2. a. $\mathbb{P}\left(\exists a \in A \text{ s.t } |\bar{r}_a - \hat{r}_a| > \sqrt{\frac{\log(2/\delta)}{2n_e}}\right)$

$$= \mathbb{P}\left(\bigcup_{a \in A} |\bar{r}_a - \hat{r}_a| > \sqrt{\frac{\log(2/\delta)}{2n_e}}\right) =$$

$$= \sum_{a \in A} \mathbb{P}\left(|\bar{r}_a - \hat{r}_a| > \sqrt{\frac{\log(2/\delta)}{2n_e}}\right) < |A| \cdot \delta$$

הספ'ע.

b.

from (a) we get that

$$\mathbb{P}\left(\forall a \in A \ |\bar{r}_a - \hat{r}_a| \le \sqrt{\frac{\log(2/\delta)}{2n_e}}\right) \ge |A|\delta$$

thus if $\sqrt{\frac{\log(2/\delta)}{2n_e}} \le \varepsilon/2$

and $|A|\delta \ge 1-\delta'$, then we get that:

$$P\left(\forall a \in A \ |\bar{r}_a - \hat{r}_a| \le \varepsilon/2\right) \ge 1-\delta'$$

Thus, in prob. of at least $1-\delta'$ $(1-|A|\delta)$ we get that:

$$\bar{r}_{a^+} \ge \hat{r}_{a^+} - \frac{\varepsilon}{2} \underset{a^+ = \arg\max_a \hat{r}_a}{\ge} \hat{r}_{a^*} - \frac{\varepsilon}{2} \ge \bar{r}_{a^*} - \varepsilon$$

Thus, if:

$$\frac{2 \cdot \log(2/\delta)}{\varepsilon^2} \le n_e$$

then: $\bar{r}_{a^+} \ge \bar{r}_{a^*} - \varepsilon$ w.P of $1-\delta|A|$