

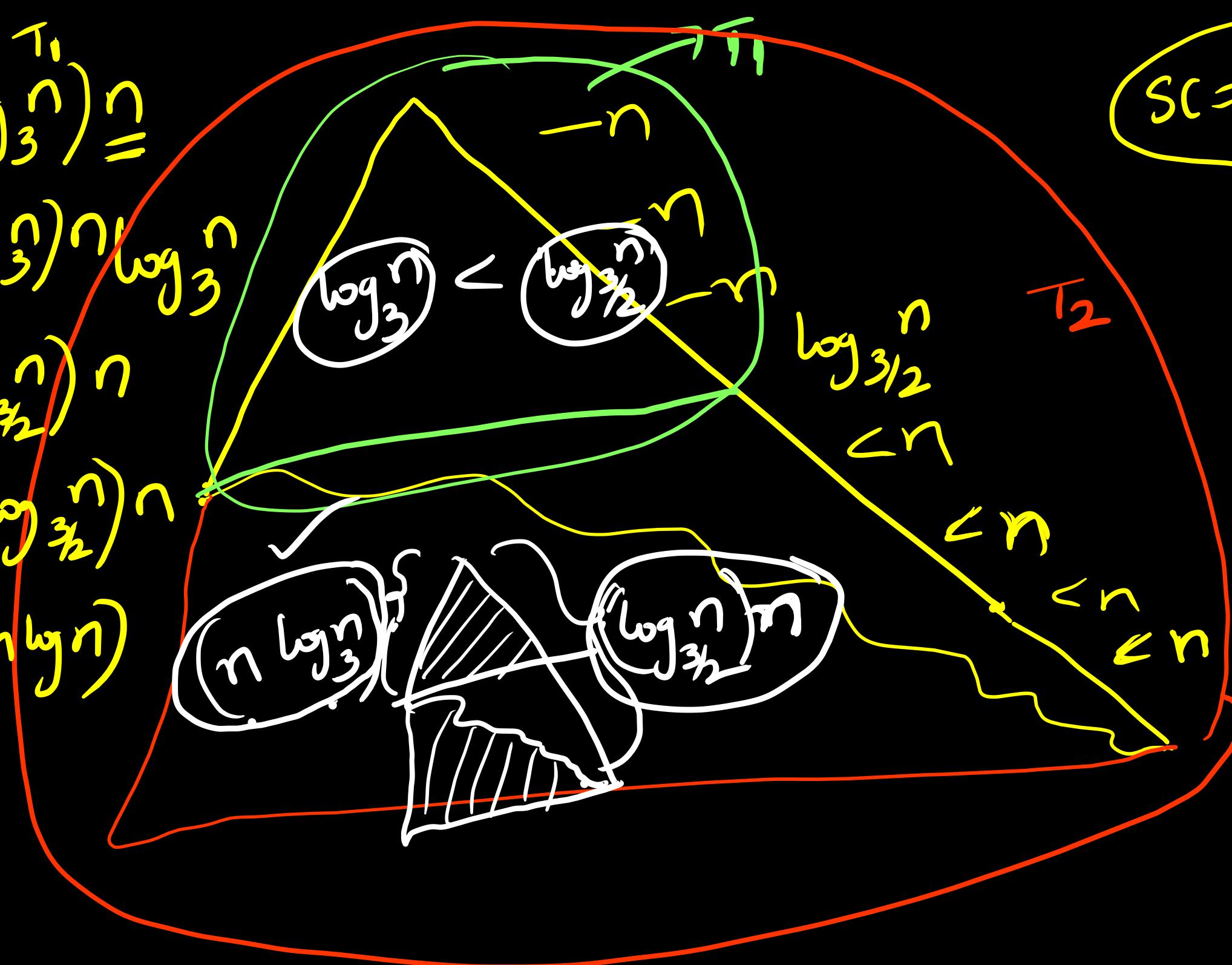
$$T(n) \geq (\log_3 n)^{T_1} \geq$$

$$\Omega(\log_3 n) \cancel{\sim} \log_3 n$$

$$T(n) \leq (\log_2 n)^c$$

$$\tau(n) = O\left(\frac{\log n}{\log \log n}\right)^c$$

$$T(n) = \Theta(n \log n)$$



$$SC = O\left(\log_{3/2} n\right) = O(\log_3 n)$$

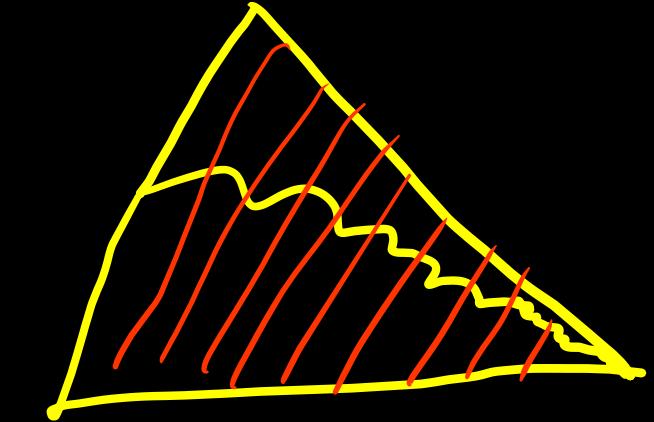
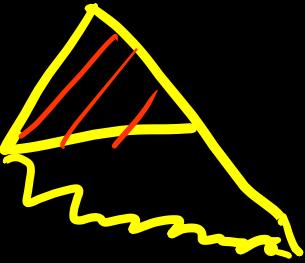
$\log_{3/2} n > \log_3 n$

lower the base  
higher the reac

$$\tau_1 < \tau < \tau_2$$

$$SC = \log_{3/2}^{\text{maximum depth}}$$

$$n \log_3 n \leq T(n) = n \log_{3/2} n$$



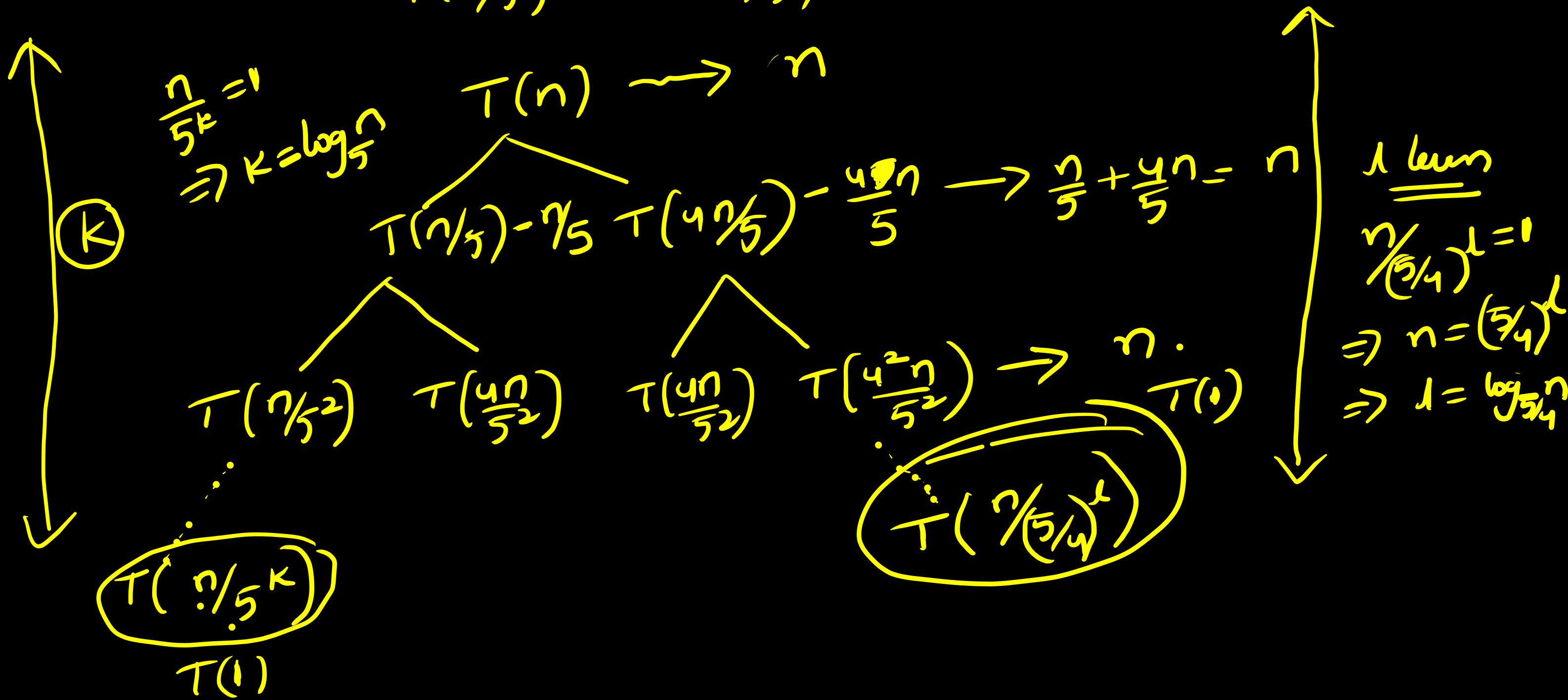
$$\begin{aligned} T(n) &= O(n \log_{3/2} n) \\ &= \Omega(n \log_3 n) \\ &= \Theta(n \log n) \end{aligned}$$

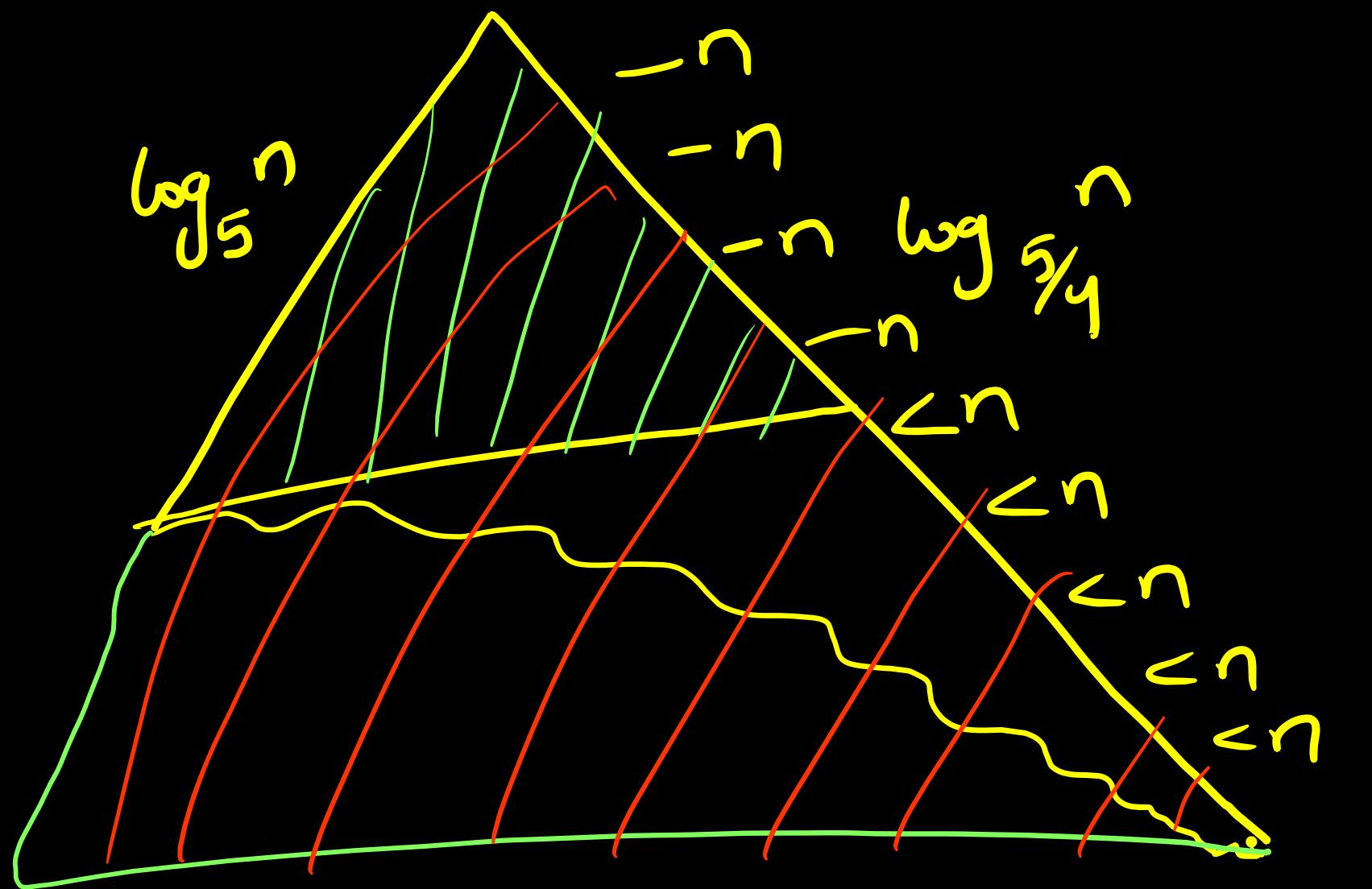
$$\text{Tree} - T(n) = \underbrace{T\left(\frac{n}{2}\right) + T\left(\frac{3n}{2}\right)}_{\text{True}} \rightarrow \text{True}$$

$$T(n) = \underbrace{T(n-1)}_{\text{one term}} + n \rightarrow \text{Subtree}$$

$$T(n) = aT\left(\frac{n}{b}\right) + n \rightarrow \text{master}.$$

$$T(n) = \begin{cases} 1 & n=1 \\ T(n/5) + T(4n/5) + n & \text{otherwise} \end{cases}$$





$$n \log_5 n \leq T(n) \leq n \log_{5/4} n$$

$$T(n) = O(n \log_{5/4} n)$$

$$T(n) = \Omega(n \log_{5/4} n)$$

$$T(n) = \Theta(n \log n)$$

$$S \in \overset{O}{\Omega}(\log_{5/4} n) = \overset{O}{\Omega}(\log n)$$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + n & \end{cases}$$

$\log_2 n = O(\log_3 n)$

$S(n) = O(\log_{10/9} n)$

$= O(\log n)$   $T(n) = O(n \log_{10/9} n)$

$T(n) = \Omega(n \log_{10} n)$

~~$\Theta$~~   $= \Theta(n \log n)$

$O(n \log_{10/9} n)$

$= O(n \log n)$

$\Omega(n \log_{10} n)$

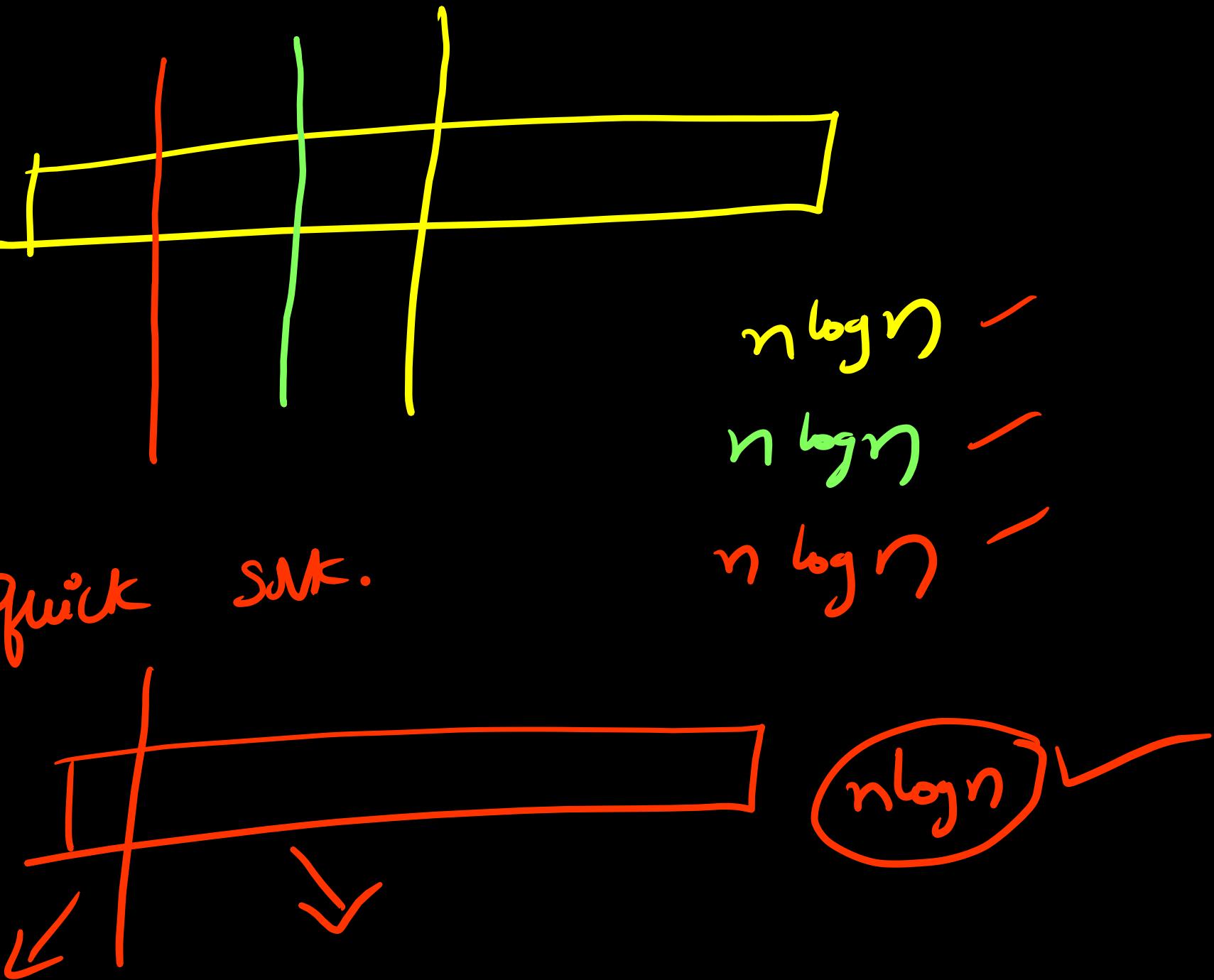
$= O(n \log n)$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T\left(\frac{n}{10}\right) + T\left(\frac{7n}{10}\right) + n & \text{otherwise} \end{cases}$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + n$$

• {  $T(c)=\Theta(n \log n)$   
 $SC=\Theta(\log n)$

$$T_C = \Theta(n \log n) \quad T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{4n}{5}\right) + n$$
$$SC = \Theta(\log n) \quad T_{AC} = \Theta(n \log n)$$
$$SC = \Theta(\log n) \quad T_{CC} = \Theta(n \log n)$$
$$T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + n$$



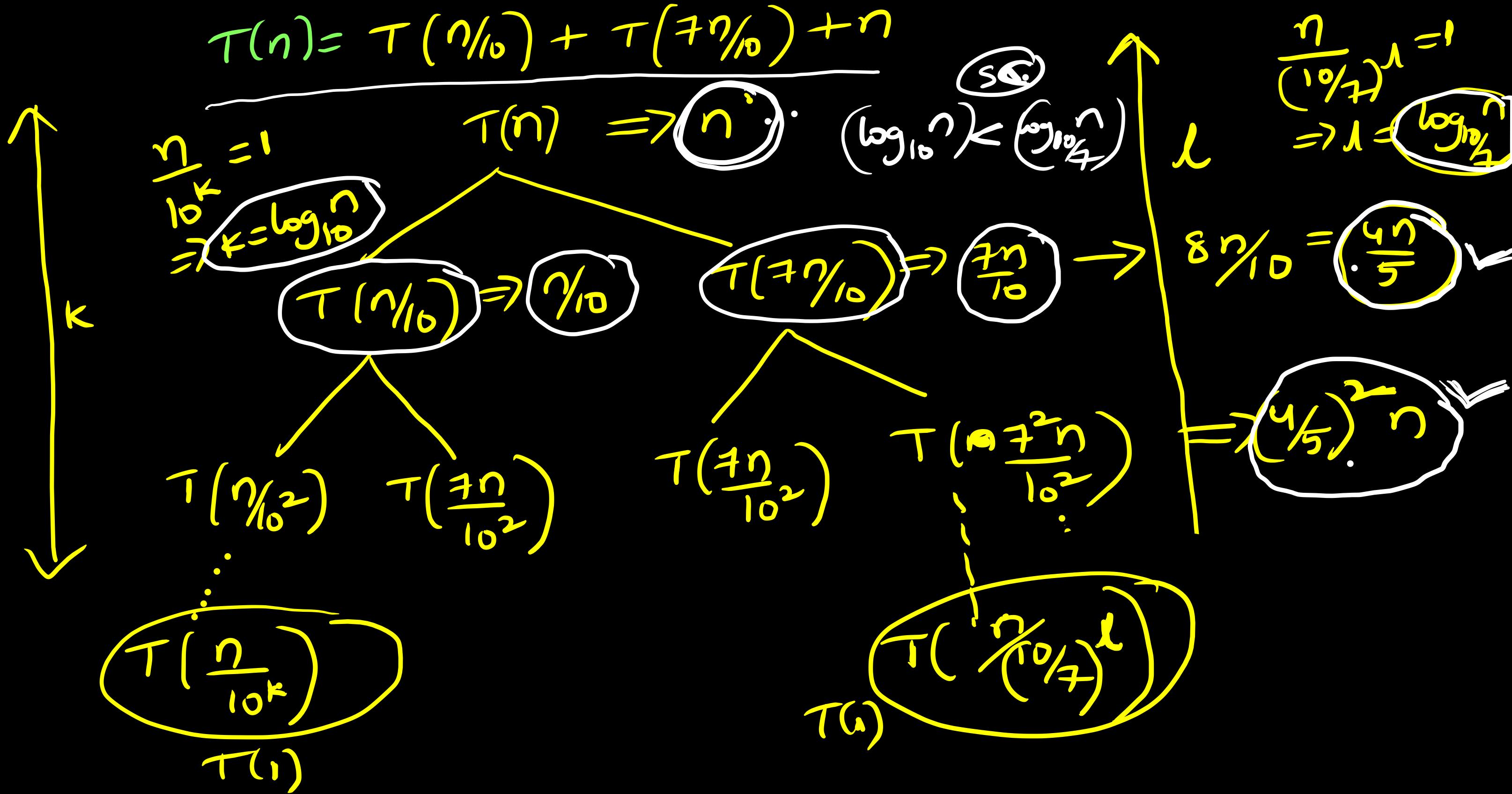
$$\tau(n) = \begin{cases} 1 & \text{if } n=1 \\ \tau(\frac{n}{10}) + \tau(7\frac{n}{10}) + n & \text{otherwise} \end{cases}$$

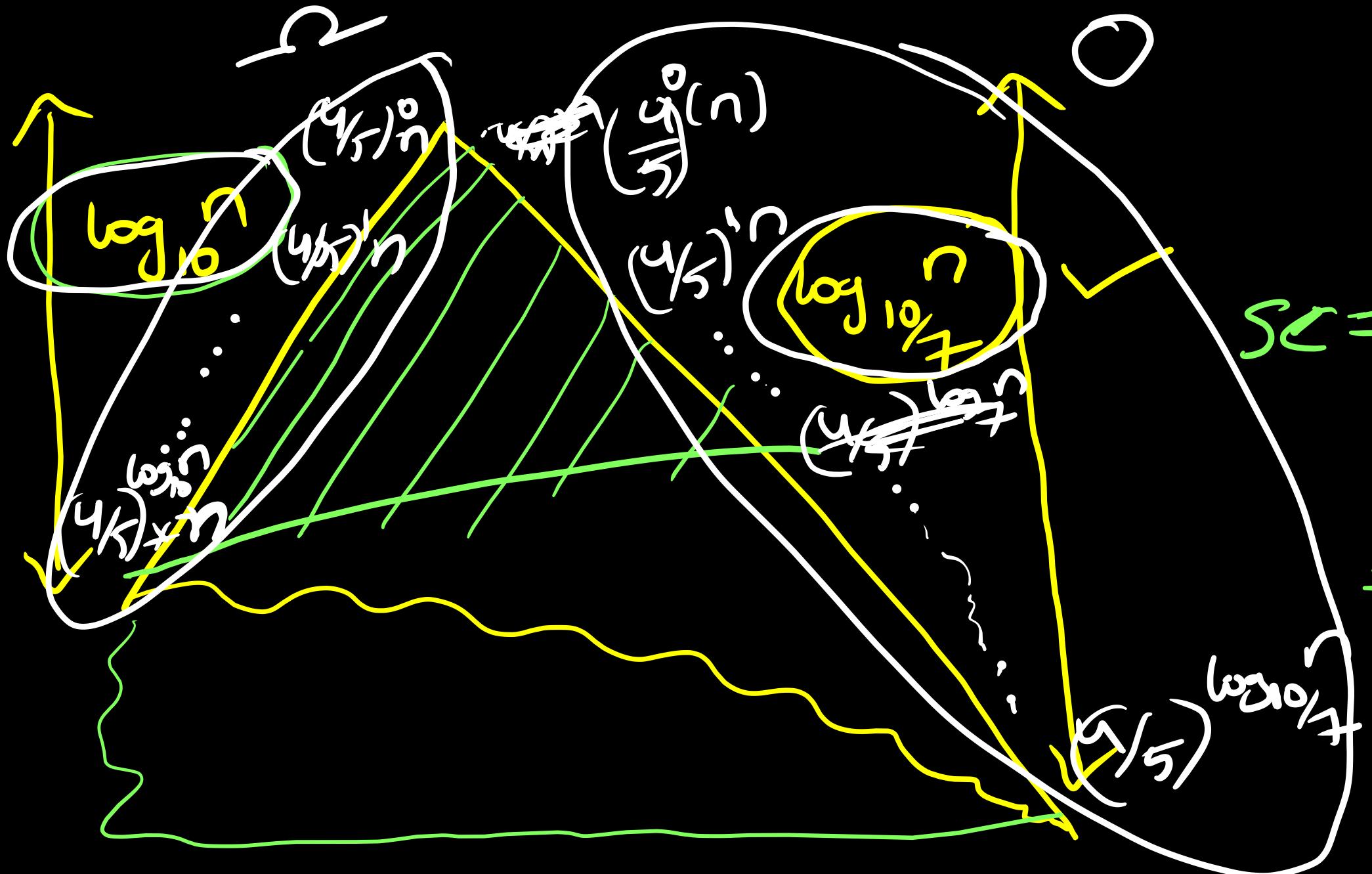
$$\frac{n}{10} + \frac{7n}{10} = \frac{8n}{10} \neq 0$$

$$\frac{n}{5} + \frac{4n}{5} = n \quad \}$$

$$\frac{n}{10} + \frac{9n}{10} = n.$$

5min block  
5min week.





$SC = \text{longest depth}$   
 $= O(\log_{10} n)$   
 $= O(\log n)$ .

$$T(n) \leq \left(\frac{4}{5}\right)^0 n + \left(\frac{4}{5}\right)^1 n + \left(\frac{4}{5}\right)^2 n + \dots + \left(\frac{4}{5}\right)^{\log_{10} \frac{n}{4}} n$$

$$n \left[ \left(\frac{4}{5}\right)^0 + \left(\frac{4}{5}\right)^1 + \left(\frac{4}{5}\right)^2 + \dots + \left(\frac{4}{5}\right)^{\log_{10} \frac{n}{4}} \right]$$

$$= n \left[ \frac{a(r^{n-1})}{r-1} \right]_{r=4/5} \quad r = 4/5 < 1 \quad \left[ \frac{a(1-r^n)}{1-r} \right]$$

$$= n \left[ \frac{1 - \left(\frac{4}{5}\right)^{\log_{10} \frac{n}{4}}}{1 - 4/5} + 1 \right] \rightarrow 0 \text{ as } n \rightarrow \infty$$

$$= O(n)$$

$$T(n) \geq (4/5)^0 n + (4/5)^1 n + (4/5)^2 n + \dots + (4/5)^{\log_{10} n}$$

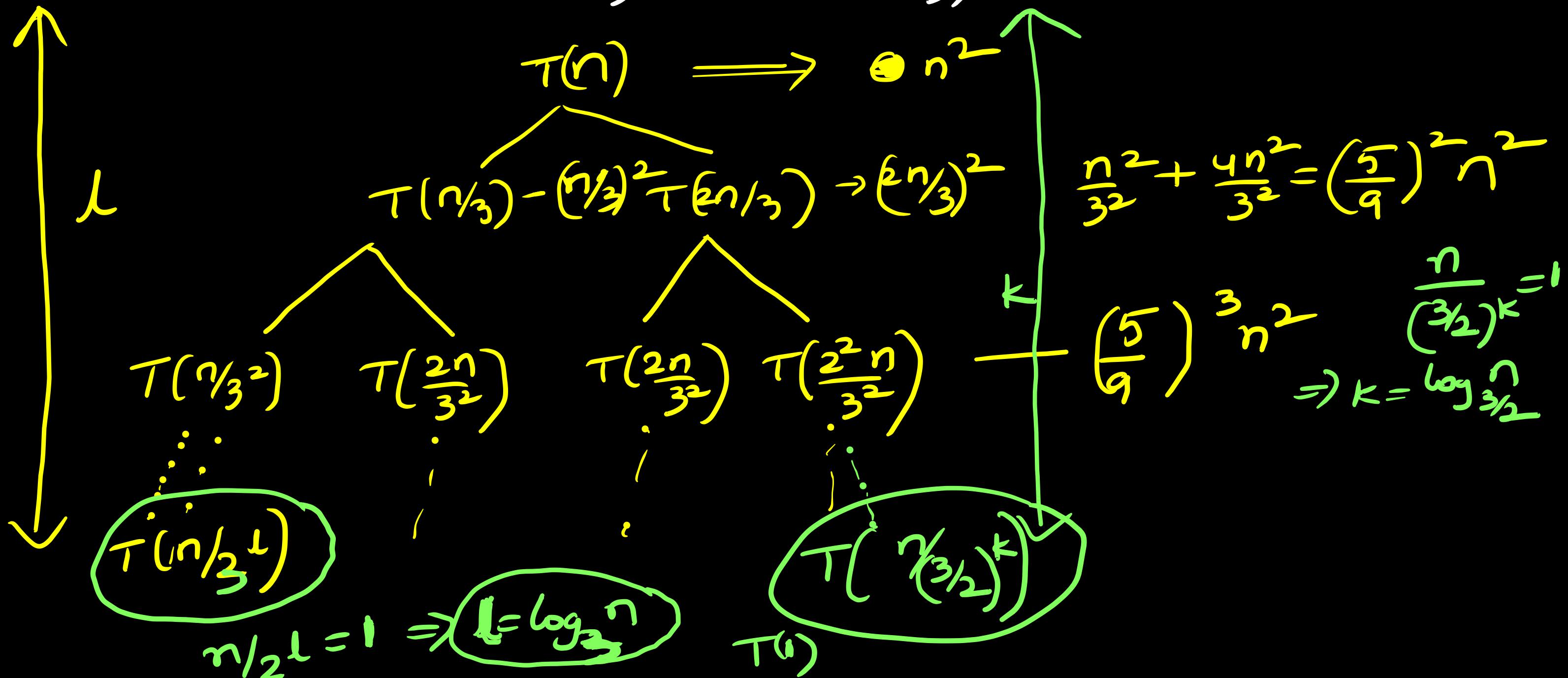
$$\begin{aligned} T(n) &= \Theta(n) \\ T(n) &\geq n((4/5)^0 + (4/5)^1 + (4/5)^2 + \dots + (4/5)^{\log_{10} n}) \\ &= \Theta(n) \\ SC &= \Theta(\log n) T(n) \geq n \left\{ \frac{1 - (1 - 4/5)^{\log_{10} n + 1}}{1 - 4/5} \right\} \rightarrow 0 \text{ as } n \rightarrow \infty \end{aligned}$$

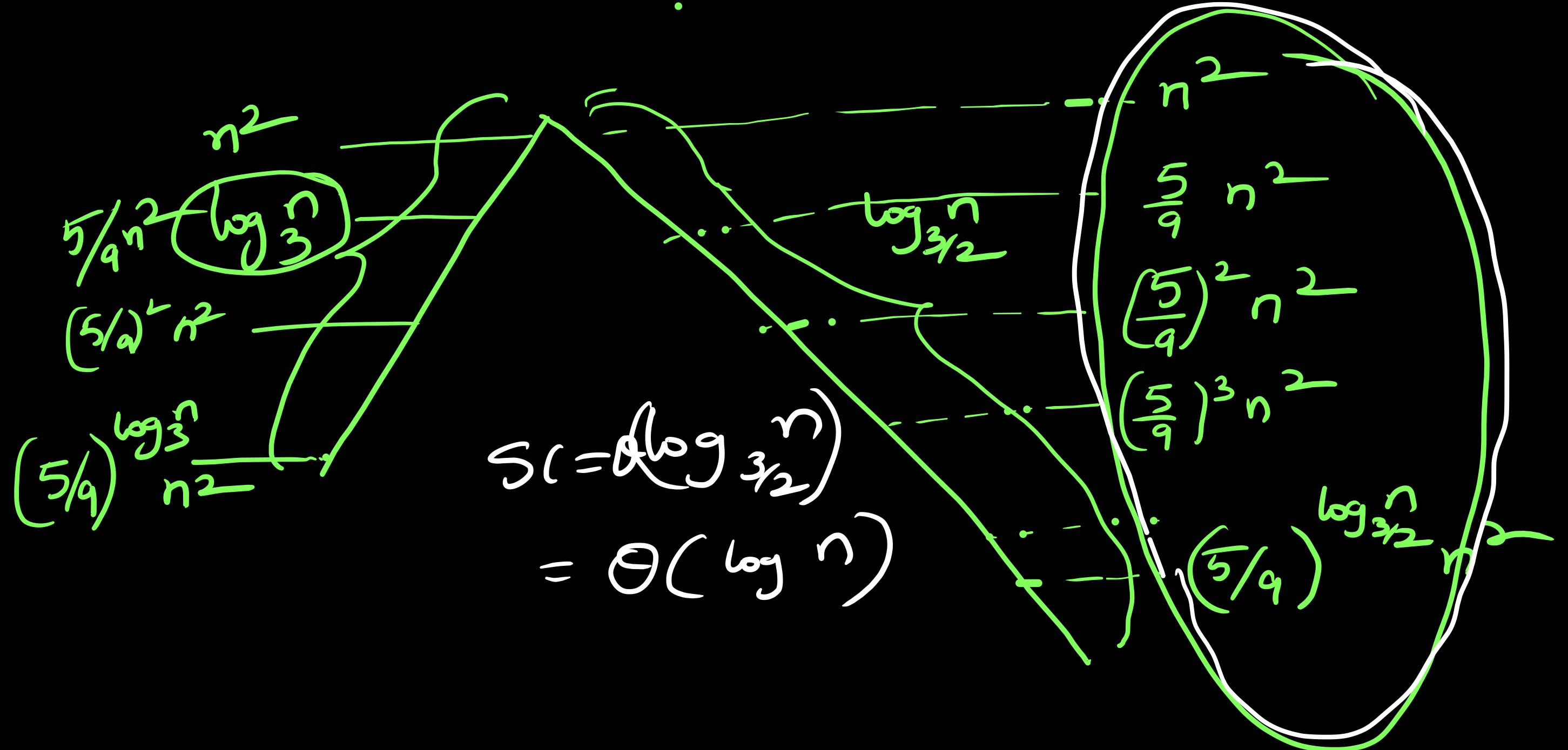
$$T(n) \geq n^c$$

$$T(n) = \Omega(n)$$

$$\begin{aligned} n^c &\geq (8/10)^n \\ \text{Let } & n \rightarrow \infty, (8/10)^n = 0 \quad r^K \rightarrow 0^- \\ & \gamma < 1 \quad \text{if } \gamma < 1 \text{ and} \\ & K \rightarrow \infty, \end{aligned}$$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n/3) + T(2n/3) + n^2 & \text{if } n>1 \end{cases}$$





$$T(n) \leq n^2 \left( (5/9)^0 + (5/9)^1 + \left(\frac{5}{9}\right)^2 + \dots + \left(\frac{5}{9}\right)^{\log_{3/2} n} \right)$$

GP  $\gamma < 1$

$$n^2 \left( 1 - \frac{(5/9)^{\log_{3/2} n} + 1}{1 - 5/9} \right) \rightarrow 0 \text{ as } n \rightarrow \infty$$

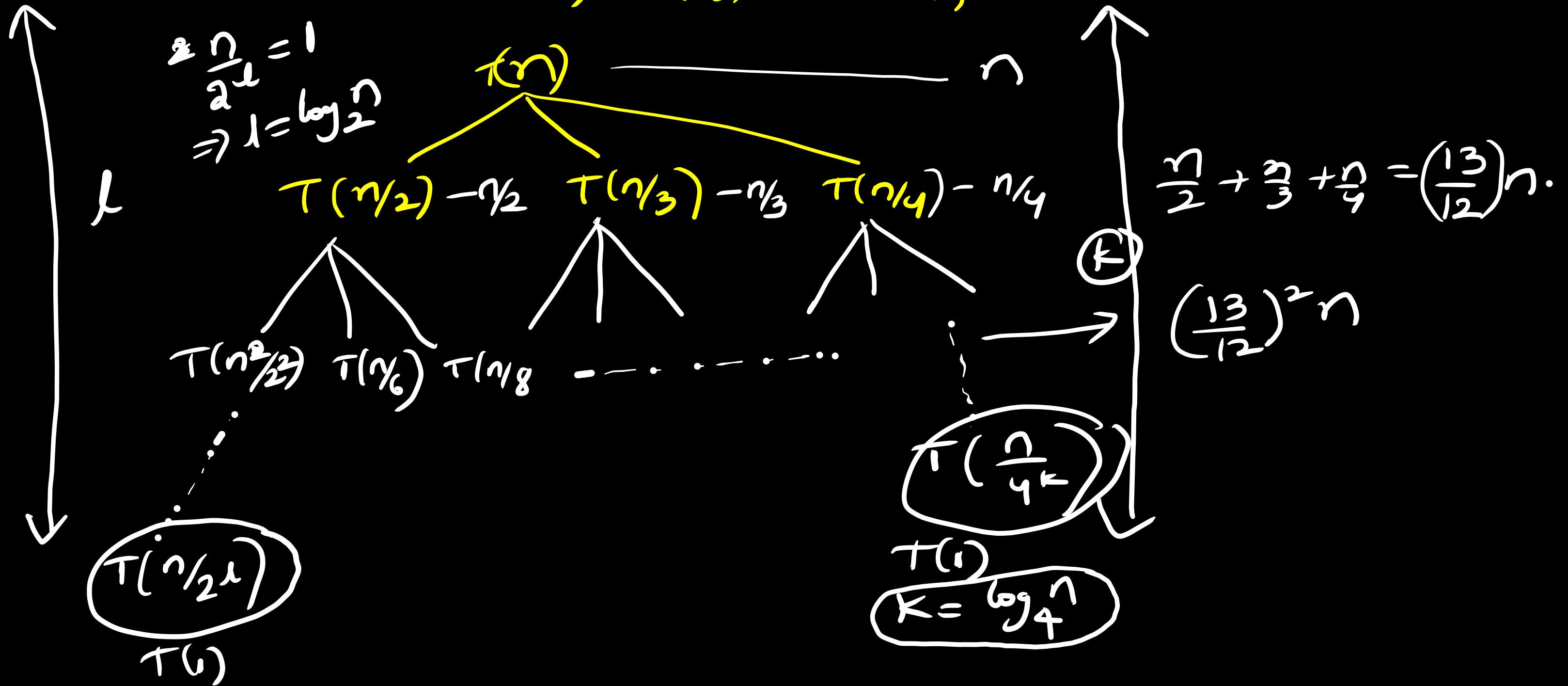
$$T(n) = O(n^2)$$

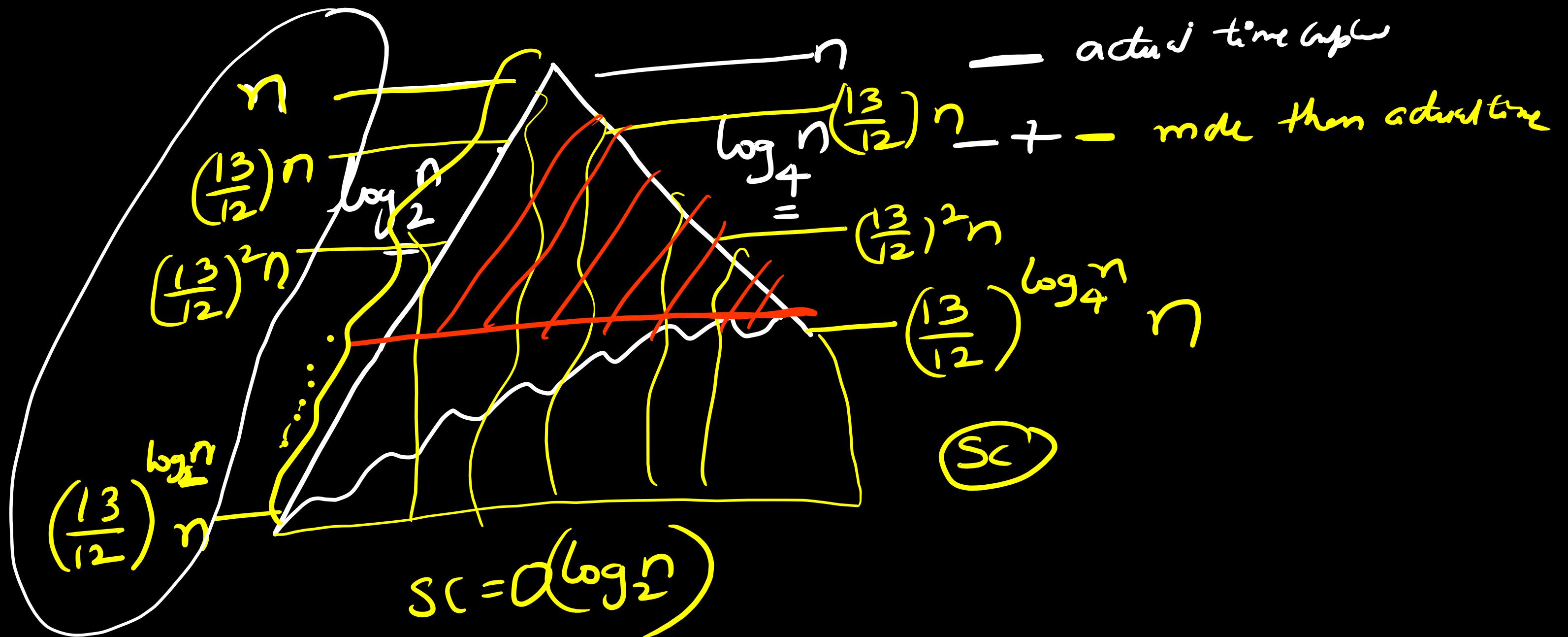
$$T(n) > n^2 \left( n(5/9)^0 + (5/9)^1 + \dots + (5/9)^{\log_{3/2} n} \right)$$

$$n^2 \left( 1 - \frac{(5/9)^{\log_{3/2} n} + 1}{1 - 5/9} \right) \rightarrow 0 \text{ as } n \rightarrow \infty$$

$$\underline{\Omega}(n^2)$$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n/2) + T(n/3) + T(n/4) + n & \text{if } n>1 \end{cases}$$





$$T(n) \leq n \left[ \left(\frac{13}{12}\right)^0 + \left(\frac{13}{12}\right)^1 + \dots + \left(\frac{13}{12}\right)^{\log_2 n} \right]$$

$$n \left[ \frac{a(r^n - 1)}{r - 1} \right] = n \left[ \frac{1 \left( \frac{13}{12}^{\log_2 n + 1} - 1 \right)}{\frac{13}{12} - 1} \right]$$

$$\begin{aligned} \log_3 n &= \log_{13} n \\ &= \log n \end{aligned}$$

~~a~~ dominating term  
 ~~$\frac{13}{12} \log_2 n$~~

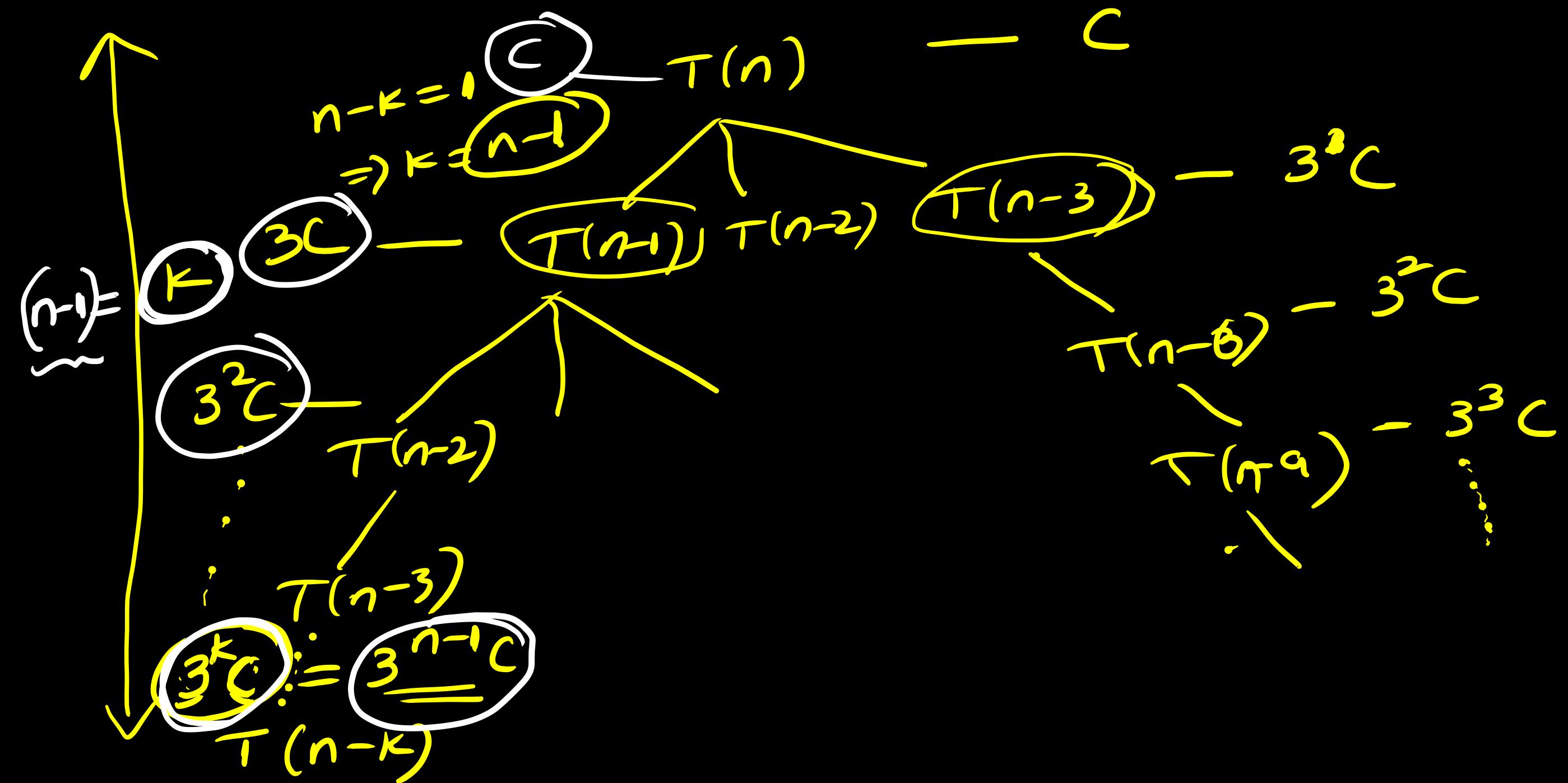
$$a^{\log_c b} = b^{\log_c a}$$

$$n \left( \frac{13}{12} \right)^{\log_2 n} = (n n^{\log_2 (\frac{13}{12})})$$

$$T(n) = O(n n^{\log_2 \frac{13}{12}}) \quad \checkmark$$

$$\begin{aligned}
T(n) &\geq n \left\{ \left(\frac{13}{12}\right)^0 + \left(\frac{13}{12}\right)^1 + \dots + \left(\frac{13}{12}\right)^{\log_4 n} \right\} \\
&\geq n \left( \frac{1 - \left(\frac{13}{12}\right)^{\log_4 n + 1}}{\frac{13}{12} - 1} \right) \\
&\geq n \left( \frac{13}{12} \right)^{\log_4 n} \\
&\geq n \cdot n^{\log_4 \frac{13}{12}} \\
&= \Omega(n \cdot n^{\log_4 \frac{13}{12}})
\end{aligned}$$

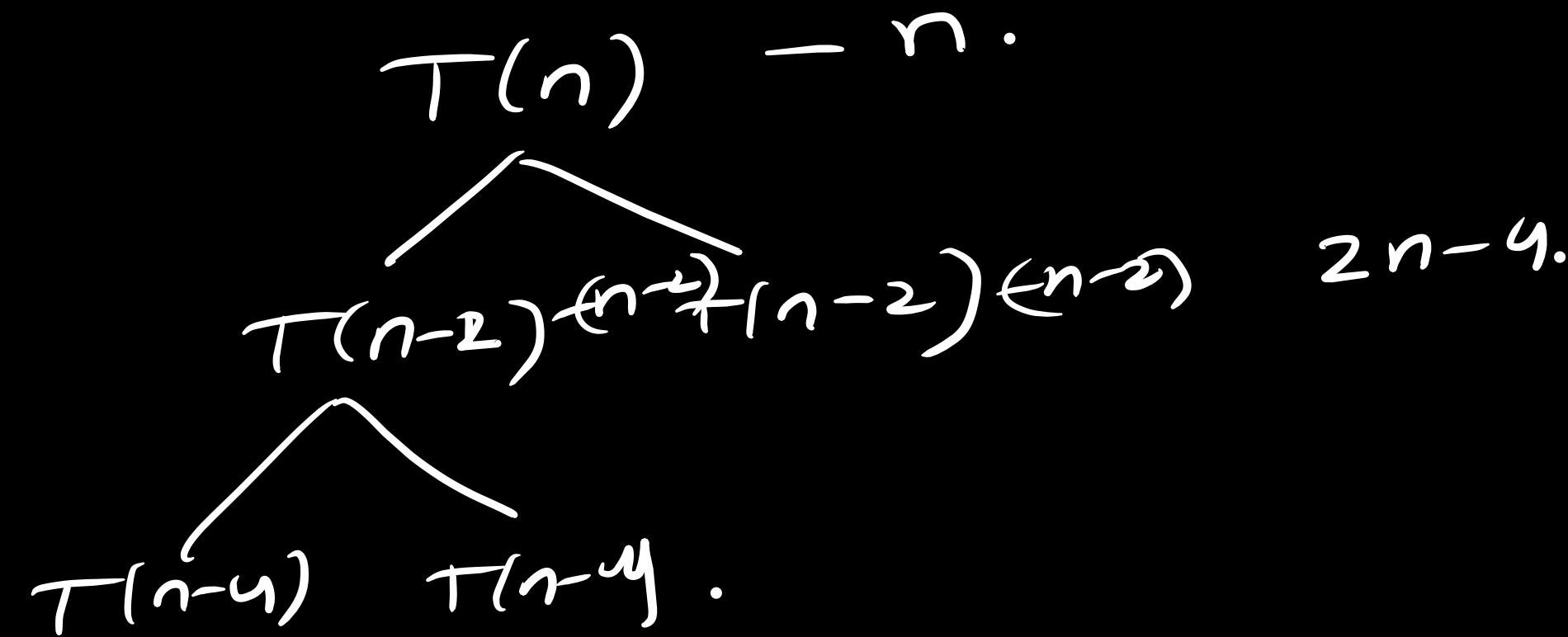
$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n-1) + T(n-2) + T(n-3) + C & \text{if } n>1 \end{cases}$$



$$\underline{\underline{SC = O(n)}}$$

$$\begin{aligned}
 T(n) &\leq 3^0 c + 3^1 c + 3^2 c + \dots + 3^{n-1} c \\
 &= c(3^0 + 3^1 + 3^2 + \dots + 3^{n-1}) \\
 &= c \left( \frac{3^n - 1}{3 - 1} \right) = c 3^n \\
 &\stackrel{\textcircled{S}\textcircled{C}}{=} O(3^n) \\
 T(n) &= O(3^n) \\
 &= \Omega(3^n) \\
 &= \Theta(3^n).
 \end{aligned}$$

$$T(n) = 2 * T(n-2) + n.$$



$$\begin{aligned} T(n) &= T(n-1) + T(n-2) + C \quad n \geq 1 \\ &= 1 \quad ; \quad n = 1. \end{aligned}$$