Functional Brain Imaging with MEG (Magnetoencephalography) and EEG (Electroencephalography)

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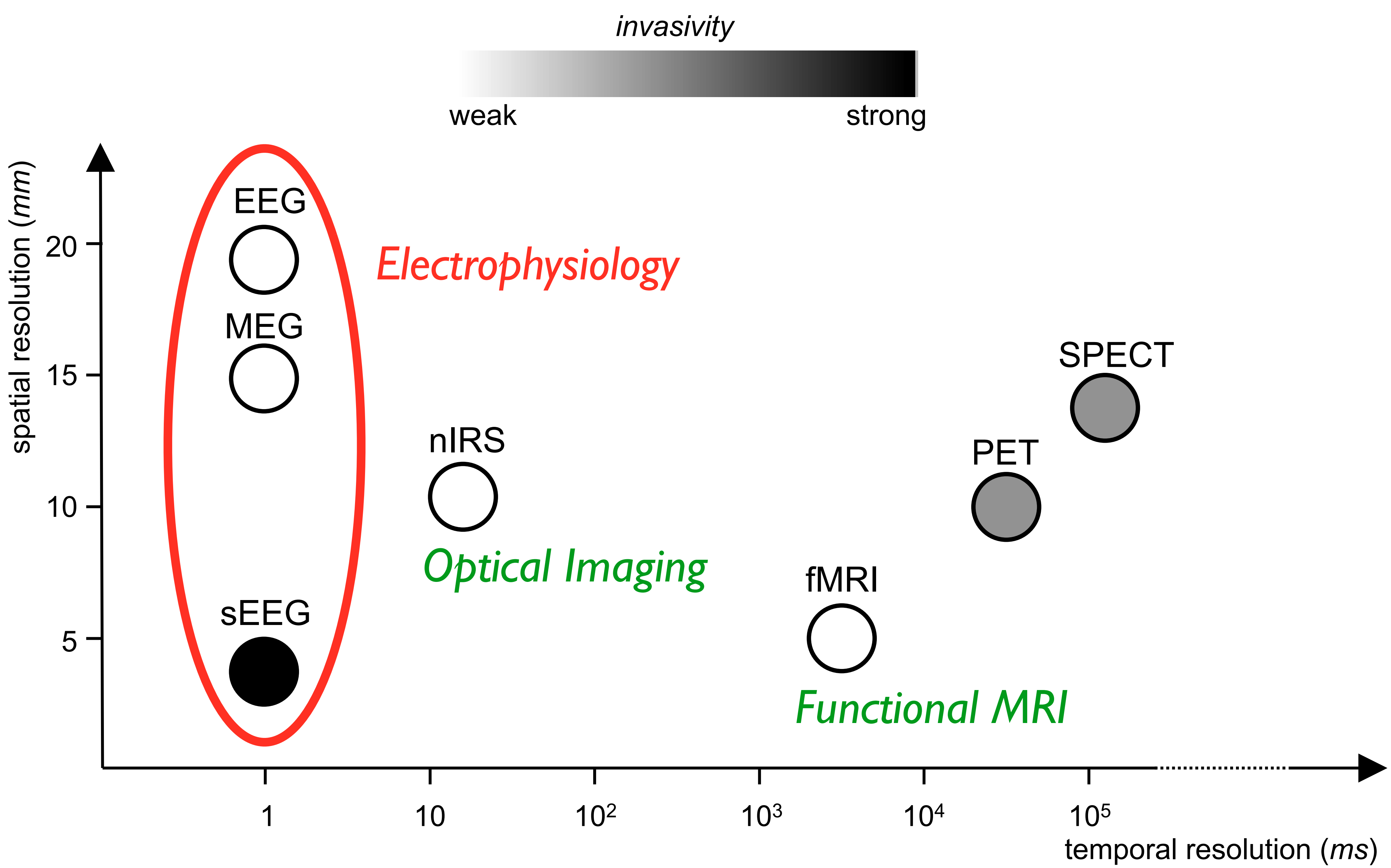
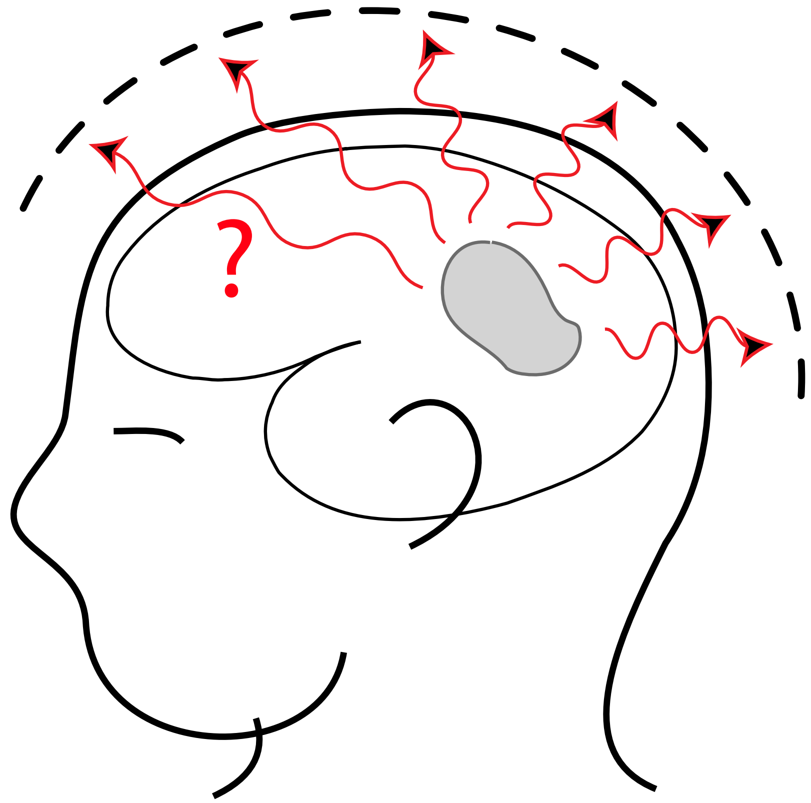
Functional neuroimaging landscape



Source localisation with M/EEG:
The **forward** and **inverse** problems

Forward problem: Objective

Predict what is the **Electric Potential** or the **Magnetic Field** produced by a current generator outside of the head



How to do it?

- Find from **Maxwell equations** the equations adapted to the problem.
- Define a **model for the current generators** (e.g., sources modeled by equivalent current dipoles).
- **Solve numerically the differential equations** obtained for a real anatomy obtained by MRI.

Maxwell

Maxwell Equations with **quasi-static** approximation

$$\begin{cases} \nabla \times \vec{E} = 0 \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{B} = \mu_0 \vec{J} \\ \nabla \cdot \vec{E} = \dfrac{\rho}{\epsilon_0} \end{cases}$$

Remark: quasi-static implies no temporal derivatives and no propagation delay

Total currents: $\vec{J} = \vec{J}_p + \vec{J}_c$

Primary currents

Conduction currents

Ohm's law: $\vec{J}_c = -\sigma \nabla V$

V Electric potential

σ Tissue conductivity

Maxwell

Potential equation (relation between the potential and the sources):

$$\nabla \cdot \nabla \times \vec{B} = 0 \Rightarrow \nabla \cdot (\vec{J}_p + \vec{J}_c) = 0$$

$$\Rightarrow \nabla \cdot \vec{J}_p = \nabla \cdot (\sigma \nabla V)$$

Poisson Equation

Remark: Relation with Kirchoff's law

Magnetic field equation:

$$\vec{B} = \frac{\mu_0}{4\pi} \int \vec{J}(r') \times \frac{r - r'}{\|r - r'\|^3} dr'$$

Biot and Savart's law

$$\Rightarrow \vec{B} = \vec{B}_0 - \frac{\mu_0}{4\pi} \int \sigma \nabla V \times \frac{r - r'}{\|r - r'\|^3} dr'$$

where $\vec{B}_0 = \frac{\mu_0}{4\pi} \int \vec{J}_p \times \frac{r - r'}{\|r - r'\|^3} dr'$

Observations:

- B is obtained after V
- B decreases in $1/R^2$
- B is due both to primary currents and volume currents

Head models

Requires to **model the properties of the different tissues**: skin, skull, brain etc.

Hypothesis: The conductivities are **piecewise constant**

Sphere models

Analytical solutions fast to compute but very **coarse** head model (esp. for EEG)

EEG : [Berg et al. 94, De Munck 93, Zhang 95]
MEG : [Sarvas 87]

Realistic models

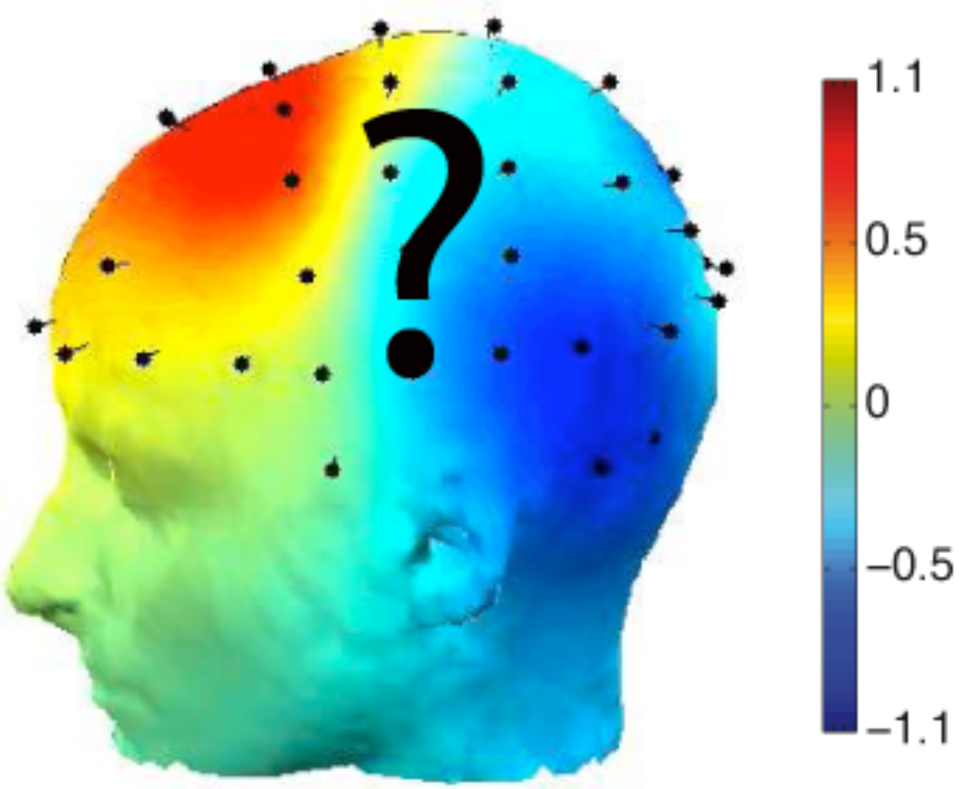
Boundary element method (BEM), i.e., numerical solver with approximate solution.

[Geselowitz 67, De Munck 92, Kybic et al. 2005]

The M/EEG inverse problem

Inverse problem: Objective

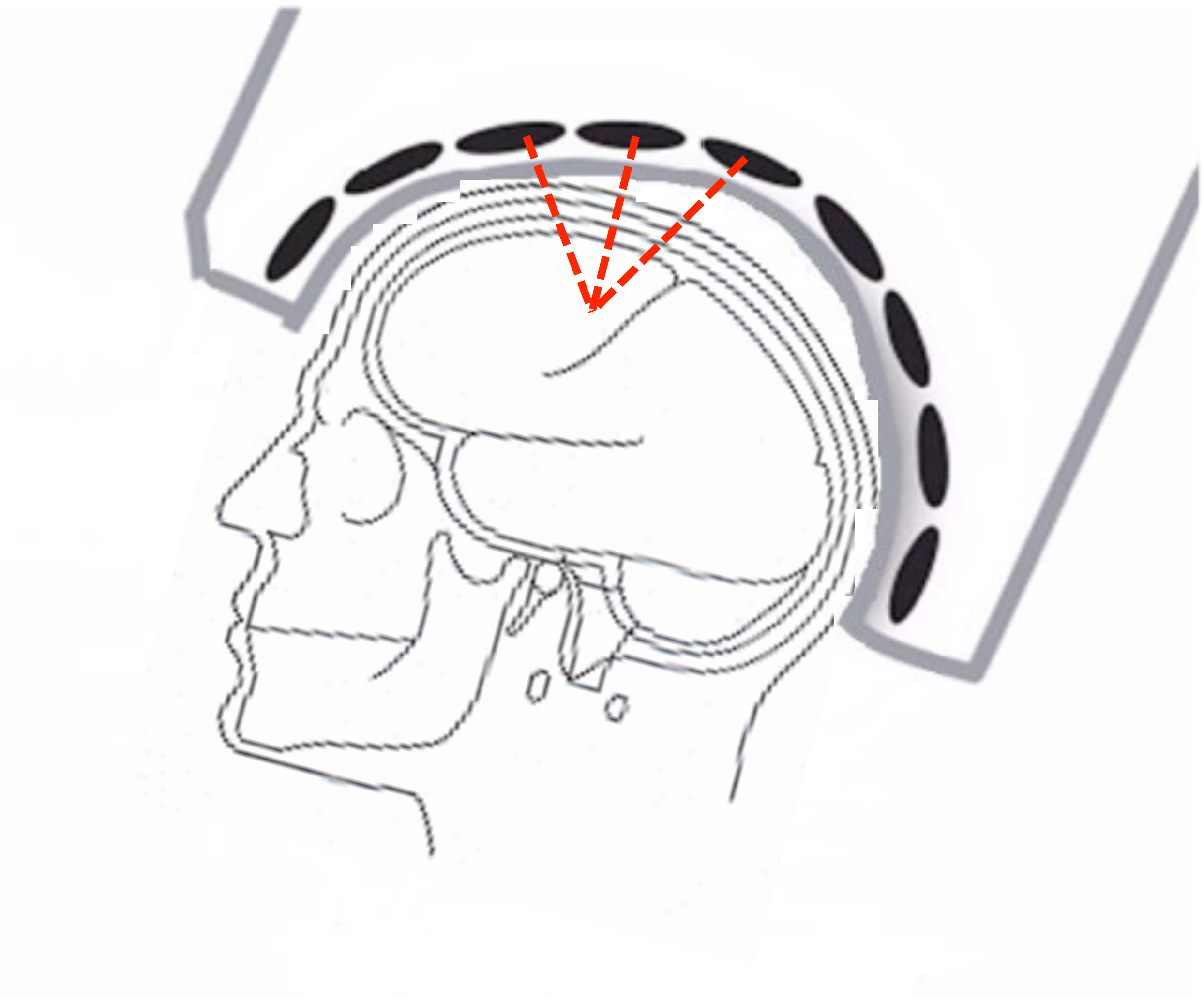
Find the current generators that produced the M/EEG measurements

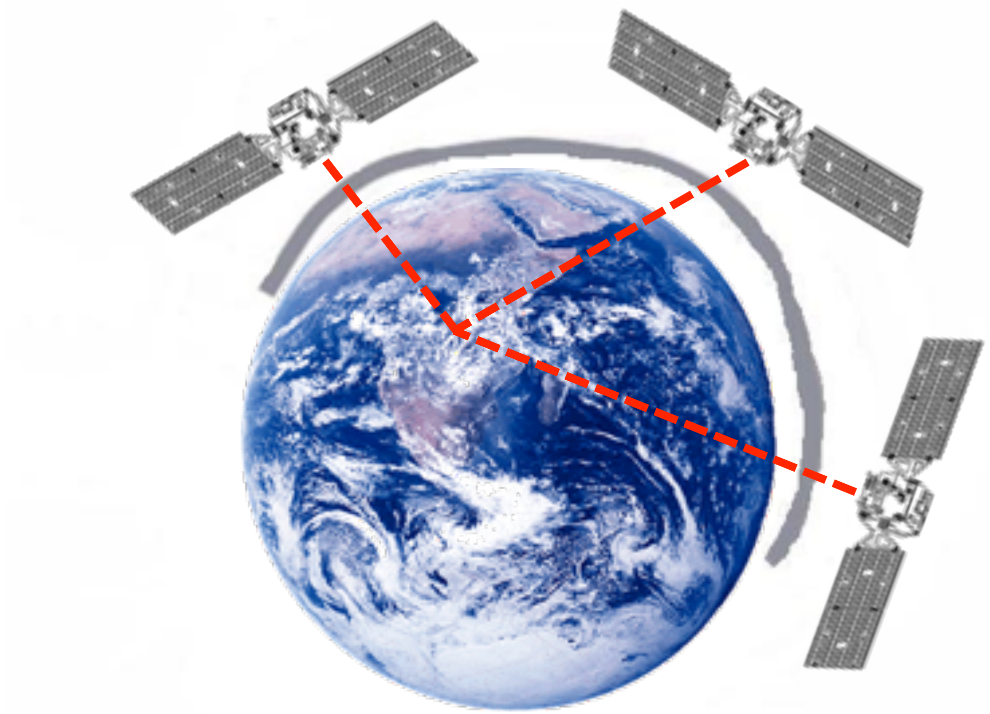


Inverse problem approaches

- Dipole fitting
- Scanning methods
- Distributed models

Dipole fitting

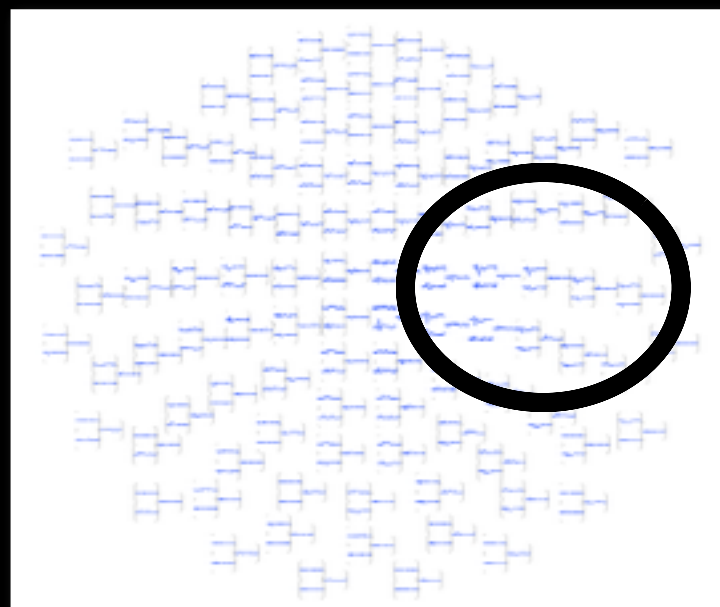




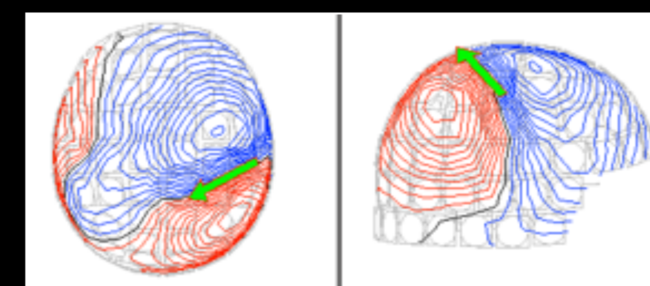
The equivalent of triangulation

Dipole fitting: procedure

1) Pick subset of sensors w/ peak



2) Pick Time Point; Observe Mag Field

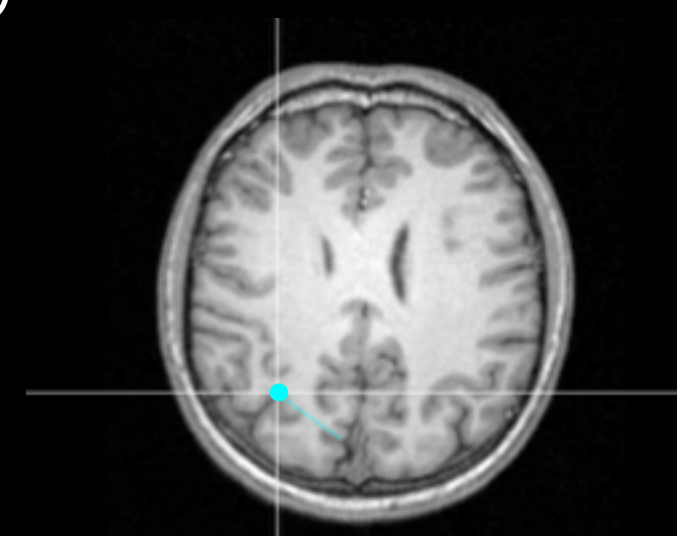


Equivalent Current Dipole Technique

Goodness of Fit
% of activity explained by forward solution based on single dipole

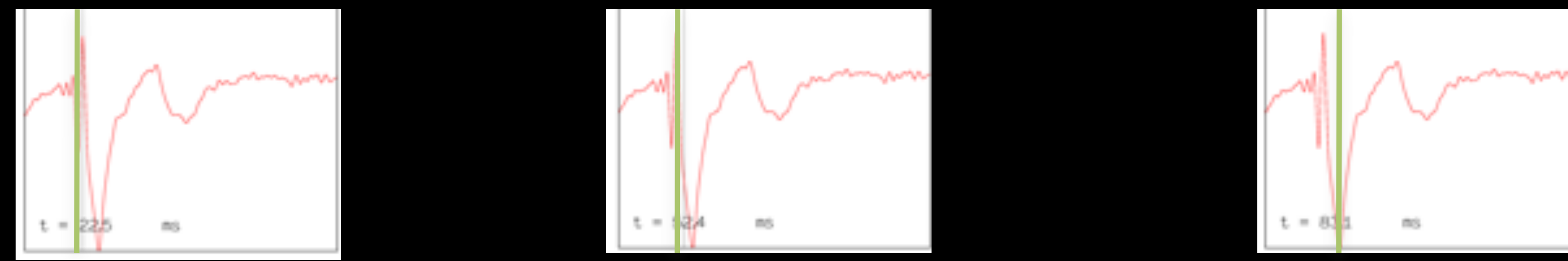
Confidence Volume
volume within which you can be 95% confident that the dipole exists

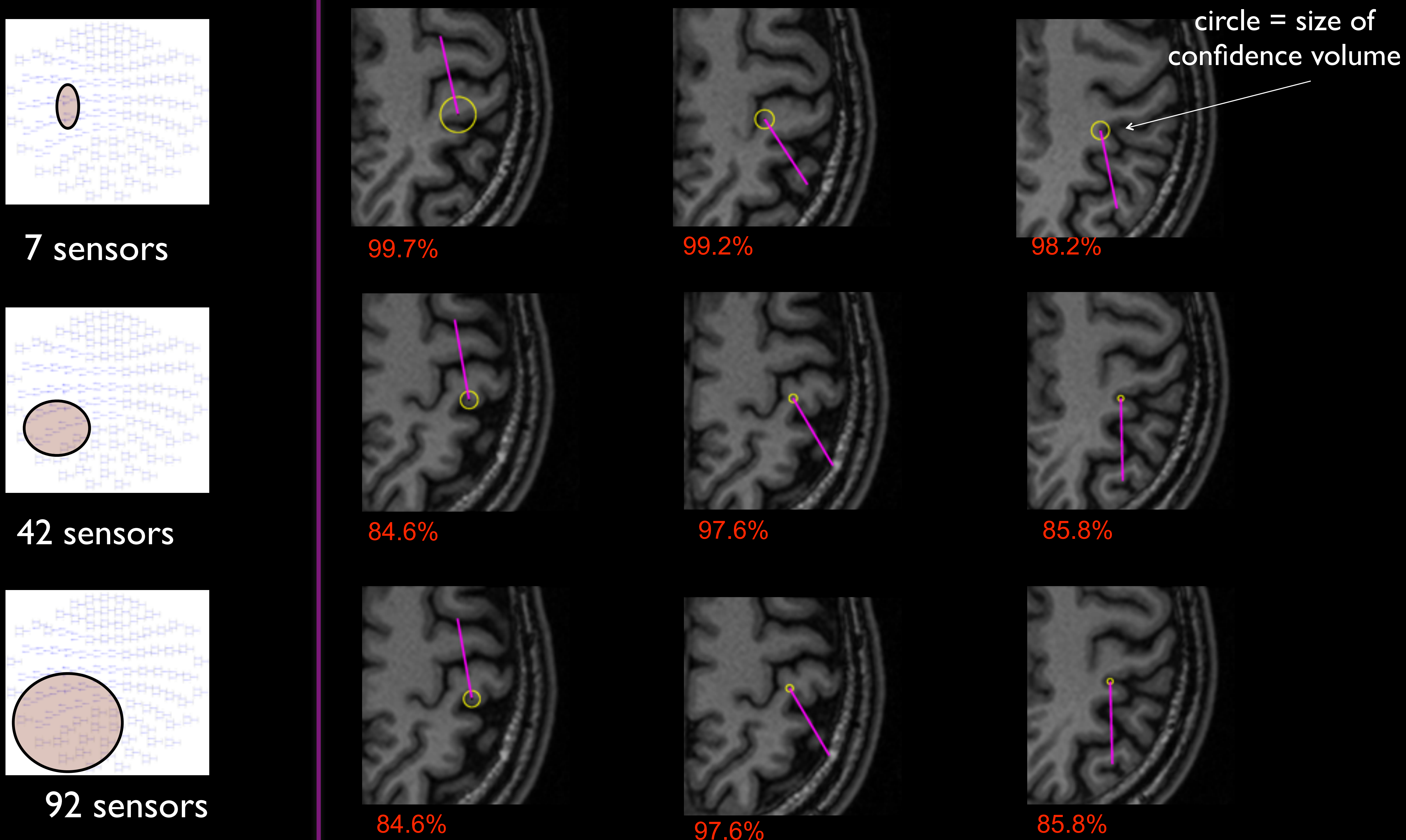
3) Measures of Quality



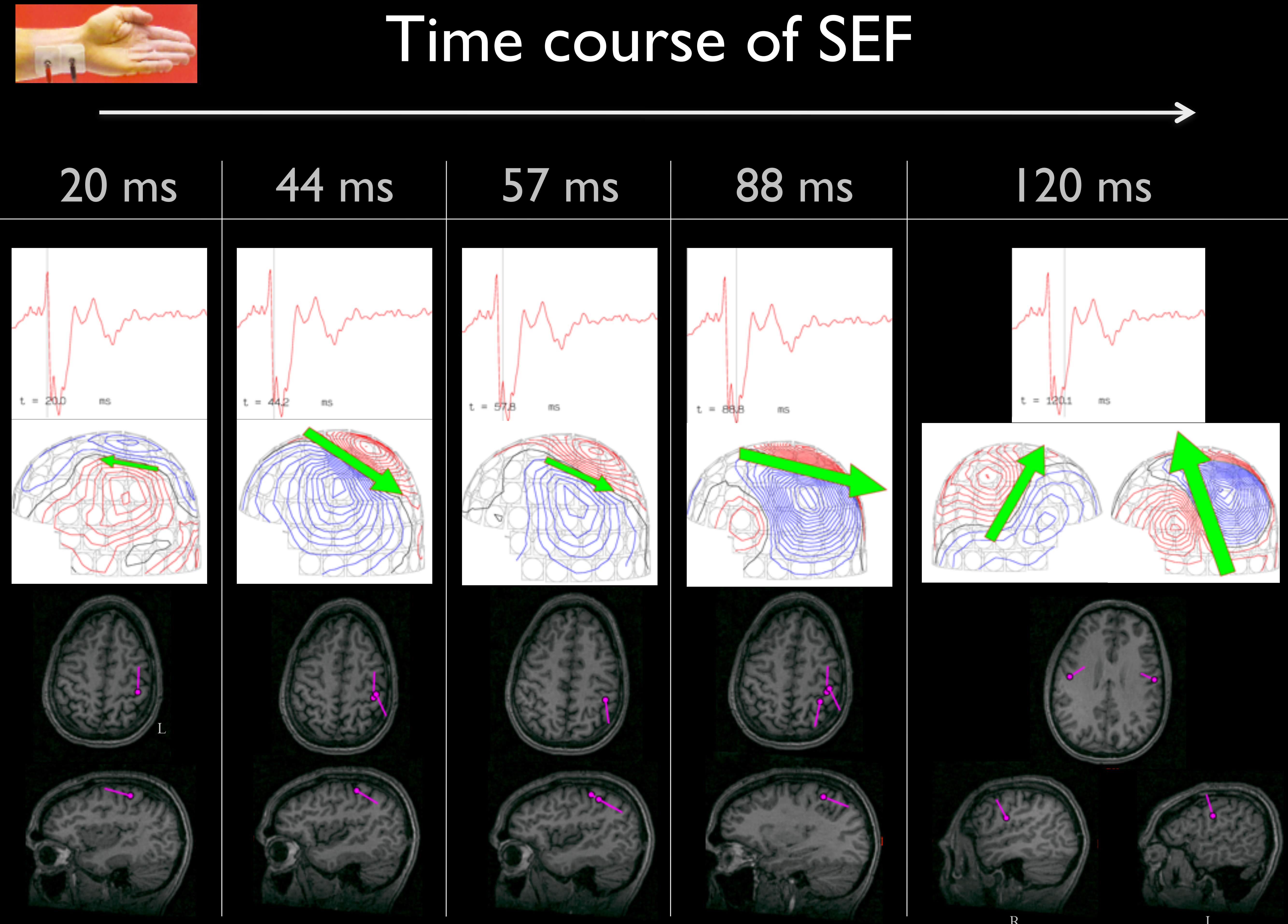
4) Map to MRI

Median Nerve Dipole Fitting Results

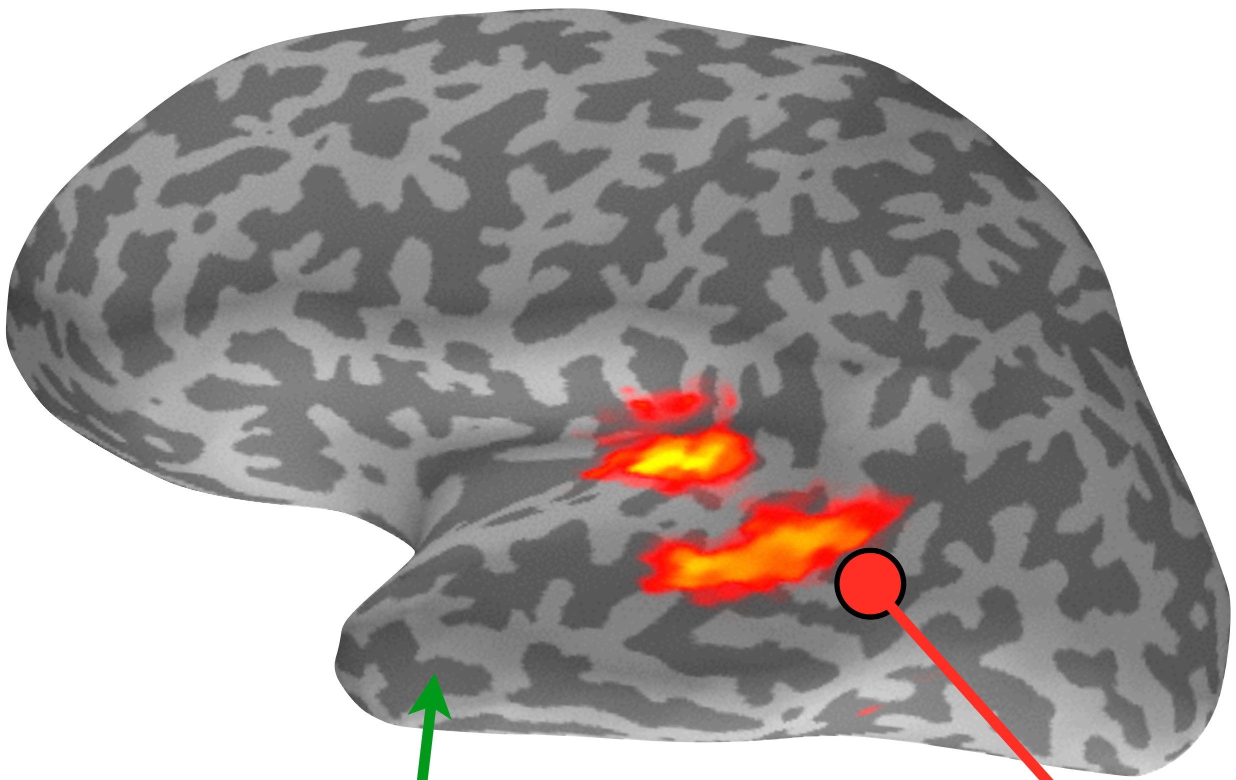




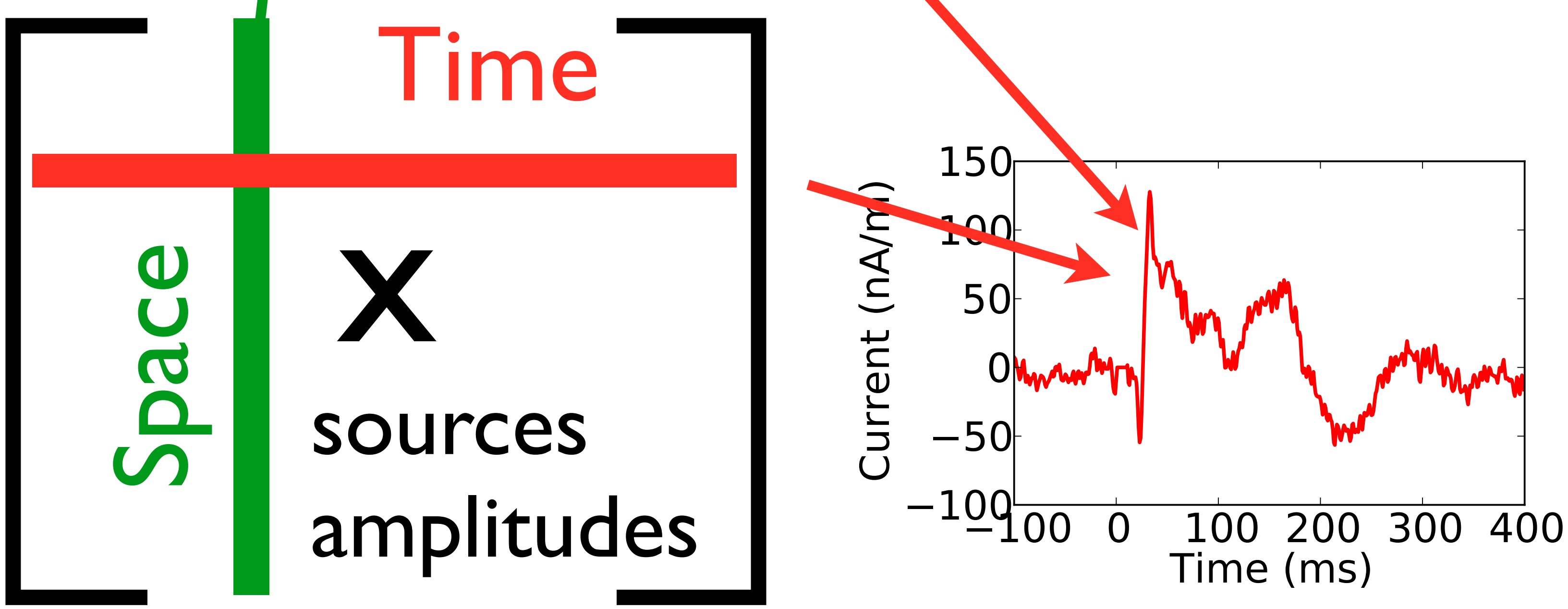
Time course of SEF



Distributed model



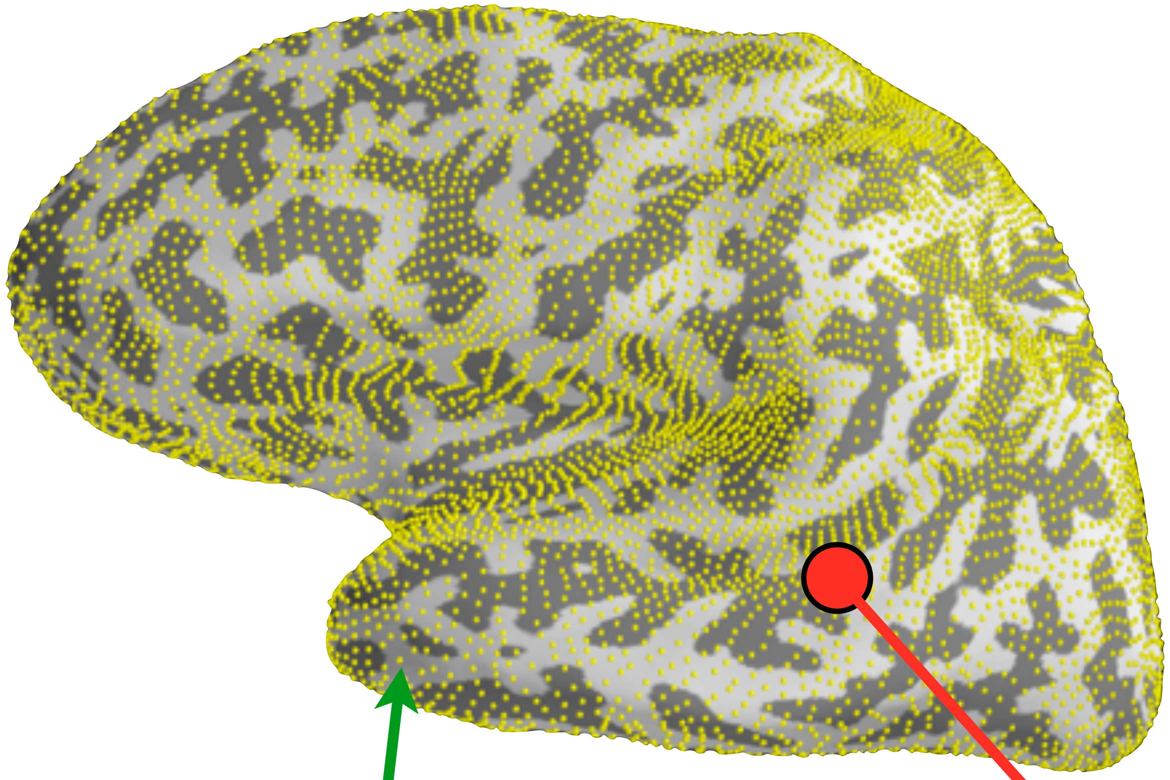
Position 5000 candidate sources over each hemisphere (e.g. every 5mm)



Scalar field defined over time

[Dale and Sereno 93]

Distributed model



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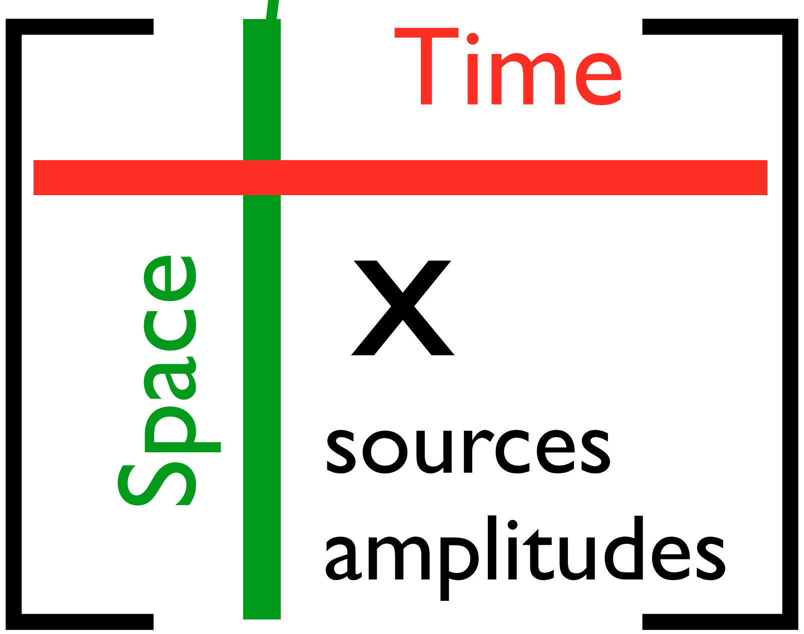

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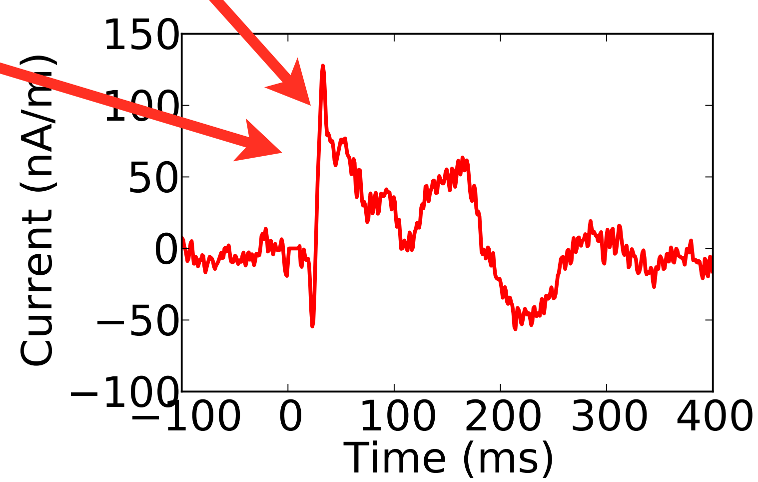
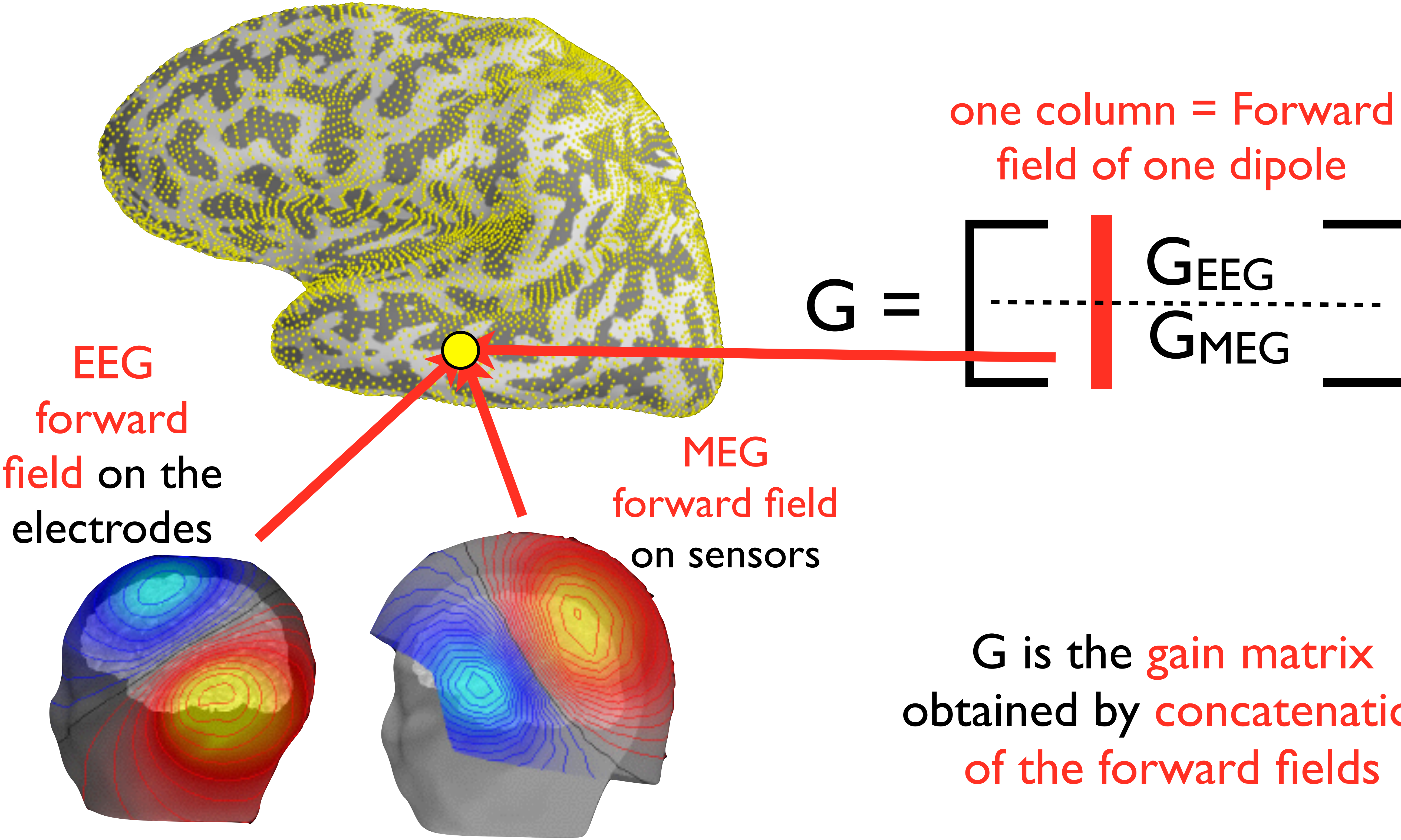


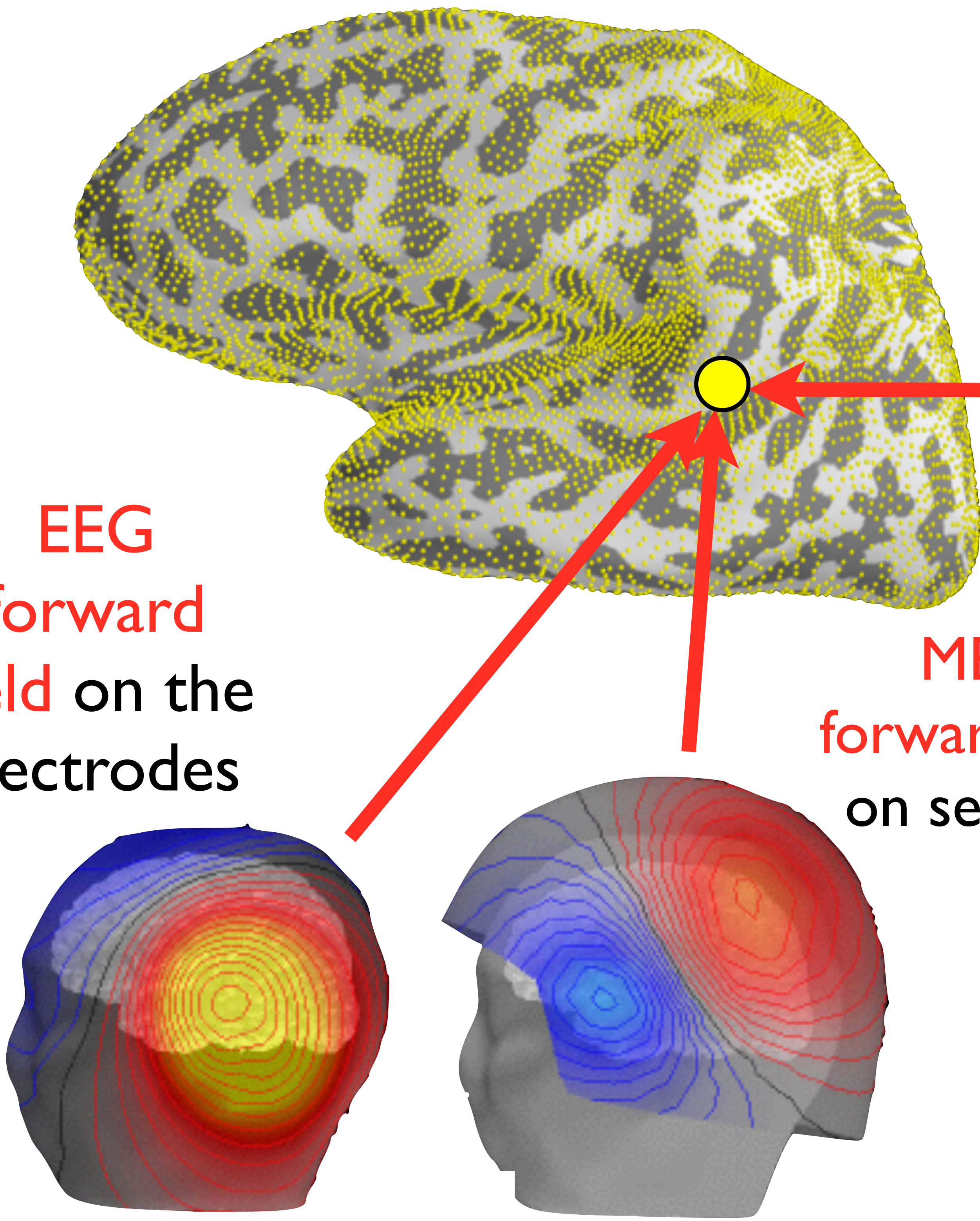
[Dale and Sereno 93]

Distributed model



G is the gain matrix obtained by concatenation of the forward fields

Distributed model



EEG forward field on the electrodes

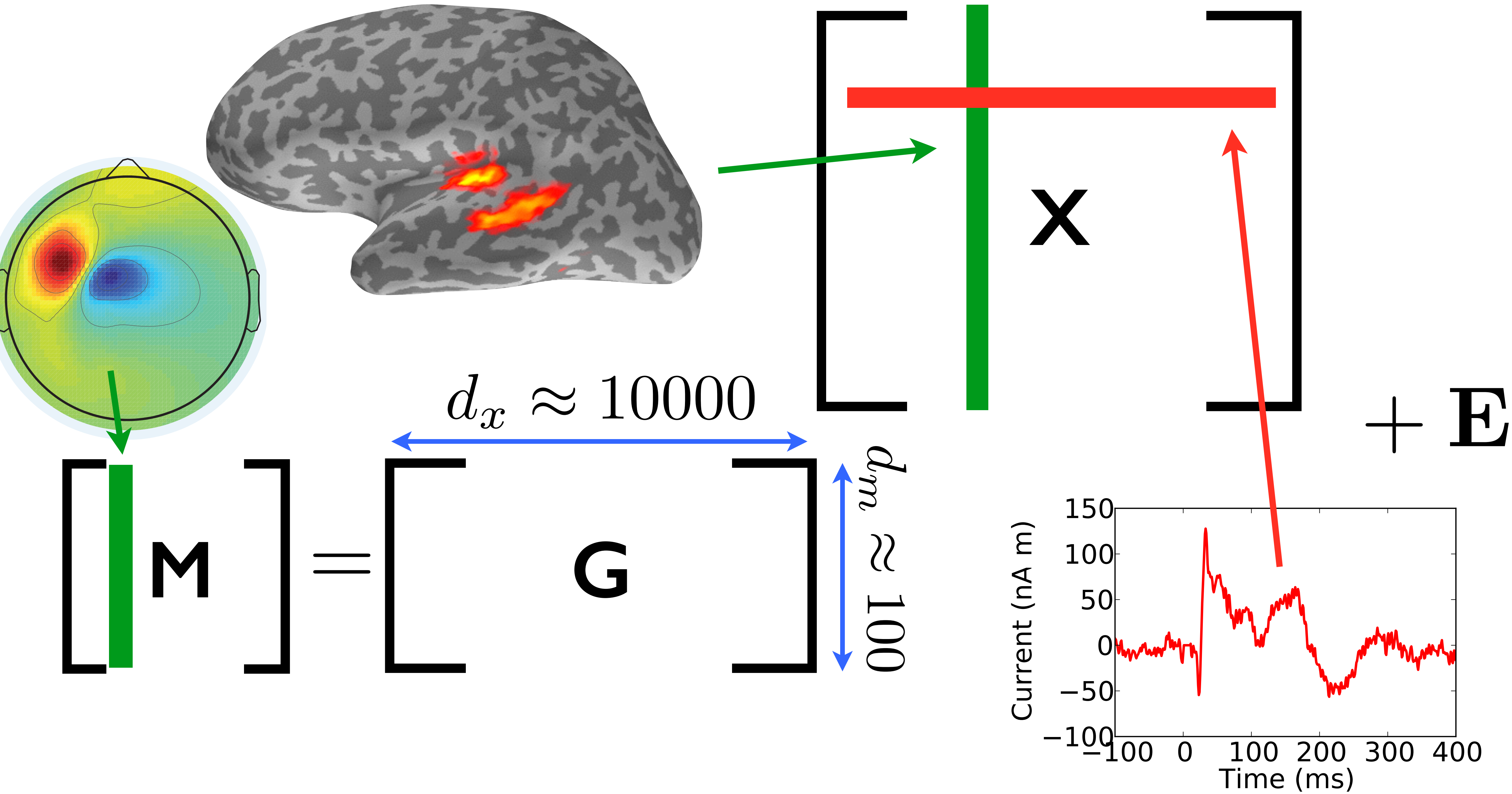
MEG forward field on sensors

one column = Forward field of one dipole

$$G = \begin{bmatrix} G_{EEG} \\ \hdashline G_{MEG} \end{bmatrix}$$

G is the gain matrix obtained by concatenation of the forward fields

M = GX+E : An ill-posed problem



Linear problem with more unknowns than the number of equations: it's ill-posed => Regularize

Inverse problem framework

An optimization problem:

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \|\mathbf{M} - \mathbf{GX}\|_F^2 + \lambda\phi(\mathbf{X}), \lambda > 0$$

Data fit — Regularization

λ : Trade-off between the data fit and the regularization

where $\|\mathbf{A}\|_F = \mathbf{tr}(\mathbf{A}^T\mathbf{A})$

$\phi(\mathbf{X})$ Measures the *complexity* of X, it's the prior.

Examples for $\phi(\mathbf{X})$: $\ell_1,\ \ell_2,\ \ell_p$ with $p \geq 1$, entropy ...

Remark: If $\phi(\mathbf{X})$ is strictly convex we have a unique minimizer (sufficient but not a necessary condition)

Inverse problem framework

Penalized (variational) formulation (with whitened data):

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \|\mathbf{M} - \mathbf{G}\mathbf{X}\|_F^2 + \lambda\phi(\mathbf{X}), \lambda > 0$$

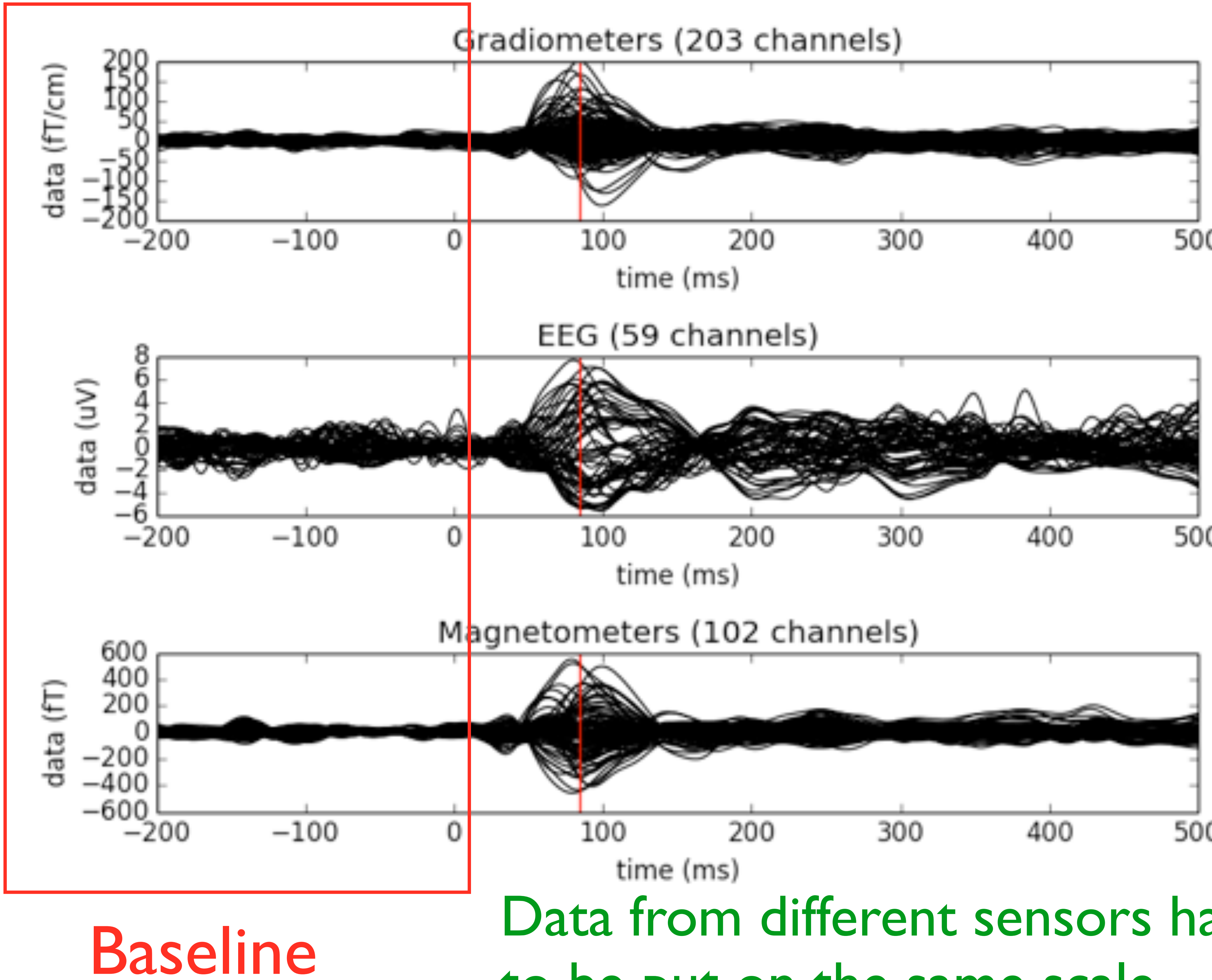
Data fit Regularization

λ : Trade-off between the data fit and the regularization

where $\|\mathbf{A}\|_F^2 = \mathbf{tr}(\mathbf{A}^T\mathbf{A})$

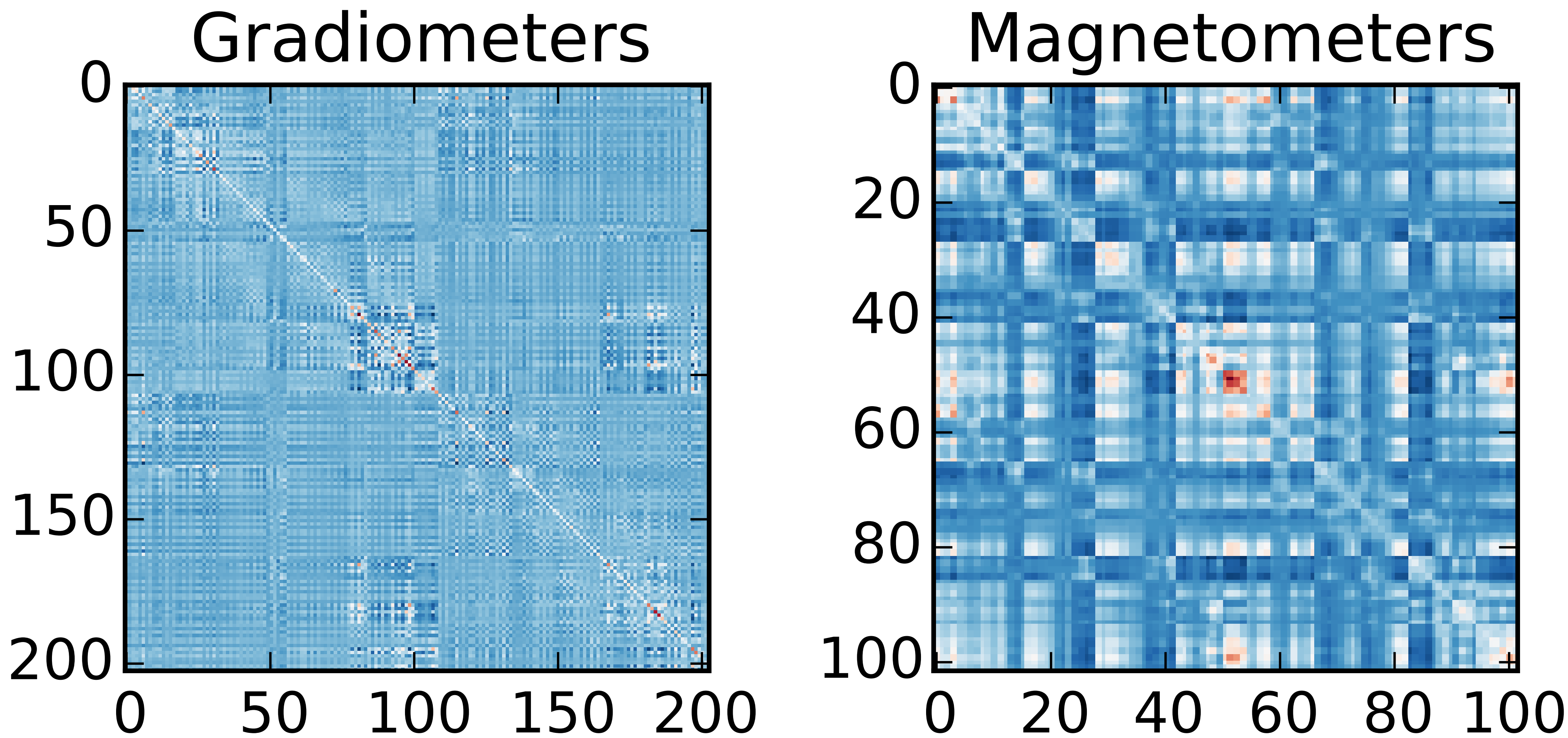
How do you whiten data?

To know what is signal look at the noise



Data from different sensors have to be put on the same scale.

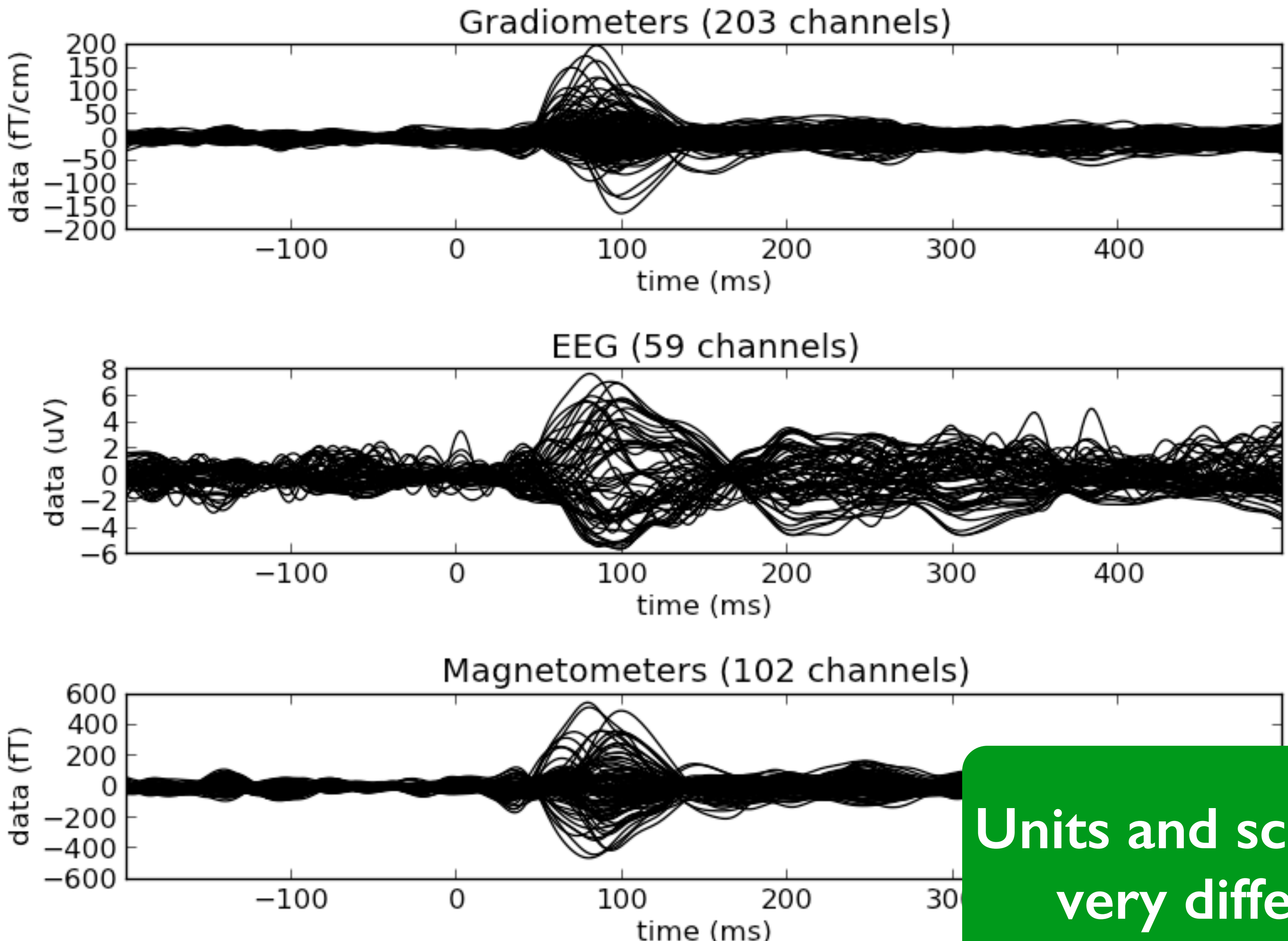
To know what is signal look at the noise



$$C = \frac{MM^T}{d_t}$$

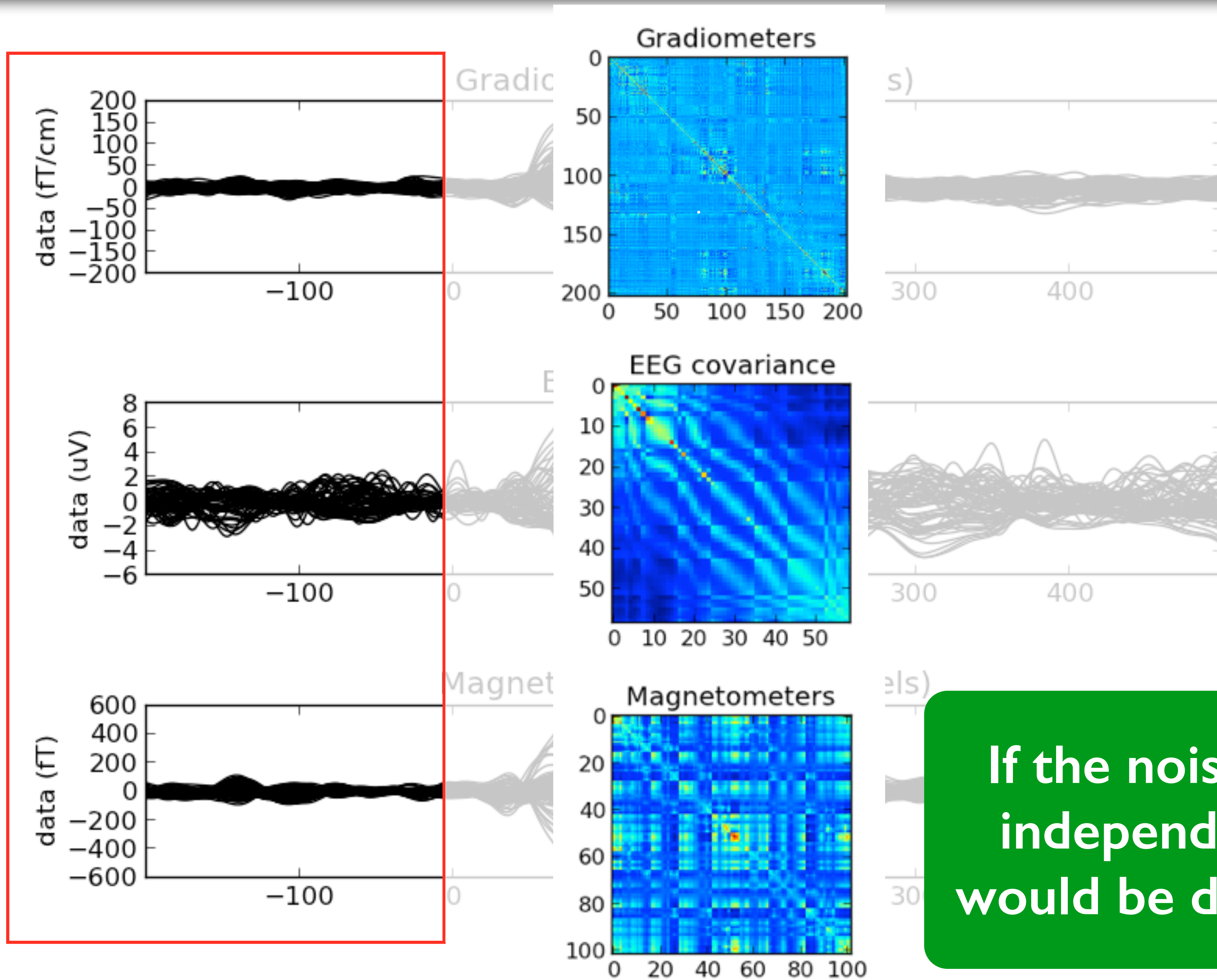
With whitened data the covariance would be diagonal

Spatial whitening



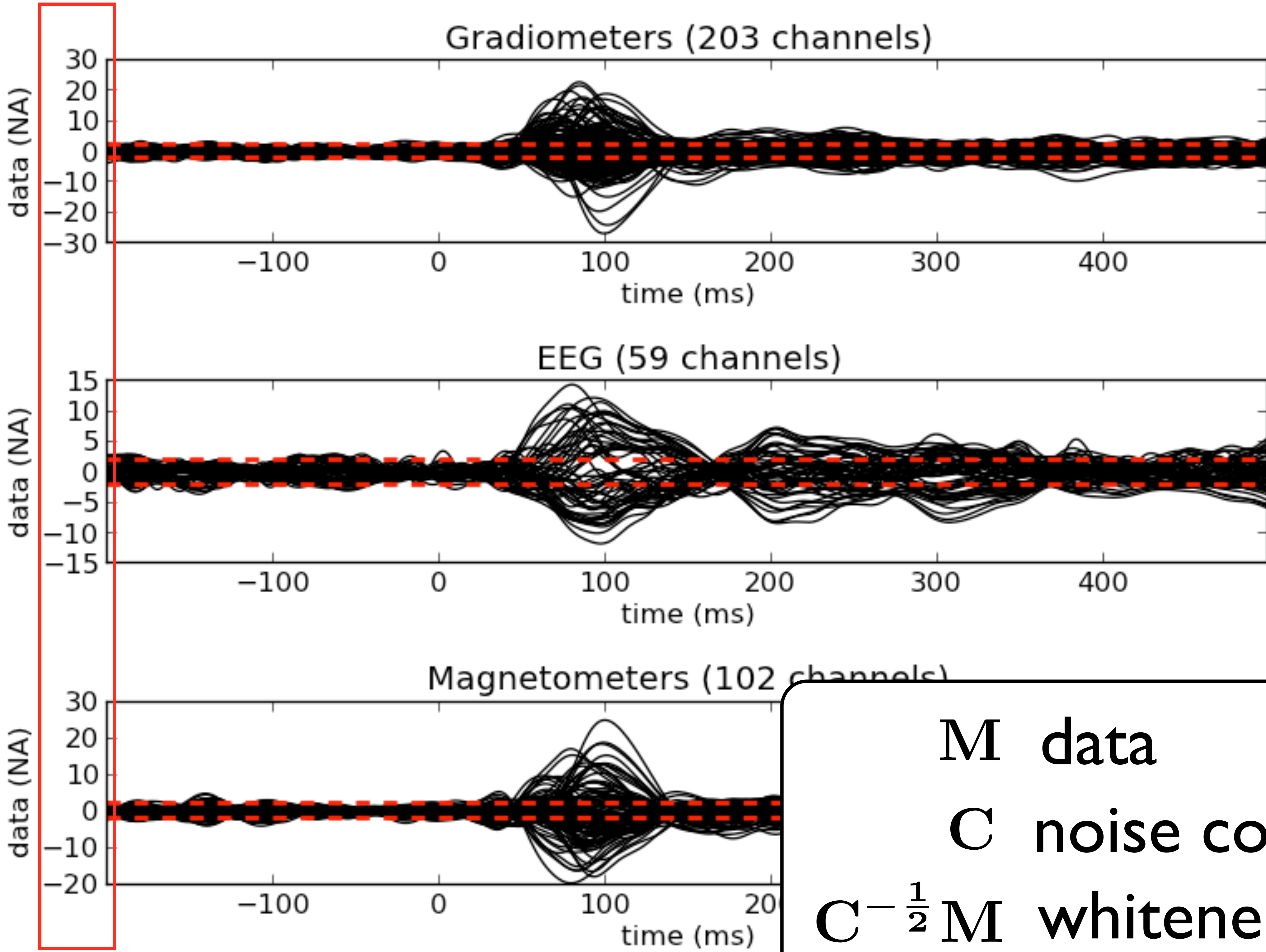
Units and scales are very different

Spatial whitening



If the noise was independent it would be diagonal

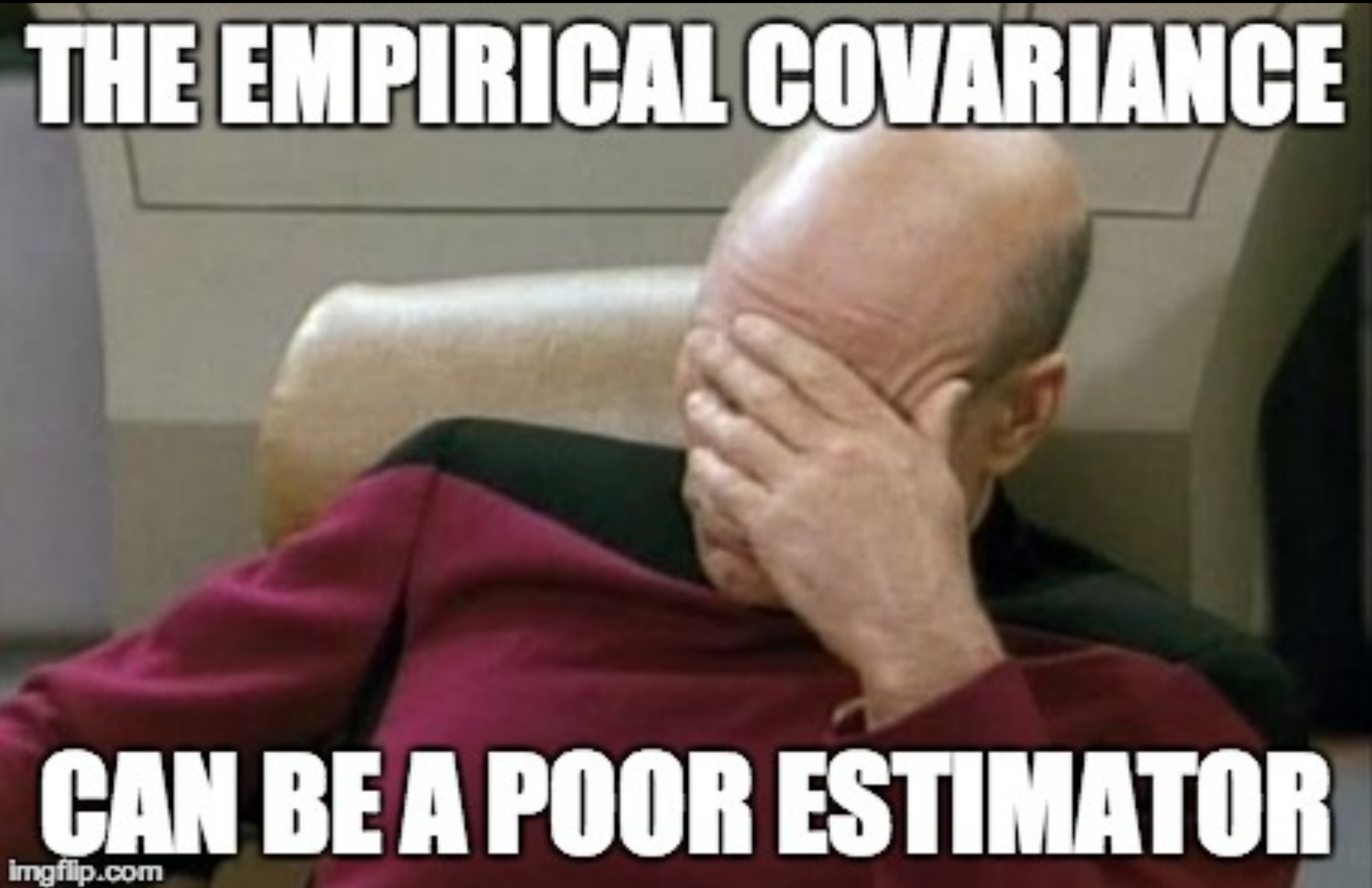
Spatial whitening



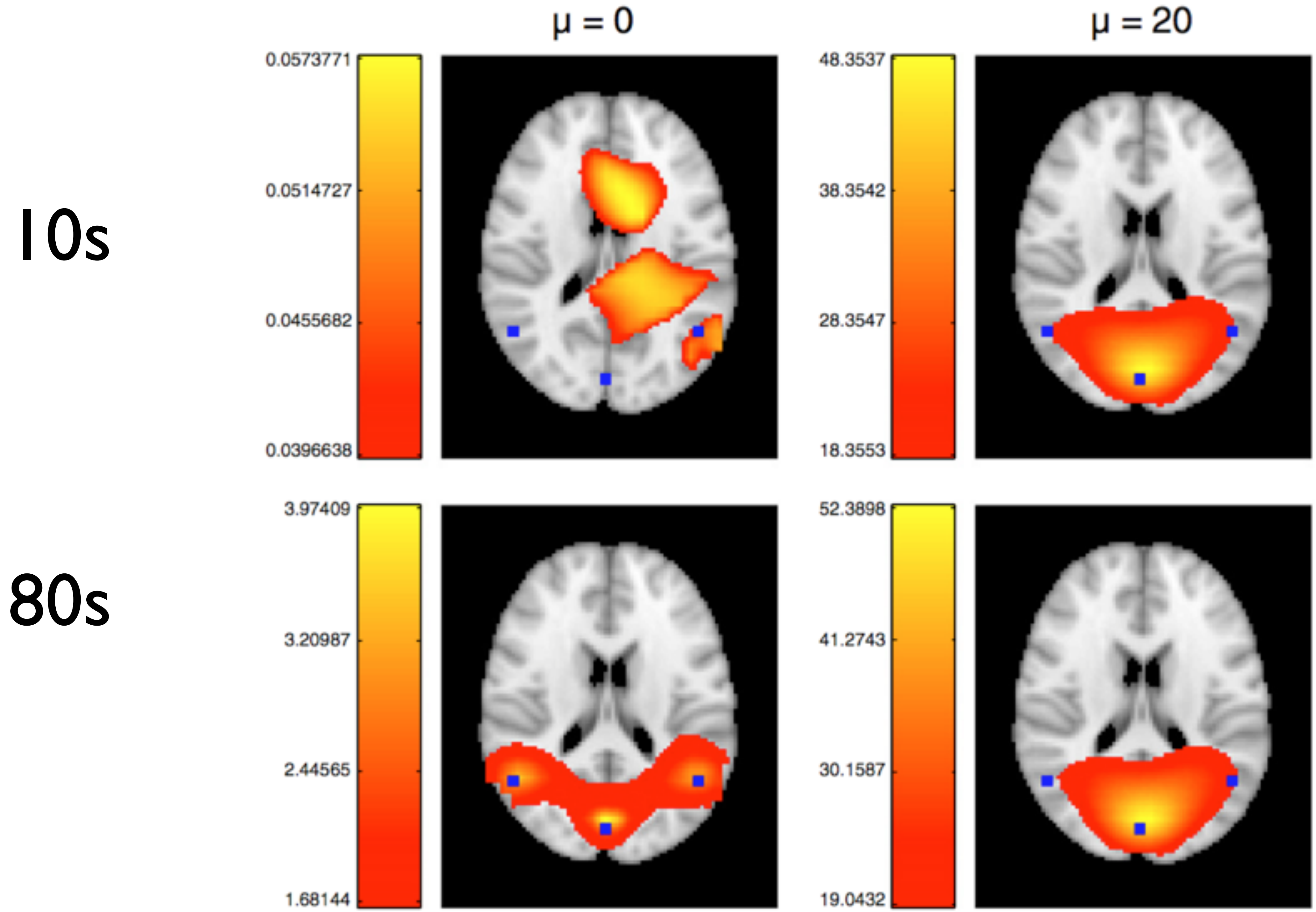
$\mathbf{M}$ data

$\mathbf{C}$ noise cov.

$\mathbf{C}^{-\frac{1}{2}}\mathbf{M}$ whitened data


THE EMPIRICAL COVARIANCE
CAN BE A POOR ESTIMATOR
imgflip.com

True sources VS LCMV Beamformer given C



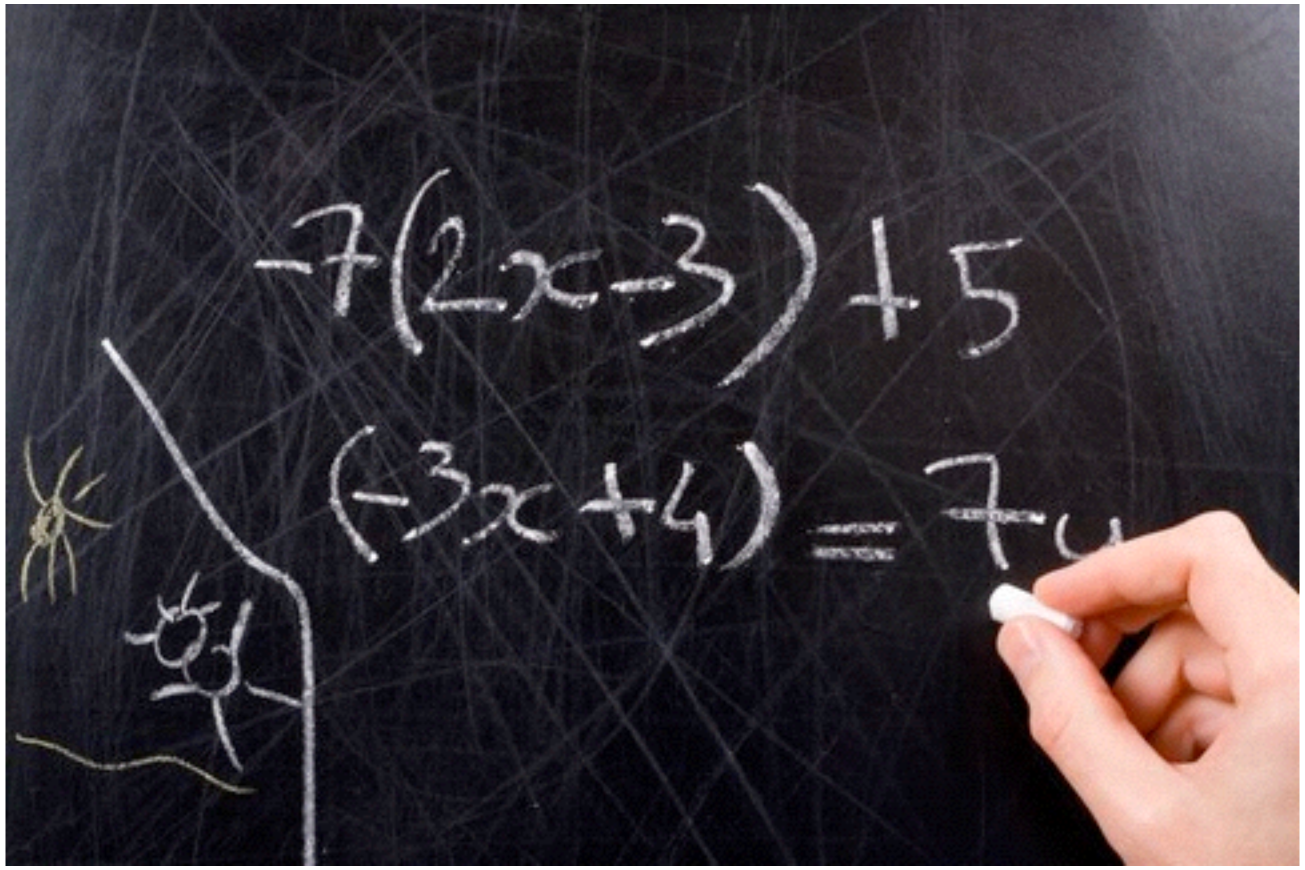
$$C_\mu = \mu\sigma_e^2 I + \frac{1}{T} M M^t$$

[Woolrich, 2011, Neuroimage]

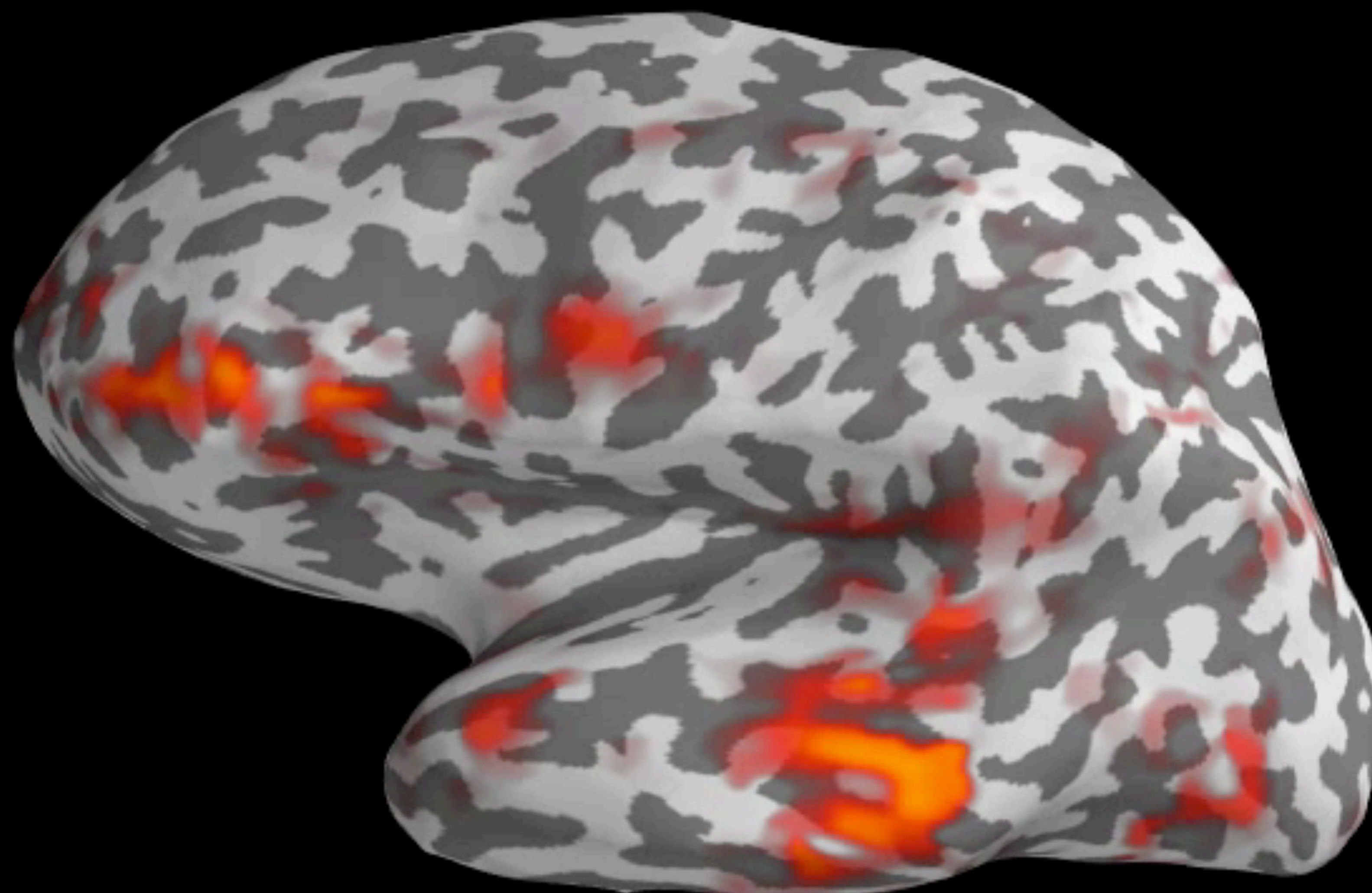
$\phi(\mathbf{X})$ with M/EEG data: L2

Simple L2 (Tikhonov):

$$\mathbf{X}^* = \arg\min_{\mathbf{X}} \mathbf{E}(\mathbf{X}) = \arg\min_{\mathbf{X}} \|\mathbf{M} - \mathbf{G}\mathbf{X}\|_F^2 + \lambda \|\mathbf{X}\|_F^2, \lambda > 0$$



http://youtu.be/Uxr5Pz7JPrs



time=0.00 ms

Alexandre Gramfort, Telecom ParisTech, CEA/Neurospin

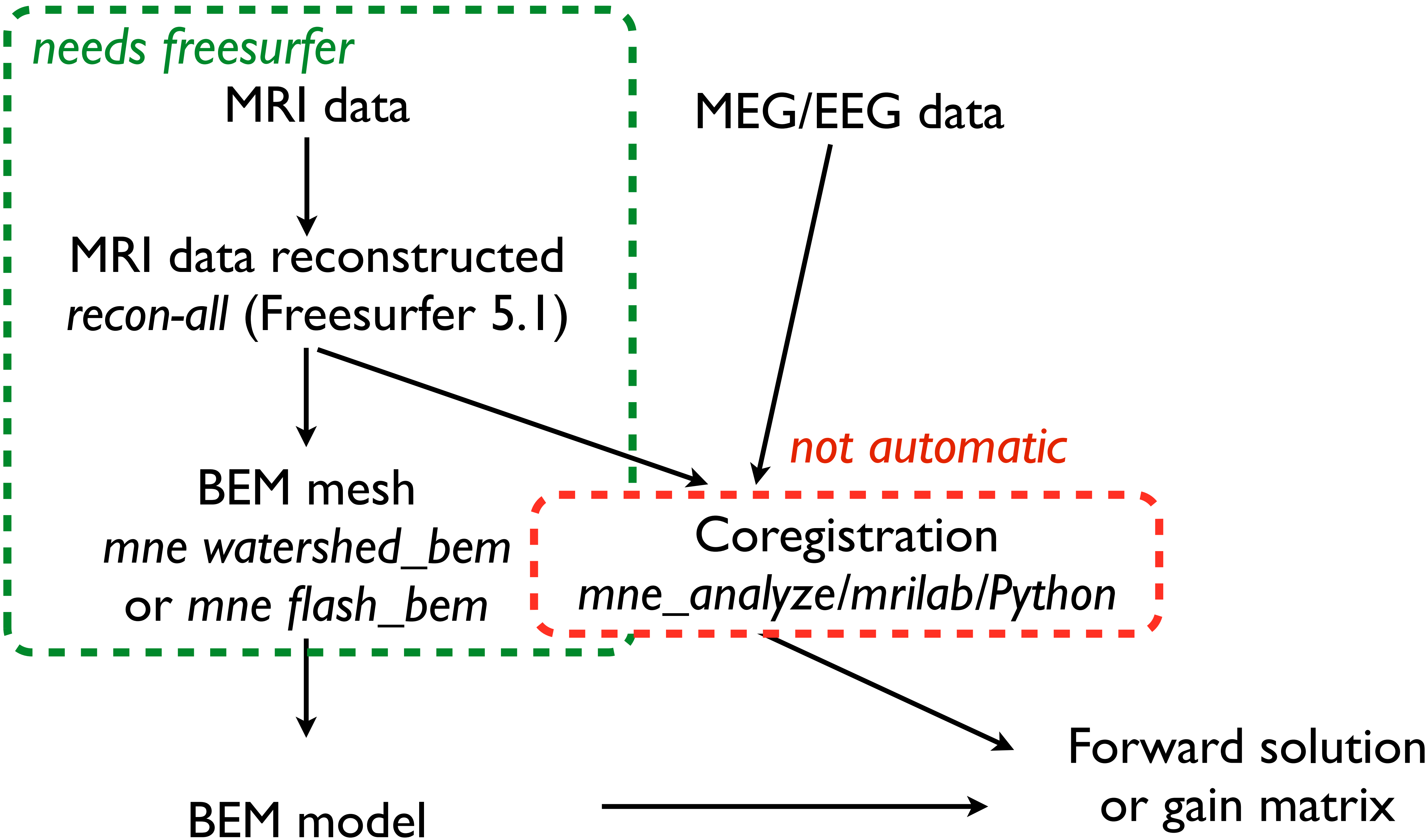
FreeSurfer recon

$ recon-all -s ${SUBJECT} -i xy000000.nii -all

and just wait…

or better go to bed and come back tomorrow…

Anatomy workflow



Demo