Intro to Functional Analysis

Lecture 3 homework

Problem 1

Consider $\mathbb{R}^2$ with standard scalar product and linear subset L defined as

$$L = \{(x, y) : 2x - y = 0\}$$

with inherited scalar product. Define following functional on L

$$\phi(x, y) = x$$

- Find the norm of $\phi : L \to \mathbb{R}$
- Prove that it can be extended uniquely to $\mathbb{R}^2$ with the same norm
- Find explicit form of the extended functional

Problem 2

Consider following Banach space:

$$B = (C^\infty[0, 1], \|f(x)\| = \max_{[0,1]} |f(x)|)$$

and linear map

$$\frac{d}{dx} : B \to B$$

Find its norm or show that it is unbounded.

Problem 3

Let K be a compact subset of $\mathbb{R}^2$. Define following linear map

$$id : C(K) \to L_2(K)$$
$$f(x, y) \to \widetilde{f(x, y)}$$

Show that this operator is bounded and find its norm.