专题七 多元函数微分学

1. 设$z=xf(\frac{x}{y})+2yf(\frac{y}{x})$,其中$f(x)$在$(-\infty,+\infty)$上由连续的二阶导数,求$z_{xy}$

2. 设f二阶可微, $u(x,y,z)=f(r), r=\sqrt{x^2+y^2+z^2}$. 若$u$满足方程

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$, 试求函数$u$.

3. 设$f(x,y)$为n次齐次方程, 即$\forall t>0, f(tx,ty)=t^n f(x,y)$, 且$f$可微. 证明：$(x,y)\neq(0,0)$处有$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf(x,y)$

4. 设$f(u,v)$有二阶连续偏导数, 且满足$Laplace$方程$\frac{\partial^2 f}{\partial u^2}+\frac{\partial^2 f}{\partial v^2}=0$.证：

$z=f(x^2-y^2,2xy)$也满足$Laplace$方程$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$

5. 证：在$u=\frac{x}{y}, v=x, w=xz-y$之下，方程$yz_{yy}+2z_y=\frac{2}{x}$可变成$W_{uu}=0$

6. 设$z=u(x,y)e^{\alpha x+\beta y}$,其中$u(x,y)$满足$u_{xy}=0$,确定$\alpha,\beta$使下述等式成立：

$z_{xy}-z_x-z_y+z=0$

7. 方程$F(\frac{x}{z},\frac{y}{z})=0$确定隐函数$z=z(x,y)$. 求$xz_x+yz_y$

8. 求$f(x,y)=(x+y)e^{-x^2-y^2}$的最大值与最小值

9. 求$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$在第一卦限的切平面与三个坐标平面围成的四面体的最小体积

10. 求坐标原点到曲线$\Gamma:\begin{cases}x^2+y^2-z^2=1\\2x-y-z=1\end{cases}$的最短距离

11. 求$f(x,y)=x^2+\sqrt{2}xy+2y^2$在$x^2+2y^2\leq 4$上的最大值与最小值

12. 证：当$x \geq 0, y \geq 0$时，$e^{x+y-2} \geq \frac{1}{12}(x^2+2y^2)$