Midterm Examination (Close Book)

Exam date & time: May 16, 2014, 10:10AM–11:50AM (100 minutes)

Do the following problems. The points are specified in the brackets (i.e., []). There are 100 points in total. If a problem consists of one mathematical statement and two questions. For the first question, you have to answer ○ for "**true**" or × for "**false**" to indicate your judgement on the validity of the given statement. For the second question, you have to prove the statement if your answer is ○ for the first question or disprove the statement if your answer is ×. If the answer for the first question is incorrect, there will be no points awarded to the second question (as well as the first question).

All of the mathematical statements that have been proved in the textbook as theorems, corollaries, and lemmas can be used.

1. [10] "A language is Turing-recognizable iff some enumerator enumerates it" has been proved in the textbook. Why didn't we use the following simpler algorithm for the forward direction of the proof? $s_1, s_2, \ldots$ is a list of all strings in Σ^*.

   E = "Ignore the input.
   1) Repeat the following for $i = 1, 2, 3, \ldots$
   2) Run M on s_i.
   3) If it accepts, print out s_i."

2. [10] Let a k-PDA be a pushdown automaton that has k stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. You already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs. So, 2-PDAs are more powerful than 1-PDAs.

   (a) [3] True (○) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

3. [10] 3-PDAs are more powerful than 2-PDAs.

   (a) [3] True (○) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

4. [10] The collection of decidable languages is closed under the operation of complementation.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

5. [10] The set of all infinite sequences over $\{ \mathtt{0}, \mathtt{1} \}$ is countable.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

6. [10] $L_6 = \{ \langle G \rangle \mid G$ is a CFG over Σ_{01} and $\mathtt{1}^* \cap L(G) \neq \emptyset \}$ is a decidable language.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

7. [10] $L_7 = \{ \langle R \rangle \mid R$ is a regular expression describing a language containing at least one string w that has 111 as a substring (i.e., $w = x\mathtt{111}y$ for some x and y) $\}$ is a decidable language.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

8. [10] $L_8 = \{ \langle M_1, M_2 \rangle \mid M_1$ and M_2 are DFAs, and $L(M_1) = L(M_2) \}$ is decidable by testing the two DFAs on a finite set of strings.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

9. [10] $L_9 = \{ \langle M \rangle \mid M$ is a TM that has useless states $\}$ is decidable.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

10. [10] In the *silly Post Correspondence Problem*, $SPCP$, in each pair the top string has the same length as the bottom string. $SPCP$ is undecidable.

    (a) [3] True (◯) or false (×)?

    (b) [7] Show that your answer for (a) is correct.