Stochsatic gradient descent for logistic regression smoothing

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1 The log-logistic regression model

We consider the four parameter log-logistic dose-response model of the real-valued random variable, Y given x. The model is defined as

$$f(x|\alpha,\beta,\gamma,\rho) = \gamma + \frac{\rho-\gamma}{1+e^{\beta\log(x)-\alpha}}$$

With $\alpha,\beta,\gamma,\rho \in \mathbb{R}$. We observe the noised variables

$$Y_i = f(x_i|\alpha,\beta,\gamma,\rho) + \varepsilon_i$$

for $i = 1,\dots,N$ and $\varepsilon_i \sim \mathcal{N}\left(0,\sigma^2\right)$ independent. We estimate $(\alpha,\beta,\gamma,\rho) \in \mathbb{R}^4$ by minimizing the non-linear least squares

$$H_N = \frac{1}{N}\sum_{i=1}^{N}(y_i - f(x_i|\alpha,\beta,\gamma,\rho))^2 = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \gamma + \frac{\gamma-\rho}{1+e^{\beta\log(x_i)-\alpha}}\right)^2$$

We calculate the gradient of H_N by computing the partial derivatives w.r.t the all the parameters.

$$\begin{aligned}
\frac{\partial H_N}{\partial\alpha} &= \frac{2}{N}\sum_{i=1}^{N}\left(y_i - \gamma + \frac{\gamma-\rho}{1+e^{\beta\cdot\log(x_i)-\alpha}}\right)\frac{(\gamma-\rho)e^{\beta\log(x_i)-\alpha}}{\left(e^{\beta\log(x_i)-\alpha}+1\right)^2} \\
\frac{\partial H_N}{\partial\beta} &= -\frac{2}{N}\sum_{i=1}^{N}\left(y_i - \gamma + \frac{\gamma-\rho}{1+e^{\beta\cdot\log(x_i)-\alpha}}\right)\frac{(\gamma-\rho)\log(x_i)e^{\beta\log(x_i)-\alpha}}{\left(e^{\beta\log(x_i)-\alpha}+1\right)^2} \\
\frac{\partial H_N}{\partial\gamma} &= \frac{2}{N}\sum_{i=1}^{N}\left(y_i - \gamma + \frac{\gamma-\rho}{1+e^{\beta\cdot\log(x_i)-\alpha}}\right)\left(\frac{1}{1+e^{\beta\log(x_i)-\alpha}} - 1\right) \\
\frac{\partial H_N}{\partial\rho} &= -\frac{2}{N}\sum_{i=1}^{N}\left(y_i - \gamma + \frac{\gamma-\rho}{1+e^{\beta\cdot\log(x_i)-\alpha}}\right)\left(\frac{1}{1+e^{\beta\log(x_i)-\alpha}}\right)
\end{aligned}$$