{\mathcal{N}_k}$ is a subset of $\mathbf{X}_k$, which contains the atoms close to the test sample. Similar to (3), the solution of (6) is obtained by solving the inverse problem,

$$\hat{\beta}_k = (\mathbf{X}_{\mathcal{N}_k}{}^T \mathbf{X}_{\mathcal{N}_k})^{-1} \mathbf{X}_{\mathcal{N}_k}{}^T \beta_k. \quad (7)$$

Besides improving the reconstruction accuracy, LLRC also alleviates the computational cost, because less predictors are used during signal reconstruction. Local linear regression model is favorable to the big sample size condition, because it neglects those elements far from the test. The final decision is reached similarly to (4).

To delineate two modified version of LRC, a simple example is given in Fig.2. There is a total of three classes, BMP2 infantry vehicle, BTR70 armored carrier and T72 tank. A test sample (BMP2) is recovered using LRC, NNLRC and LLRC, respectively. The weight coefficients are shown in the first row, with the reconstruction error shown in the second row. As can be seen , both the positive and negative coefficients are produced by LRC, and the test is misclassified as BTR70. The shortage is covered by nonnegative constraint on the representation, as displayed in Fig.2 (b), where only the positive and zero weights are generated by NNLC. Moreover, the sparse representations are obtained using LLRC, as given in Fig.2 (c). Both the two proposed methods give the correct class label, BMP2.

3. EXPERIMENTS AND DISCUSSIONS

This section verifies the proposed methods on MSTAR database, a collection done using a 10 GHz SAR sensor in one-foot resolution spotlight mode for several targets. For each target, images are captured at different depression angles over a full 0-359 degrees range of aspect view. Three vehicle targets, BMP2, BTR70 and T72 are employed. Following the official guideline, images taken at $17°$ depression are used for training, while images collected at $15°$ depression are used for testing. The number of aspect view available for 3 targtes is listed in TABLE.1. The center 80×80 pixels patch of the chip image are cropped to exclude the clutter background. The simple down-sampling by a factor of $1/4$ and $1/16$, are employed to make the high-dimensional space computationally tractable. Three representative algorithms, LRC, SRC and SVM are utilized to provide the standard comparison with the proposed methods.

TABLE 2 gives the recognition rates obtained using the methods to be studied, with the corresponding performance pictorially drawn in Fig.3. As can be seen, the significant improvement in recognition accuracy has been obtained by LRC, SRC, and NNLRC with the down-sampling factor increased from $1/16$ to $1/4$, while similar performance is obtained by SVM, KSVM and LLRC. The recognition accuracies for NNLRC with the feature space of $1600D$ and $400D$ are 0.9319 and 0.9509, 3.99%, 2.72%, 3.52% better in average than LRC, SRC and SVM. The performance of NNLSRC is much similar compared to KSVM. Another propsoed method, LLRC, obtains the highest accuracy amongst all the algorithms to be studied. The rate for LLRC with the feature space of $1600D$ is 0.9523, 3.07%, 2.49%, 5.19% and 1.17% better than LRC, SRC, SVM and KSVM. Even with the feature space of $400D$, LLRC still achieves the recognition accuracy of 0.9531, compared to 0.9120 for SVM, 0.9010 for SRC, and 0.8813 for LRC. More interestingly, LLRC exhibits very good recognition accuracy as well as other attractive properties: low computational cost and simplicity of feature selection.

Table 1. The number of aspect view images available for different targets

Depr.	BMP2	T72	BTR70	SUM
$17°$	**233**(9563)	**232**(132)	233	698
$15°$	195(9563) 196(9566) 196(c21)	196(132) 195(812) 191(s7)	196	1365

Table 2. The accuracies with the feature space of $1600D$ and $400D$ in 3-target recognition.

Alg.	LRC	SRC	SVM	KSVM	NNLRC	LLRC
40×40	0.9216	0.9274	0.9004	0.9406	0.9319	0.9523
20×20	0.8813	0.9010	0.9120	0.9531	0.9509	0.9531

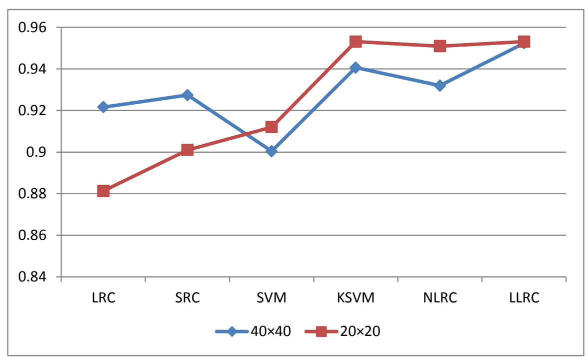


Fig. 3. The recognition rates of the methods to be studied with the feature space of $1600D$ and $400D$, respectively.

4. CONCLUSION

To deal with the basic limitations of LRC, the classification via nonnegative and local linear regression model is presented. Since the negative weights play the counteractive role during reconstruction, this paper propose to generate the nonnegative pattern. Moreover, only the elements close to the test are used to improve the recovery accuracy. Extensive experimental comparisons of the proposed methods with the standard LRC, as well as other popular methods are conducted on MSTAR database. The results demonstrate that the proposed methods significantly improve the recognition accuracy.

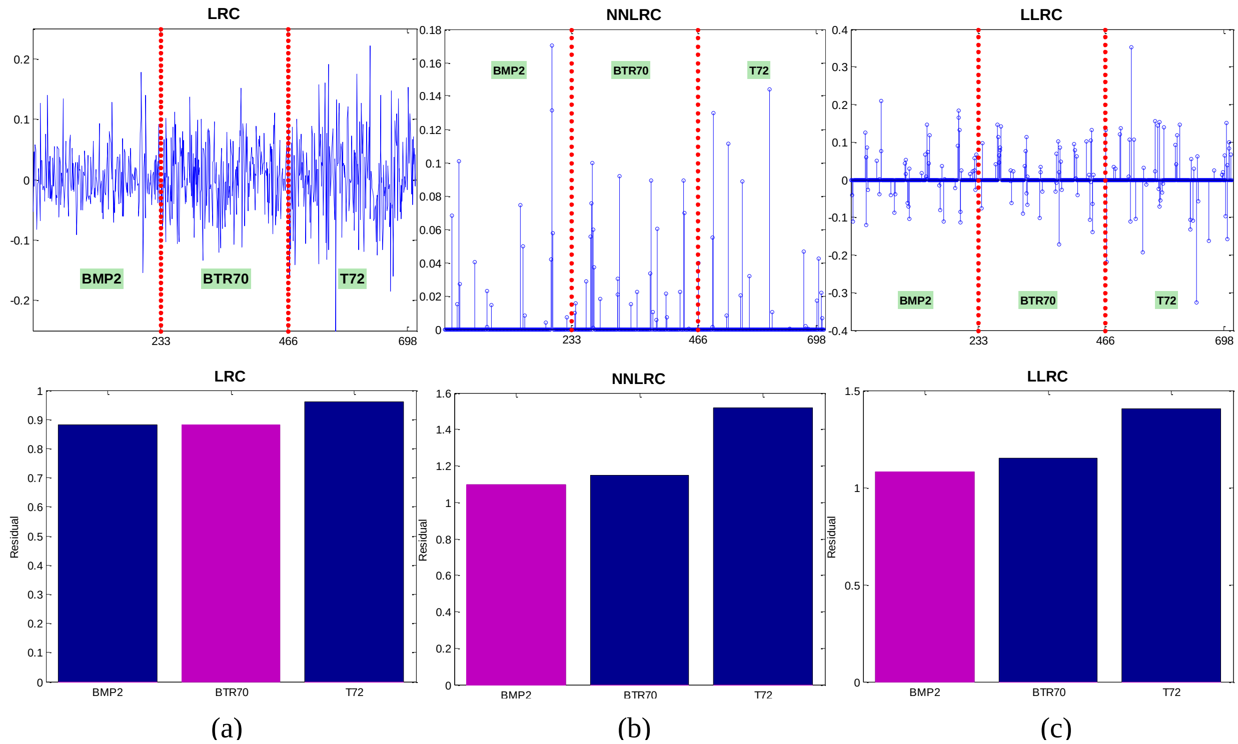


Fig. 2. Comparison of (a) LRC, (b) NNLRC and (c) LLRC. The first row gives the regression coefficients, while the second row display the reconstruction error.

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