# homework 5

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## Problem 1

This is a P problem.

The given DNF Boolean formula is $DNF$.

```
for each clause in DNF:
    for each term in clause:
        if term conflicts with any other terms in this clause:
            then: jumpout this for loop
        else:
            if current term is the last term in this clause:
                return here is a truth assignment for X, such that DNF evaluates
to be 1
```

## Problem 2

This is  a **NP-complete problem**.

1. Prove it is in **NP**

   The given graph $G = (V, E)$. Suppose the test solution is $S$.

   ```
   k = 5
   for each node n in S:
       remove every edge adjacent to v from set E
       k = k - 1
       if k >= 0 and E is empty:
           then: S is a vertex cover in G
       else: S is not a vertex cover in G
   ```

2. Prove it is in **NPC**

> I have read note "16 Some Examples on Polynomial Reduction" when I solve this problem.

We'll use a polynomial-time reduction from Independent Set to Vertex Cover.

- Input reduction

  Given a graph $G = (V, E)$, independent set's parameter is $k$ and vertex cover's parameter is $k' = n - k$.

- Proof of "equivalence"

  $\rightarrow$: Suppose that $S$ is an independent set in $G$. For any edge $e = (u, v) \in E$, we know that $u$ and $v$ will not be both in $S$. Namely, for any edge $e = (u, v) \in E$, either $u$ in $V - S$ or $v$ in $V - S$, which means that $V - S$ is a vertex cover.

  $\leftarrow$: Suppose that $V - S$ is an vertex cover in $G$. Suppose that there is an edge $e = (u, v) \in E$, and both $u, v$ are in $S$. Then there is a contradiction, because at least one of the nodes on either side of any edge should be in $V - S$. Only this way, $V - S$ will be a vertex cover. As a result, $V$ is a independent set.

# Problem 3

1. Prove it is in **NP**

   Suppose the test solution is $C$ and the length of $C$ is larger than $K$. The verification algorithm is very simple:

```
for each product in store:
    for each customer in C:
        if there are more than one customers in C buy this product:
            return wrong soluion
return true solution
```

2. Prove it is in **NPC**

   We'll use a polynomial-time reduction from Independent Set to this problem.

   - Input Reduction

     Given a graph $G = (V, E)$, each customer has a vertex. For each product, connect customers who have bought this product with each other in $G$. The parameter is $k$.

   - Proof of "equivalence"

     $\rightarrow$: Suppose that $S$ is an independent set in $G$. For any two vertexes $a, b$ in $S$, we know that there will not be an edge between $a$ and $b$, which means that for any two customers in $S$, they have not bought same products.

     $\leftarrow$: Suppose that $S$ is a solution of this problem. For any two customer in $S$, we know that there will not be an edge between them, which means that this is an independent set.

# Problem 4

1. Prove it is **NP**

   Given a graph $G = (V, E)$. Suppose the test solution is $S$.

```
for each node n in S:
    for each node n' in V:
        if e = (n, n') in E:
            for each node n'' in S:
                if e = (n', n'') in E:
                    return wrong solution
return true solution
```

2. Prove it is in **NPC**

   For every edge $e = (u, v) \in E$, we can find a set $S_v \in V$ that for every $s_{vi}$ in $S_v$, there exist $e_i = (v, s_{vi}) \in E$. Then for every $s_{vi}$ in $S_v$, we'll add an edge $(u, S_{vi})$ to E. Finally we will have a new graph $G' = (V, E')$.

   Then apparently, when we try to find an independent set in $G'$, we are finding a strongly independent set in G.

# Problem 5

1. Prove it is **NP**

   Suppose the test solution is $S$.

   1. Check if $S$ contains more than $k$ jobs.
   2. Check if any two jobs in $S$ have a contradiction.
2. Prove it is in **NPC**

   We'll use a polynomial-time reduction from Independent Set to this problem.

   - Input Reduction

     Given a graph $G = (V, E)$, each jobs has a vertex. For each job, connect it to every other jobs in $G$ which has contradiction with this job. The parameter is $k$.

   - Proof of "equivalence"

     $\rightarrow$: Suppose that $S$ is an independent set in $G$. For any two vertexes $a, b$ in $S$, we know that there will not be an edge between $a$ and $b$, which means that for any two jobs in $S$, they will not have contradiction.

     $\leftarrow$: Suppose that $S$ is a solution of this problem. For any two jobs in $S$, we know that there will not be an edge between them, which means that this is an independent set.

# Problem 6

- Algorithm

  Given a graph $G = (V, E)$ and a black-box function $f(g)$, that $g$ is a graph as a parameter of $f$ and $f(g)$ return if there exist a Hamiltonian cycle in $g$. $|V| = m, |E| = n$

```
G = (V, E)
E' = E
G' = (V, E')
for each edge e in E:
    // in this case, G' is a Hamiltonian cycle.
    if |E'| == |V|:
        then return G' = (V, E')

    // try to remove every edge and test if there still exist a Hamiltonian
cycle.
    G_test = (V, (E-e))
    if f(G_test) == true:
        then: remove e in E'
    else: continue to next round
```

- Time complexity

  Suppose the black box function $f(g)$'s time complexity is x. Then the overall time complexity is $O(nx)$.