Fall 2019 B503 Homework 1

Due date: September 14, 11:59PM   Lecturer: Qin Zhang

Your Name:________________   Your University ID:________________

Instruction: Please submit your homework solution **before due date, via Canvas**. Homework solution must be typesetted in PDF format via LaTeX (preferred) or Word. Please add references to ALL the resources you have used for completing the assignment. You are allowed to discuss the assignment with other students, and if you do so, please list their names in the submission.

Due: September 14, Sunday, 11:59PM

Total points: 70

Late Policy: No extensions or late homeworks will be granted, unless a request is made to the course instructor before due date and written documents are provided to support the reason for late submission.

Problem 1 (10 points). Please prove the following claim: When applying the algorithm we learned form the class to solve stable matching problem, if men propose, every woman gets her worst valid partner (according to her own preference).

Problem 2 (10 points). A polygon is convex if all of its internal angles are less than 180 degree (and none of the edges cross each other). Figure 1 shows an example.

We represent a convex polygon as an array $V[1, \ldots, n]$ where each element of the array represents a vertex of the polygon in the form of a coordinate pair (x, y). We are told vertex $V[1]$ is the vertex with the minimum x coordinate and that the vertices $V[1...n]$ are ordered counterclockwise, as in the figure. You may also assume that the x coordinates of the vertices are all distinct, as are the y coordinates of the vertices.

Give an algorithm to find the vertex with the maximum y coordinate in $O(\log n)$ time.

Problem 3 (10 points). You are given a binary tree $T = (V, E)$, along with a designated root node $r \in V$. You wish to preprocess the tree so that queries of the form "is u an ancestor of v?" can be answered in $O(1)$ time. The preprocessing itself should take $O(n)$ (linear in terms of the number of vertices n) time. Please describe and analyze your preprocessing and query algorithm.

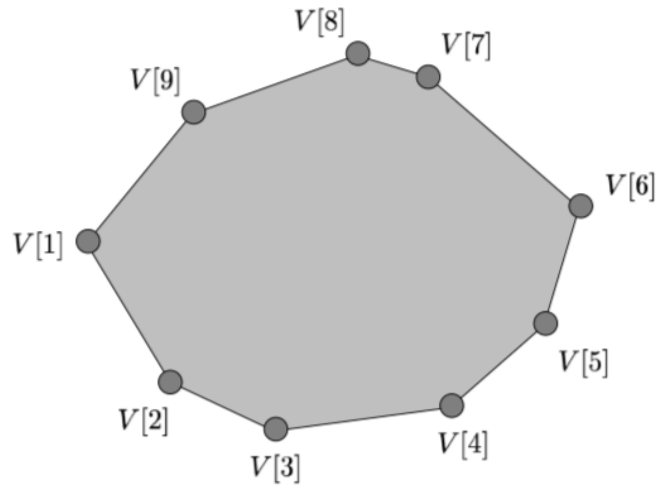

Figure 1: An example of a convex polygon represented by the array $V[1..9]$. $V[1]$ is the vertex with the minimum x-coordinate, and $V[1..9]$ are ordered counterclockwise.

Problem 4 (10 points). Judge Jill are dealing with tons of complains, each complaint containing exactly two names: that of the person who filled it and that of the person he or she is complaining about. Jill would like an automated approach to deal with such large amount of complaints.

She decides to try to label each person involved in the complaint as either good or evil. She only needs the labeling scheme to be consistent, not necessarily correct. A labeling is consistent if every complaint labels one person as good and the other person as evil, and no person gets labeled both as good and evil in different complaints.

(a) Please model this problem as a graph model and propose an efficient algorithm to consistently label all the names as good or evil, or to decide that no such labeling exists. Your algorithm should run in $O(m+n)$ time (linear in terms of the number of vertices n and edges m).

(b) Later, Judge Jill wants more than a consistent labeling. She will interview some people to figure out his or her true label, as she can always determine whether a person is good or evil by interviewing him or her. Assuming that there exits a correct labeling such that one person in every complaint is either good or evil, what is the minimum number of people she needs to interview to correctly classify all the people named in the complaints?

Problem 5 (10 points). Given an undirected graph, please give an $O(n)$ time (linear in terms of the number of vertices n) algorithm to detect whether the given graph contains a cycle.

Problem 6 (10 points). Suppose that an n-node undirected graph $G = (V, E)$ contains two nodes s and t such that the distance between s and t is strictly greater than $n/2$.

(a) Show that there must exist some node v, not equal to either s or t, such that deleting v from G destroys all s-t paths. (In other words, the graph obtained from G by deleting v contains no path from s to t.)

(b) Give an algorithm to find such a node v with $O(m + n)$ running time (linear in terms of the number of vertices n and edges m).

Problem 7 (10 points). The police department in the city of the Wonder Land has made every street in Wonder Land one-way. Despite widespread complaints from confused motorists, the mayor claims that it is possible to legally drive from any intersection to any other intersection.

(a) The city needs to either verify or refute the mayor's claim. Formalize this problem in terms of graphs, and then describe and analyze an algorithm to solve it.

(b) After running your algorithm from part (a), the mayor reluctantly admits that she was misinformed. Call an intersection x good if, for any intersection y that one can legally reach from x, it is possible to legally drive from y back to x. Now the mayor claims that over 95% of the intersections in Wonder Land are good. Describe and analyze an efficient algorithm to verify or refute her claim.

For full credit, both algorithms should run in $O(m + n)$ time (linear in terms of the number of vertices n and edges m).