Fast and Memory Optimal Low-Rank Matrix Approximation

Team #17

Team members:

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Background

Low-rank approximation of matrix $M \in [0,1]^{m \times n}$ is a matrix **Z** such that $\min_{\text{rank}(Z)=k} \|M - Z\|_F$

Possible applications:

- Image compression
- Noise reduction
- Latent semantic indexing

Algorithm: Streaming Low-Rank Approximation (SLA)

Idea:

1. Independently sample entries from M to A_1, A_2
2. Q is k-rank PCA for first l columns of A_1
3. Trim some rows and columns of A_2
4. Create sketch of the matrix based on basis Q, remove A_1, A_2, Q from memory
5. Iteratively update sketch for every new column of M

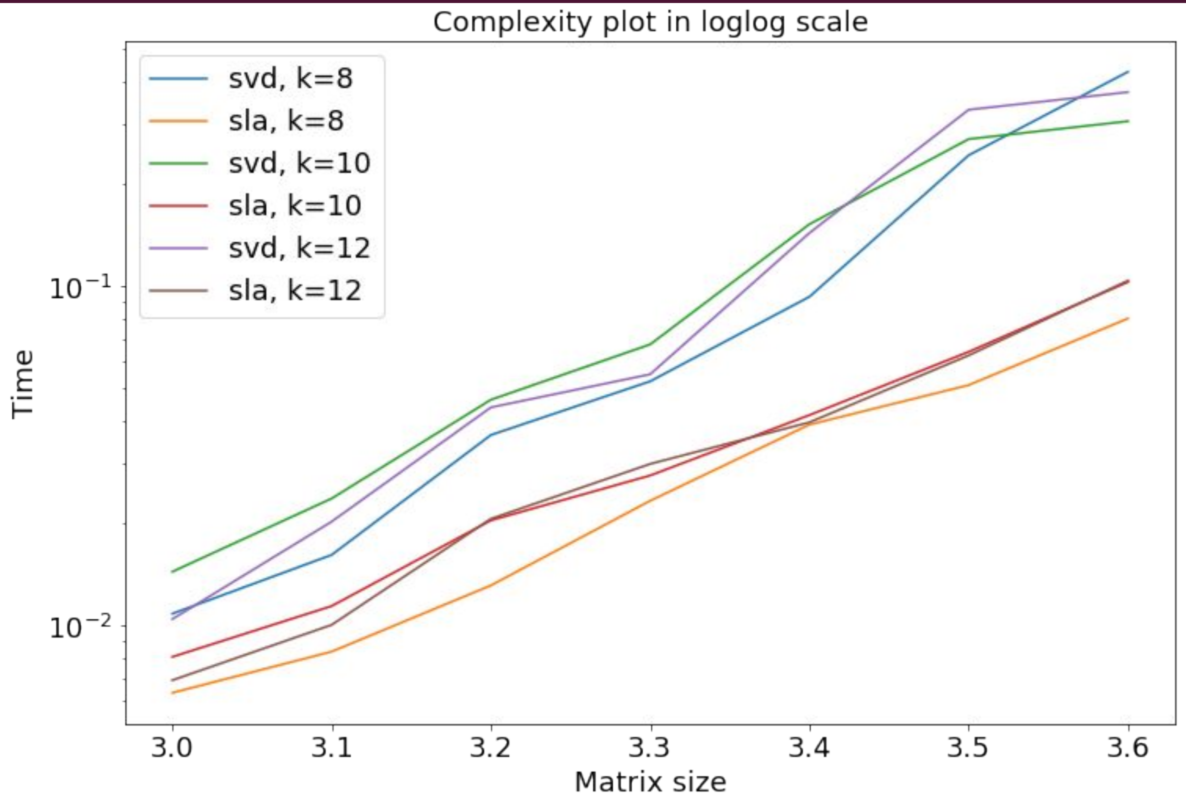
Key features:

- Streaming: matrix is observed in a sequential order in a single pass
- Only one observed column of size m is needed at each iteration
- Memory complexity is $O(k(n+m))$

Se-Young Yun, Marc Lelarge, Alexandre Proutiere. [Fast and Memory Optimal Low-Rank Matrix Approximation]

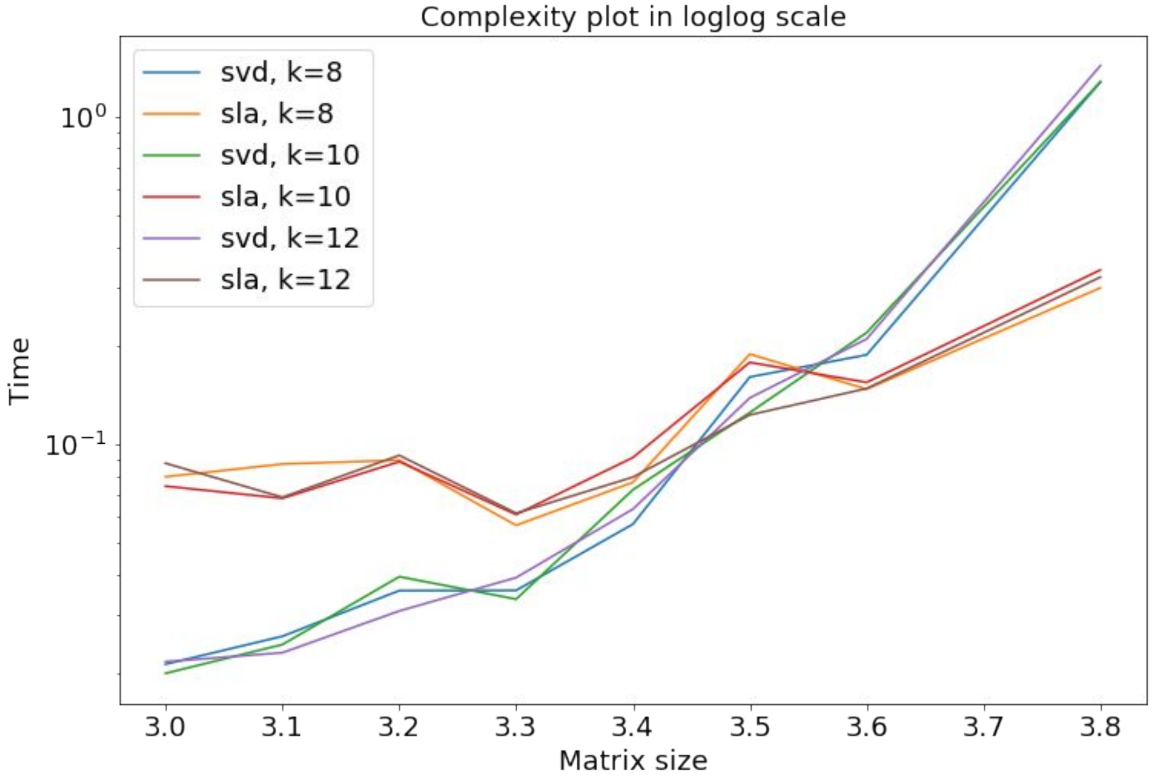
Computational complexity

Dense matrices

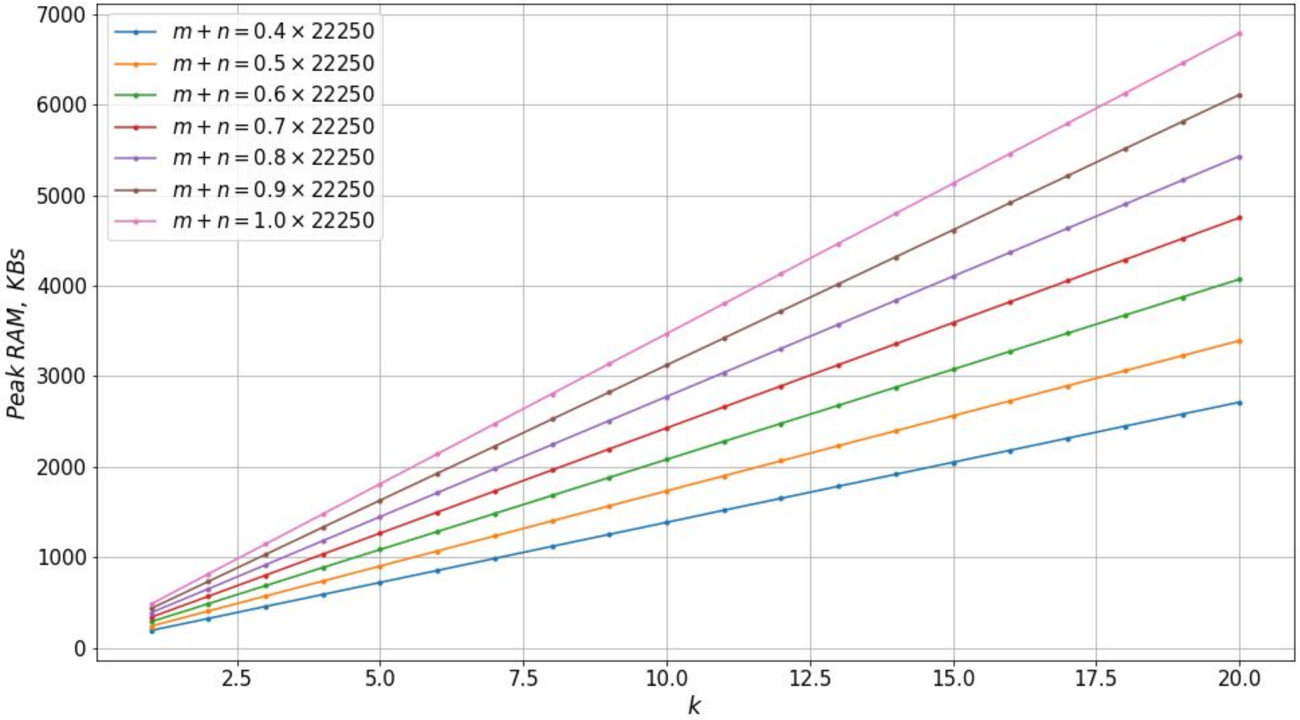


Computational complexity

Sparse matrices



Memory consumption



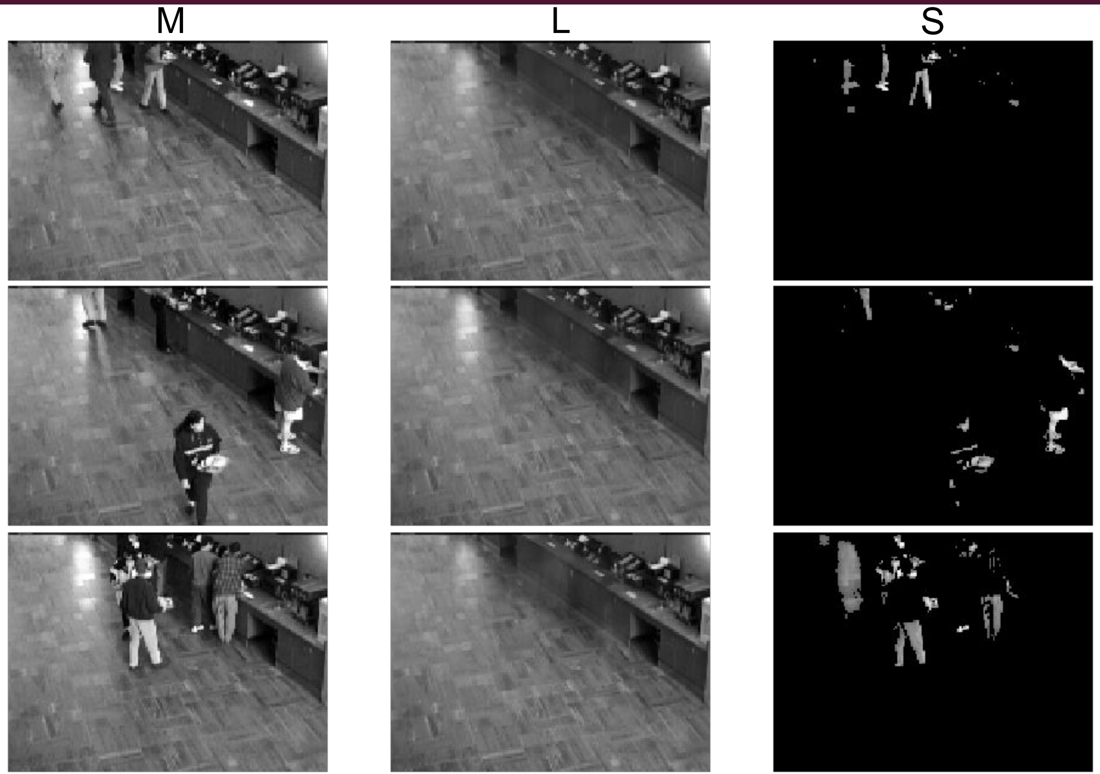
- All memory allocations for matrices were tracked
- Memory complexity is `O(k(n+m))` — as in theory

Applications

Video foreground/background separation

- `M = L + S`
- **L** — low-rank matrix
- **L** — static background (scene)
- **S** — sparse matrix
- **S** — foreground (moving objects)

- Video: [link]
- **~3000** frames
- **1-2** seconds to process



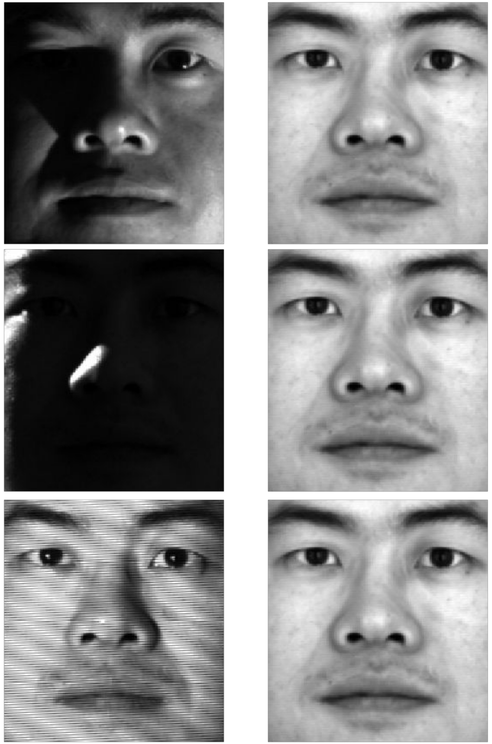
Dataset: K. Toyama. Wallflower: Principles and Practice of Background Maintenance

Applications
Shadow removing

- `M = L + S`, the same as in the previous slide
- 58 images of the same person
- Different illumination conditions
- ~ `100ms` per person

References:

- E. J. Candes. Robust PCA
  (~ 5 min for video, 1.5 min for faces)
- A. Yurtsever. Sketchy Decisions: Convex Low-Rank Matrix Optimization with Optimal Storage
  (~ 30 min for video)
- [Yale Face Database]



M L

Conclusion

Streaming Low-Rank Approximation (SLA) method:

- Optimal memory consumption
- Real-time data processing
- Good performance in comparison with SVD, Robust PCA method, Sketchy Conditional Gradient Method (SketchyCGM)

Contribution

- Semyon Abramov
  - SLA implementation
  - Application: faces shadow removing
- Denis Koposov
  - SLA implementation
  - Application: faces shadow removing
- Daniil Lopatkin
  - SLA implementation
  - Application: video foreground/background separation
- Albert Nagapetyan
  - Performance analysis
- Viktor Prutyanov
  - Application: video foreground/background separation
  - Memory consumption analysis