Time Series Analysis

Homework4

Chenyin Gao*

(Dept. Mathematics School Major Statistics 15338047)

5.1

The data set arf is 1000 simulated observations from an ARFIMA(1, 1, 0) model with $\phi = 0.75$ and $d = 0.4$

(a) Plot the data and comment.

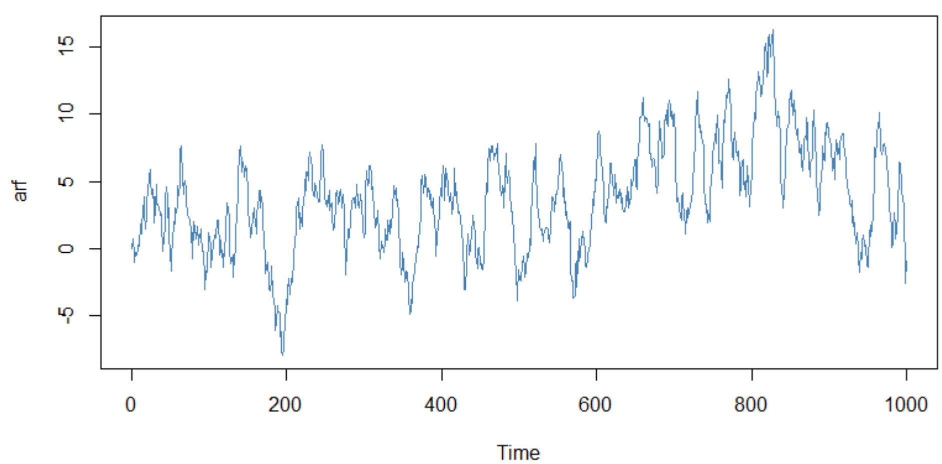

Fig 1. x_t produced by ARFIMA(1,1,0) process

*gaochy5@mail2.sysu.edu.cn

From Fig 1, we notice the time series is stationary and similar to a random walk with underlying cycle.

(b) Plot the ACF and PACF of the data and comment.

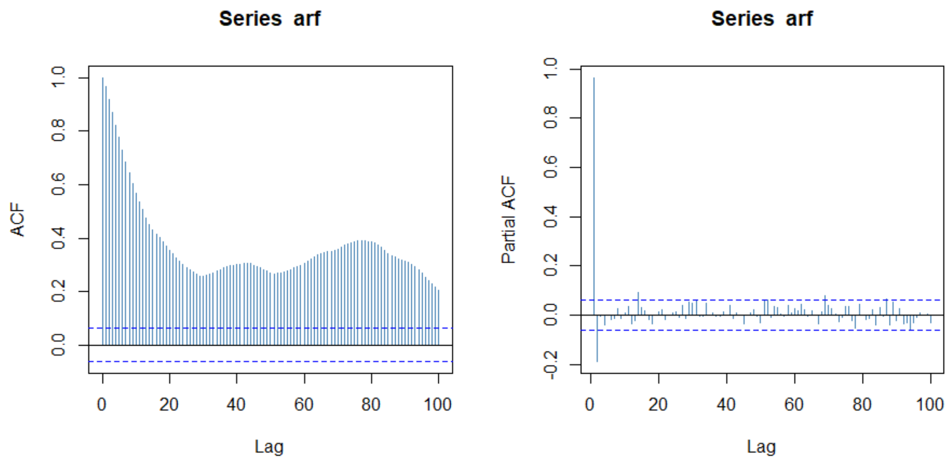

Fig 2. ACF (left) and PACF (right) of x_t

In Fig 2, we find a autocorrelation with lone memory, which sample ACF are not necessarily large but persist for a long time. However, the PACF of x_t cut off at a low lag and remain the level in the high lags.

(c) Estimate the parameters and test for the significance of the estimates $\hat{\phi}$ and $\hat{d}$.

Assume x_t is a fractionally differenced series with fractional values of d, so a basic long memory series gets generated as

$$\phi(1-\mathbf{B})^d x_t = w_t \tag{1}$$

To investigate the property, we can use the binomial expansion to write Eq 1 as

$$w_t = \phi(1-\mathbf{B})^d x_t = \phi\sum_{j=0}^{\infty}\pi_j B^j x_t = \phi\sum_{j=0}^{\infty}\pi_j x_{t-j} \tag{2}$$

where

$$\pi_j = \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} \tag{3}$$

Using Eq 3, it is easy to derive the recursions

$$\pi_{j+1}(d) = \frac{(j-d)\pi_j(d)}{j+1} \tag{4}$$

Then we maximizing the joint likelihood of w_t under normality assumption. First, we show the joint likelihood in Eq 5.

$$L(d|x_t) = \prod_{t=1}^{n} w_t(d) = \frac{1}{\sqrt{2\pi}} \exp(\frac{1}{2}\sum_{t=1}^{n} w_t(d)^2) \propto \exp(\sum_{t=1}^{n} w_t(d)^2) \tag{5}$$

Then we use the Gauss-Newton method to maximize the log-likelihood in Eq 5 given initial value $d = d_0$

$$d = d_0 - \frac{\sum_t w_t'(d_0)w_t(d_0)}{\sum_t w_t'(d_0)^2} \tag{6}$$

Last, we form an approximation to Eq 2, namely,

$$w_t(d) = \phi \sum_{j=0}^{t} \pi_j(d) x_{t-j} \tag{7}$$

We perform our analysis in R to the mean-adjusted series, $x_t - \bar{x}$ with initial $\hat{\phi}_0 = 1, \hat{h}_0 = 0$ and truncation value M=500. We show the estimated parameters after N times recursion in Tab 1

Tab 1. The estimateed parameters and significance values of $\hat{d}_N$ and $\hat{\phi}_N$

	Estimate	Std. Error	z value	Pr(>|z|)
$\hat{d}_N$	0.2637	0.0097	27.2293	0.0000
$\hat{\phi}_N$	0.8623	0.0170	50.8284	0.0000

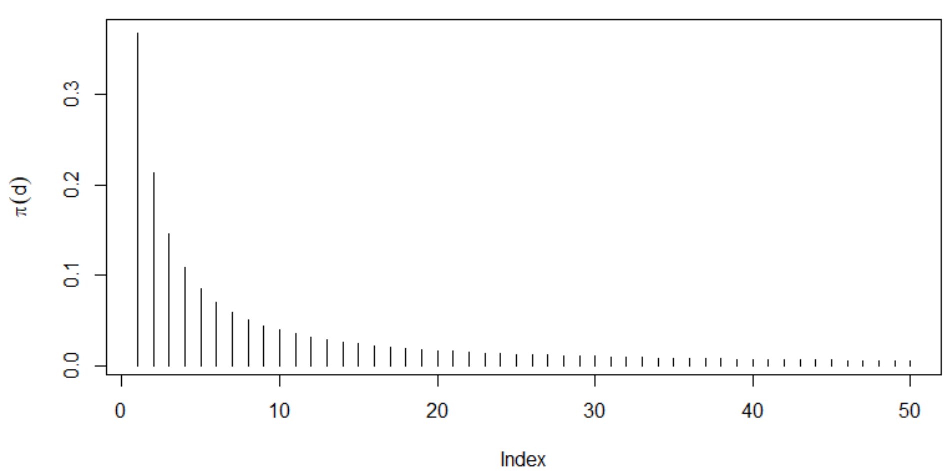

Fig 3. Coefficients $\pi(0.2636)$, j=1,2,$\cdots$,50 in representation Eq 2

(d) Explain why, using the results of parts (a) and (b), it would seem reasonable to difference the data prior to the analysis. That is, if x_t represents the data, explain why we might choose to fit an ARMA model to ∇x_t

In (a) and (b), we notice that the sample ACF of a time series decays slowly, so we assume the underlying model of x_t is random walk with a drift as

$$x_t = \mu_t + s_t + w_t \tag{8}$$

So with the first difference of the logarithms of x_t, we could show the stationarity of the first differenced series as

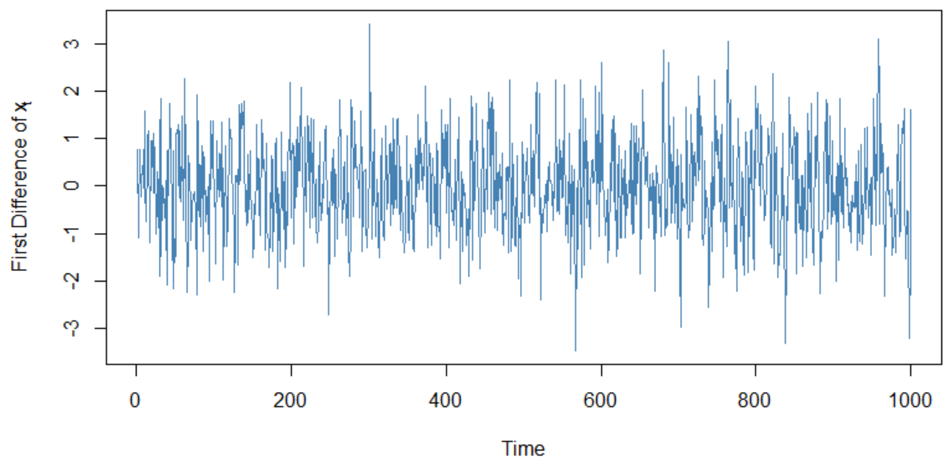

Fig 4. The first differenced values of x_t

After the differenced transformation of x_t in Fig 4, we could notice a clear stationarity of ∇x_t. So, we might choose to fit an ARMA model to ∇x_t.

(e) Plot the ACF and PACF of ∇x_t and comment.

We plot the ACF and PACF of ∇x_t in Fig 5

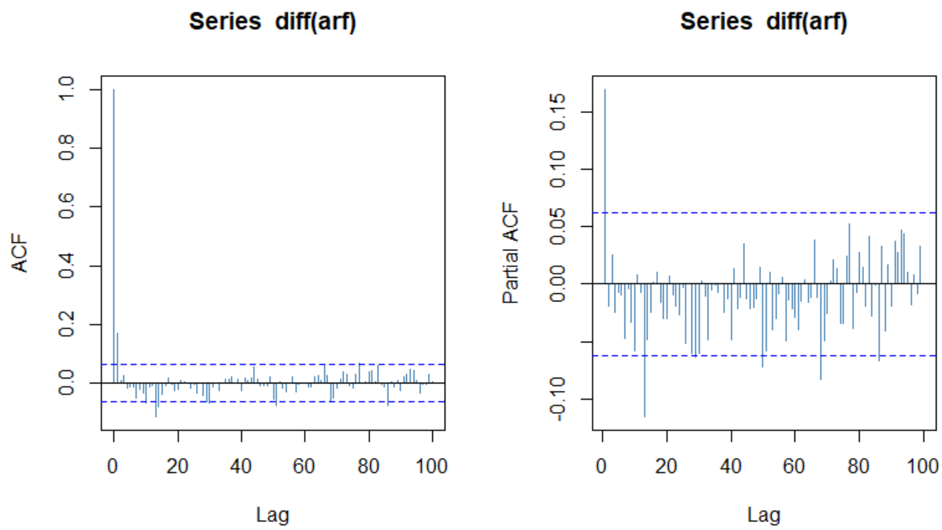

Fig 5. The ACF (left)and PACF (right) of ∇x_t

We find that the ACF and PACF both cut off at a low lag and stay in the confidence interval since then. As a result, from Fig 5, we could conclude the stationarity of ∇x_t.

(f) Fit an ARMA model to ∇x_t and comment.

As usual, we fit an ARMA(1,1) to ∇x_t

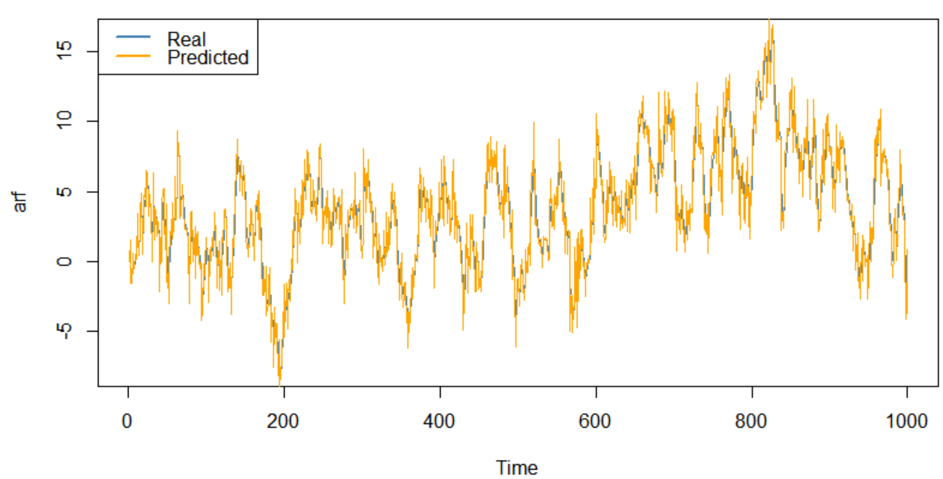

Fig 6. The original time series of ∇x_t(blue solid line) with the ARMA(1,1) prediction of ∇x_t(orange solid line)

We finish our estimation with $\hat{\phi} = 0.0031(0.22)$ and $\hat{\theta} = 0.1711(0.22)$. When compared to the real parameter where $\phi = 0.75$ and $\theta = 0$, it seems the use of the first difference $\nabla x_t = (1-\mathbf{B})x_t$, however, can sometimes be too severe a modification in the sense that the non-stationary model might represent an overdifferencing of the original process. Consequently, we might consider long memory time series as an intermediate compromises in (c).

5.5

Verify (5.33)

The DF test was extended to accommodate AR(p) models,

$$x_t = \sum_{j=1}^{p} \phi_j x_{t-j} + w_t \tag{9}$$

$$\begin{aligned} x_t &= (\phi_1 + \phi_2 + \cdots + \phi_p) x_{t-1} \\ &\quad - \phi_2 (\underbrace{x_{t-1} - x_{t-2}}_{\nabla x_{t-1}}) \\ &\quad - \phi_3 (\underbrace{x_{t-1} - x_{t-2}}_{\nabla x_{t-1}} + \underbrace{x_{t-2} - x_{t-3}}_{\nabla x_{t-2}}) \\ &\quad \ldots \\ &\quad - \phi_p (\underbrace{x_{t-1} - x_{t-2}}_{\nabla x_{t-1}} + \cdots + \underbrace{x_{t-p+1} - x_{t-p}}_{\nabla x_{t-p+1}}) + w_t \\ &= \sum_{j=1}^{p} x_{t-1} - \underbrace{\sum_{j=2}^{p} \phi_j}_{\psi_1} \nabla x_{t-1} - \cdots - \underbrace{\sum_{j=p}^{p} \phi_j}_{\psi_{p-1}} \nabla x_{t-p+1} + w_t \\ &= \sum_{j=1}^{p} x_{t-1} + \sum_{j=1}^{p-1} \psi_j \nabla x_{t-j} + w_t \end{aligned} \tag{10}$$

Then subtract x_{t1} from both sides to obtain as follows and thereby complete the verification.

$$\begin{aligned} \underbrace{x_t - x_{t-1}}_{\nabla x_t} &= \underbrace{(\sum_{j=1}^{p} -1)}_{\gamma} x_{t-1} + \sum_{j=1}^{p-1} \psi_j \nabla x_{t-j} + w_t \\ \nabla x_t &= \gamma x_{t-1} + {}_{j=1}^{p-1} \psi_j \nabla x_{t-j} + w_t \end{aligned} \tag{11}$$

5.7

The stats package of R contains the daily closing prices of four major European stock indices; type help(EuStockMarkets) for details. Fit a GARCH model to the returns of one of these series and discuss your findings. (Note: The data set contains actual values, and not returns. Hence, the data must be transformed prior to the model fitting.)

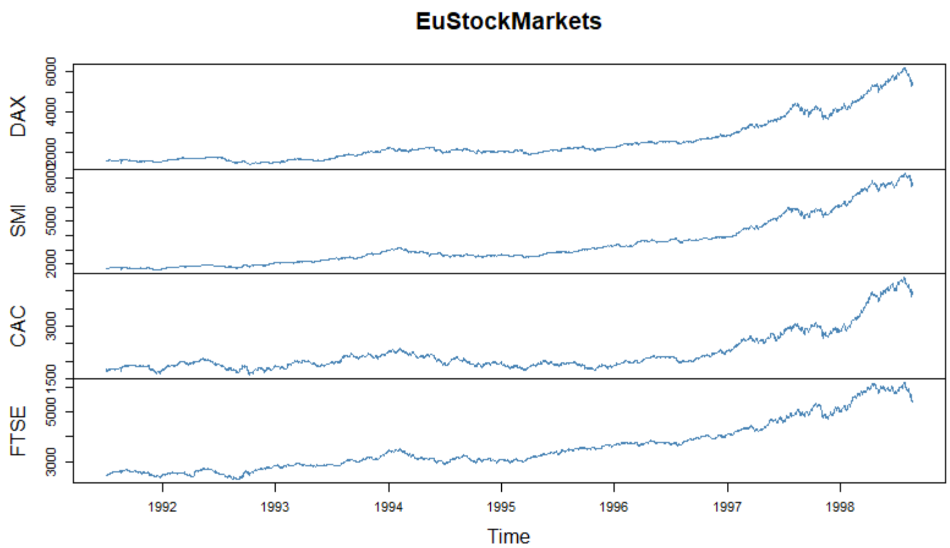

Fig 7. The timeline plot of EuStockMarkets series from 1991 to 1998

We chose DAX and fit a GARCH(1,1) model to the return of the price. We plot the ACF and PACF by assuming

$$\begin{aligned} r_t &= \sigma_t \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \end{aligned} \tag{12}$$

where $\varepsilon_t \overset{i.i.d.}{\sim} N(0,1)$.

Therefore, we form the conditional likelihood of the data $r_2, \cdots, r_n$ given r_1 after $\nabla \log$ transformation.

$$\begin{aligned} L(\alpha_0, \alpha_1, \beta_1 | r_1, \sigma_1^2) &= \prod_{t=2}^{n} f_{\alpha_0,\alpha_1,\beta_1}(r_t | r_{t-1}, \sigma_{t-1}^2) \\ &= \frac{1}{\sqrt{\left[2\pi(\alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2)\right]}} \exp\left[\sum_{t=2}^{n} \frac{r_t^2}{2(\alpha_0 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1}^2)}\right] \end{aligned} \tag{13}$$

Tab 2. GARCH(1,1) parameters with its significant values

	Estimate	Std. Error	t value	Pr(>|t|)
α_0	0.0007	0.0002	3.0289	0.0025
ω	0.0000	0.0000	3.7601	0.0002
α_1	0.0684	0.0148	4.6299	0.0000
β_1	0.8876	0.0236	37.6768	0.0000

Last, we analysis Eq 13 in R with Newton-Raphson method to obtain mle as follow in Tab 2. After 29 iterative estimation of the conditional likelihood, the result, mles, converge, as follow.

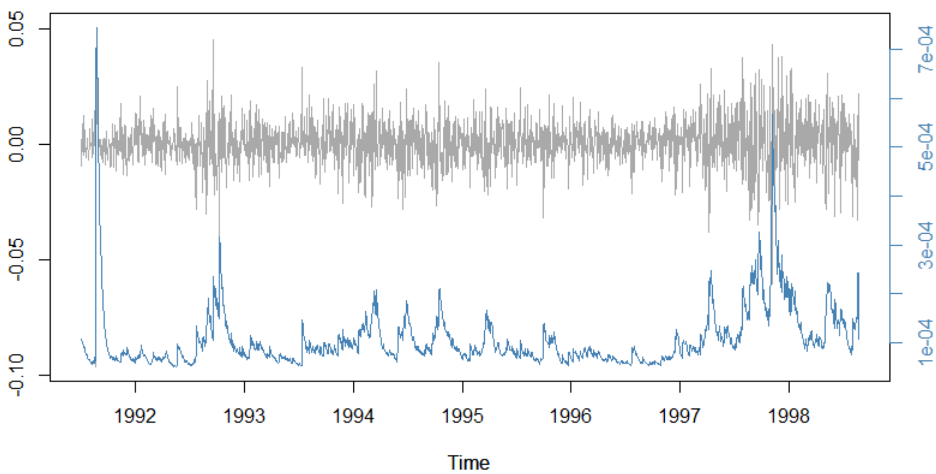

Fig 8. GARCH(1,1) one-step-ahead predictions (grey solid line) of the DXA volatility,$\hat{\sigma}_t^2$ (blue solid line)

To further explore the GARCH(1,1) predictions of volatility, we calculated and plotted the prediction in Fig 8. We notice that the first peak of volatility in 1991and another peak in late 1997, perhaps due to financial crisis in European market.

5.8

The 2×1 gradient vector, $l^{(1)}(\alpha_0, \alpha_1)$, given for an ARCH(1) model was displayed in (5.47). Verify (5.47) and then use the result to calculate the 2×2 Hessian matrix

First, we show the conditional likelihood of data $r_2, \cdots, r_n$ given r_1, is given by

$$L(\alpha_0, \alpha_1 | r_1) = \prod_{t=2}^{n} f_{\alpha_0,\alpha_1}(r_t | r_{t-1}) \tag{14}$$

where $r_t | r_{t-1} \sim N(0, \alpha_0 + \alpha_1 r_{t-1}^2)$. Hence, the criterion function to be minimized, $l(\alpha_0, \alpha_1) \propto -\ln L(\alpha_0, \alpha_1 | r_1)$ is given by

$$l(\alpha_0, \alpha_1) = \frac{1}{2}\sum_{t=2}^{n} \ln(\alpha_0 + \alpha_1 r_{t-1}^2) + \frac{1}{2}\sum_{t=2}^{n}\left(\frac{r_t^2}{\alpha_0 + \alpha_1 r_{t-1}^2}\right) \tag{15}$$

We show the analytic expressions for the gradient vector, $l^{(1)}(\alpha_0, \alpha_1)$ by straight-forward calculations

$$\begin{aligned}\frac{\partial l}{\partial \alpha_0} &= \frac{1}{2}\sum_{t=2}^{n} \frac{1}{\alpha_0 + \alpha_1 r_{t-1}^2} + \frac{1}{2}\sum_{t=2}^{n} r_t^2 \left[\frac{-1}{(\alpha_0 + \alpha_1 r_{t-1}^2)^2}\right] \\ &= \sum_{t=2}^{n} \frac{\alpha_0 + \alpha_1 r_{t-1}^2 - r_t^2}{2(\alpha_0 + \alpha_1 r_{t-1}^2)^2}\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\partial l}{\partial \alpha_1} &= \frac{1}{2}\sum_{t=2}^{n} \frac{r_{t-1}^2}{\alpha_0 + \alpha_1 r_{t-1}^2} + \frac{1}{2}\sum_{t=2}^{n} r_t^2 \left[\frac{-r_{t-1}^2}{(\alpha_0 + \alpha_1 r_{t-1}^2)^2}\right] \\ &= \sum_{t=2}^{n} r_{t-1}^2 \frac{\alpha_0 + \alpha_1 r_{t-1}^2 - r_t^2}{2(\alpha_0 + \alpha_1 r_{t-1}^2)^2}\end{aligned} \tag{17}$$

Based on Eq 16 and Eq 17, the 2×1 gradient vector, $l^{(1)}(\alpha_0, \alpha_1)$, given by

$$\begin{pmatrix} \partial l / \partial \alpha_0 \\ \partial l / \partial \alpha_1 \end{pmatrix} = \sum_{t=2}^{n} \begin{pmatrix} 1 \\ r_{t-1}^2 \end{pmatrix} \times \frac{\alpha_0 + \alpha_1 r_{t-1}^2 - r_t^2}{2\Big(\alpha_0 + \alpha_1 r_{t-1}^2\Big)} \tag{18}$$

We then use the result of Eq 18 to compute the 2×2 Hessian matrix. Since $\sum_{t=2}^{n} r_{t-1}^2 \frac{\alpha_0+\alpha_1 r_{t-1}^2 - r_t^2}{2(\alpha_0+\alpha_1 r_{t-1}^2)^2}$ is continuous function, $\partial^2 l / \partial \alpha_0 \alpha_1$ is equal to $\partial^2 l / \partial \alpha_1 \alpha_0$. We then break our computation step by step.

$$\begin{aligned}\frac{\partial^2 l}{\partial \alpha_0^2} &= \sum_{t=2}^{n} \frac{2(\alpha_0 + \alpha_1 r_{t-1}^2) - 4(\alpha_0 + \alpha_1 r_{t-1}^2 - r_t^2)(\alpha_0 + \alpha_1 r_{t-1}^2)}{4(\alpha_0 + \alpha_1 r_{t-1}^2)^4} \\ &= \sum_{t=2}^{n} \frac{2r_t^2 - \alpha_1 r_{t-1}^2 - \alpha_0}{2(\alpha_0 + \alpha_1 r_{t-1}^2)^3}\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial^2 l}{\partial \alpha_1^2} &= \sum_{t=2}^{n} r_{t-1}^2 \frac{2r_{t-1}^2(\alpha_0 + \alpha_1 r_{t-1}^2) - 4r_{t-1}^2(\alpha_0 + \alpha_1 r_{t-1}^2 - r_t^2)(\alpha_0 + \alpha_1 r_{t-1}^2)}{4(\alpha_0 + \alpha_1 r_{t-1}^2)^4} \\ &= \sum_{t=2}^{n} r_{t-1}^4 \frac{2r_t^2 - \alpha_1 r_{t-1}^2 - \alpha_0}{2(\alpha_0 + \alpha_1 r_{t-1}^2)^3}\end{aligned} \tag{20}$$

$$\begin{aligned}\frac{\partial^2 l}{\partial\alpha_0\alpha_1}=\frac{\partial^2 l}{\partial\alpha_1\alpha_0}&=\sum_{t=2}^{n} r_{t-1}^2\frac{2(\alpha_0+\alpha_1 r_{t-1}^2)-4(\alpha_0+\alpha_1 r_{t-1}^2-r_t^2)(\alpha_0+\alpha_1 r_{t-1}^2)}{4(\alpha_0+\alpha_1 r_{t-1}^2)^4}\\&=\sum_{t=2}^{n} r_{t-1}^2\frac{2r_t^2-\alpha_1 r_{t-1}^2-\alpha_0}{2(\alpha_0+\alpha_1 r_{t-1}^2)^3}\end{aligned}\tag{21}$$

From Eq 19,Eq 20,Eq 21, we could write the 2×2 Hessian matrix as

$$\begin{aligned}l^{(2)}(\alpha_0,\alpha_1)&=\begin{pmatrix}\partial^2 l/\partial\alpha_0^2 & \partial^2 l/\partial\alpha_0\alpha_1\\ \partial^2 l/\partial\alpha_1\alpha_0 & \partial^2 l/\partial\alpha_1^2\end{pmatrix}\\&=\sum_{t=2}^{n}\frac{2r_t^2-\alpha_1 r_{t-1}^2-\alpha_0}{2(\alpha_0+\alpha_1 r_{t-1}^2)^3}\begin{pmatrix}1 & r_{t-1}^2\\ r_{t-1}^2 & r_{t-1}^4\end{pmatrix}\end{aligned}\tag{22}$$

5.10

The data in climhyd have 454 months of measured values for the climatic variables air temperature, dew point, cloud cover, wind speed, precipitation (pt), and inflow (it), at Lake Shasta; the data are displayed in Fig 9. We would like to look at possible relations between the weather factors and the inflow to Lake Shasta

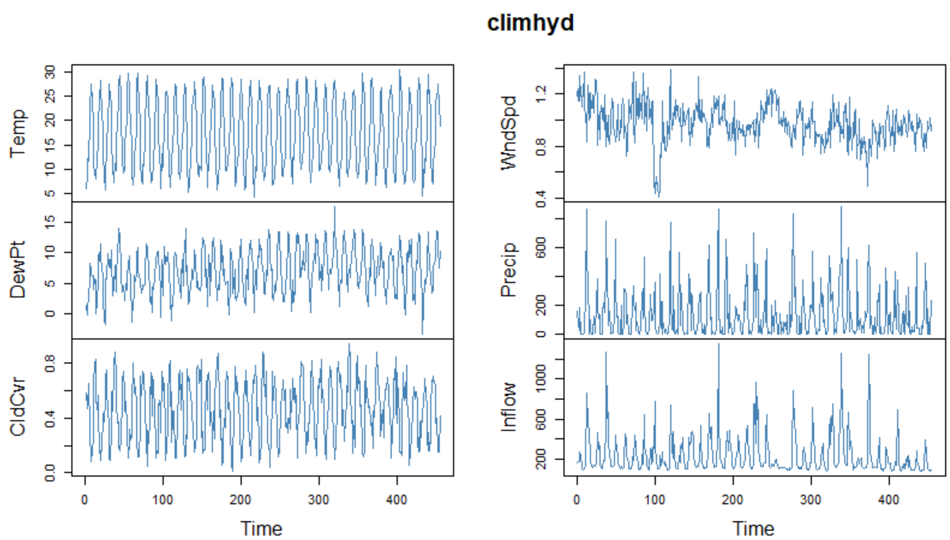

Fig 9. Monthly values of weather and inflow at Lake Shasta (climhyd)

(a) Fit ARIMA$(0,0,0)\times(0,1,1)_{12}$ models to (i) transformed precipitation $P_t = \sqrt{p_t}$ and (ii) transformed inflow $I_t = \log i_t$.

We display our model fitted values and the corresponding residual analysis in Fig 10,Fig 11,Fig 12,Fig 13.

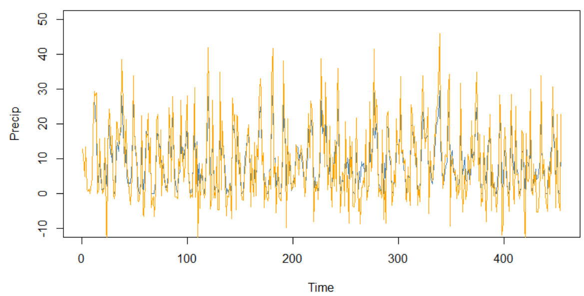

Fig 10. 454 months transformed precipitation $\sqrt{p_t}$. Blue solid line represents the true series;orange solid line represents the ARIMA model fitted values

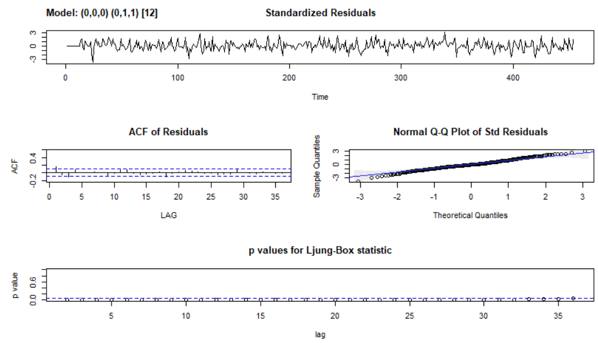

Fig 11. Residual analysis of ARIMAR(0,0,0)$\times(0,1,1)_{12}$ model fitted to $\sqrt{p_t}$

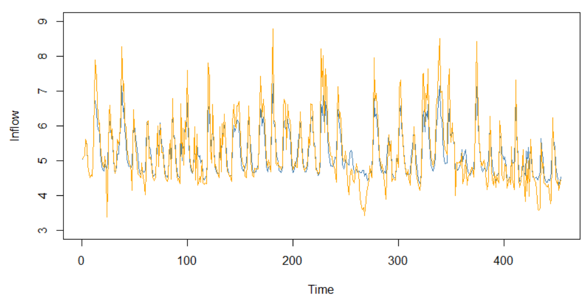

Fig 12. 454 months transformed precipitation $\log i_t$. Blue solid line represents the true series;orange solid line represents the ARIMA model fitted values

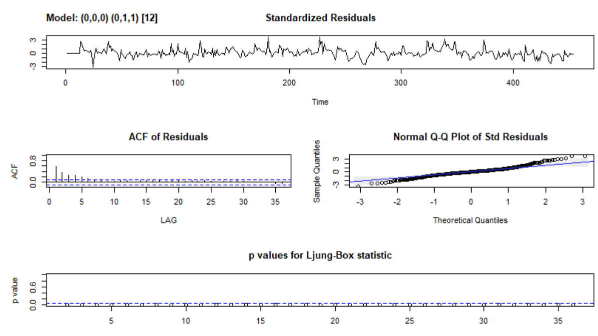

Fig 13. Residual analysis of ARIMAR(0,0,0)$\times(0,1,1)_{12}$ model fitted to $\log i_t$

(b) Apply the ARIMA model fitted in part (a) for transformed precipitation to the flow series to generate the prewhitened flow residuals assuming the precipitation model. Compute the cross-correlation between the flow residuals using the precipitation ARIMA model and the precipitation residuals using the precipitation model and interpret. Use the coefficients from the ARIMA model to construct the transformed flow residuals.

Assume the ARIMA$(0,0,0)\times(0,0,1)_{12}$ to generate the prewhitened flow residuals

$$\Delta_{12}\log i_t = \delta + (1+\Theta_1 B^{12})w_t \tag{23}$$

Fit the series with auto-correlation error, we obtain

$$\delta = -7 * 10^{-4} \approx 0 \qquad \Theta_1 = -0.9815 \tag{24}$$

And it produced the prewhitened version of $\log i_t$ as follow,

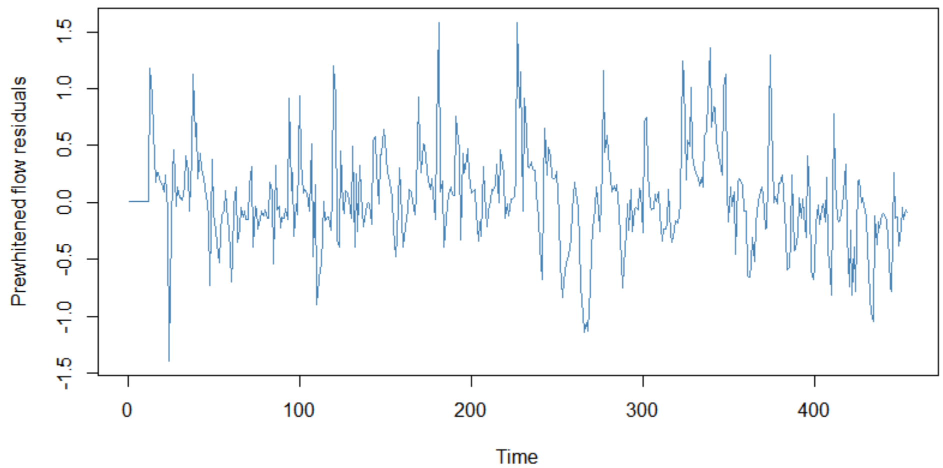

Fig 14. prewhitened flow residuals version of $\log i_t$

Then we could rewrite the final operation as

$$(1-\mathbf{B}^{12})\log i_t = (1-0.9815\mathbf{B}^{12})w_t \tag{25}$$

We applied the operator $\frac{1-\mathbf{B}^{12}}{1-0.9815\mathbf{B}^{12}}$ to both $\sqrt{p_t}$ and $\log i_t$. We start our transformation with Eq 25.

$$\begin{aligned}(1-\mathbf{B}^{12})\log i_t &= (1-0.9815\mathbf{B}^{12})w_t \\ &= (1-0.9815\mathbf{B}^{12})(1+\pi_1\mathbf{B}+\cdots+\pi_N\mathbf{B}^N)\log i_t\end{aligned} \tag{26}$$

which lead to the following equations Eq 27

$$\begin{aligned}&1 = 1 \\ &(-0.9815+\pi_{12})\mathbf{B}^{12} = -\mathbf{B}^{12} \\ &(\pi_{24}-0.9815\pi_{12})\mathbf{B}^{24} = 0 \\ &\dots \\ &(\pi_N-0.9815\pi_{N-12})\mathbf{B}^N = 0\end{aligned} \tag{27}$$

By solving the above equations, we could obtain

$$
\begin{aligned}
&1 = 1 \\
&\pi_{12} = -1 + 0.9815 = -0.0185 \\
&\pi_{24} = 0.9815\pi_{12} = 0.9815 * (-0.0185) \\
&\cdots \\
&\pi_N = \pi_{12n} = (0.9815)^{n-1}\pi_{12}
\end{aligned}
\tag{28}
$$

We applied the 10×12-lag truncated operator $\tilde{\Pi}$ to $\sqrt{p_t}$.

$$
\tilde{\Pi} = \left[1, \underbrace{0, \cdots, 0}_{11 times}, \pi_{12}, \underbrace{0, \cdots, 0}_{11 times}, \pi_{24}, \cdots, \underbrace{0, \cdots, 0}_{11 times}, \pi_{10\times12}\right]
$$

The filtered precipitation $\sqrt{p_t}$ can be shown in Fig 15

$$
\tilde{\Pi} \times \sqrt{p_t} = \sqrt{\tilde{p_t}} \tag{29}
$$

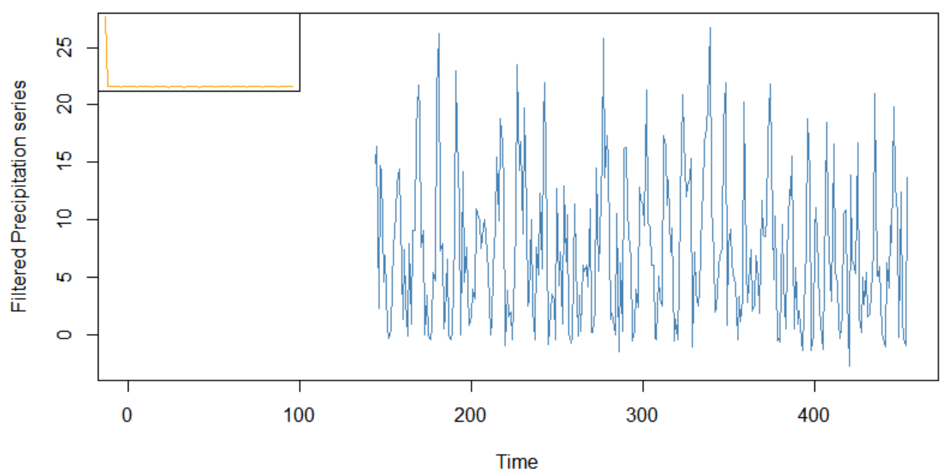

Fig 15. The filtered precipitation of $\sqrt{p_t}$. The insert shows the shapes of the AR parameters (the filter values)

We then compute the cross-correlation between the flow residuals $\sqrt{\tilde{p_t}}$ using the precipitation ARIMA model and the precipitation residuals w_t using the precipitation model. In Fig 16, noting the apparent peak at zero order and the decrease thereafter, we conclude that the precipitation and the inflow happened simultaneously.

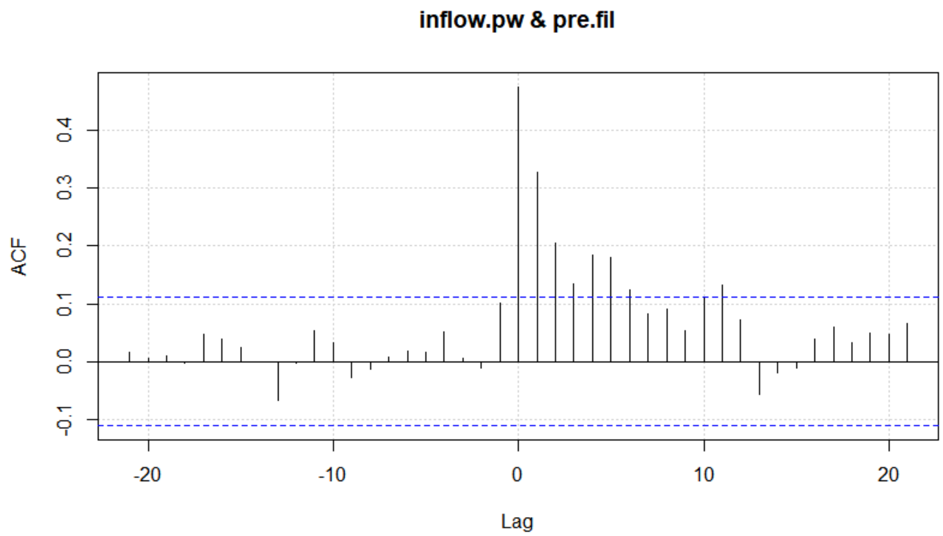

Fig 16. Sample CCF of the prewhitened of $\log i_t$ and the similarly transformed $\sqrt{p_t}$ series; negative lags indicate that i_t leads p_t

5.12

Consider the data set *econ5* containing quarterly U.S. unemployment, GNP, consumption, and government and private investment from 1948-III to 1988-II. The seasonal component has been removed from the data. Concentrating on unemployment (U_t), GNP (G_t), and consumption (C_t), fit a vector ARMA model to the data after first logging each series, and then removing the linear trend. That is, fit a vector ARMA model to $x_t = (x_{1t}, x_{2t}, x_{3t})$, where, for example, $x_{1t} = \log(U_t) - \hat{\beta}_0 - \hat{\beta}_1 t$, where $\hat{\beta}_0$ and $\hat{\beta}_1$ are the least squares estimates for the regression of $\log(U_t)$ on time, t. Run a complete set of diagnostics on the residuals.

First, we explore the original time series of U_t,G_t and C_t in Fig 17

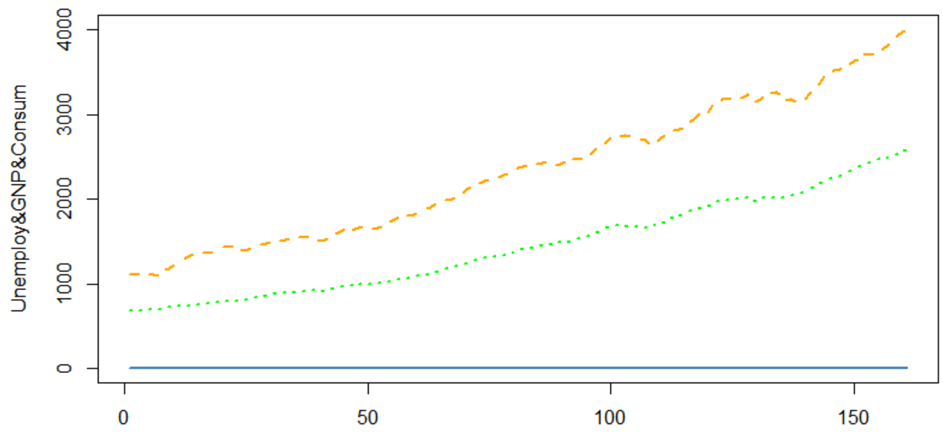

Fig 17. The original time series; The steelblue solid line represent the original unemployment, the orange solid line represent the original GNP, the green solid line represent the original consumption

We then detrend *econ5* with linear regression, for example, $x_{1t} = \log(U_t) - \hat{\beta}_0 - \hat{\beta}_1 t$. We obtain the detrended series as follow,

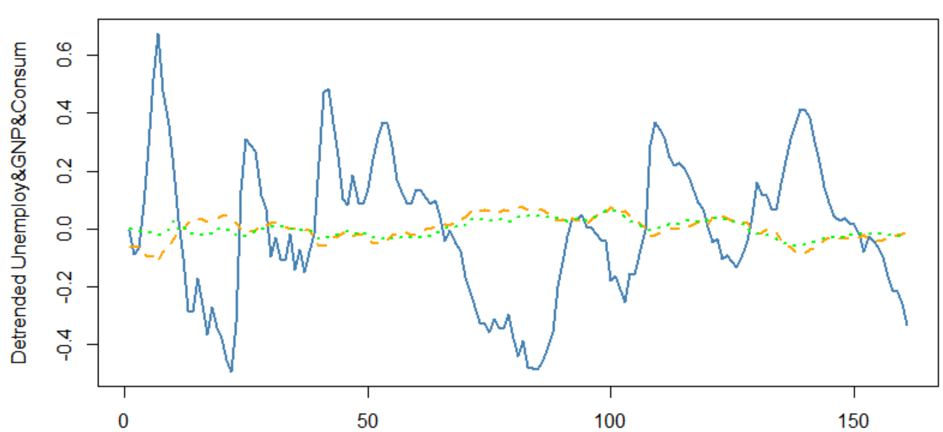

Fig 18. The detrended time series; The steelblue solid line represent the detrended unemployment, the orange solid line represent the detrended GNP, the green solid line represent the detrended consumption

We use *marima* to fit a vector ARMA(1,1) to the detrended unemployment,GNP and consumption.

$$\Phi_1(B)x_t = \alpha + \Theta_1(B)w_t \tag{30}$$

We obtain the coefficients values as follow,

$$\hat{\alpha} = (-4.328857\times 10^{-18}, 4.737707\times 10^{-19}, 7.42655\times 10^{-20})'$$
$$\hat{\Phi}_1 = \begin{pmatrix} -0.7639 & 0.9265 & -0.0530 \\ -0.0218 & -1.0089 & -0.0566 \\ -0.0129 & -0.0587 & -0.9513 \end{pmatrix} \tag{31}$$
$$\hat{\Theta}_1 = \begin{pmatrix} 0.3039 & -1.5213 & -2.1782 \\ -0.0208 & 0.1235 & 0.2150 \\ 0.0026 & 0.0950 & -0.0488 \end{pmatrix}$$

Run a complete set of diagnostics on the residuals and show the results in Fig 19

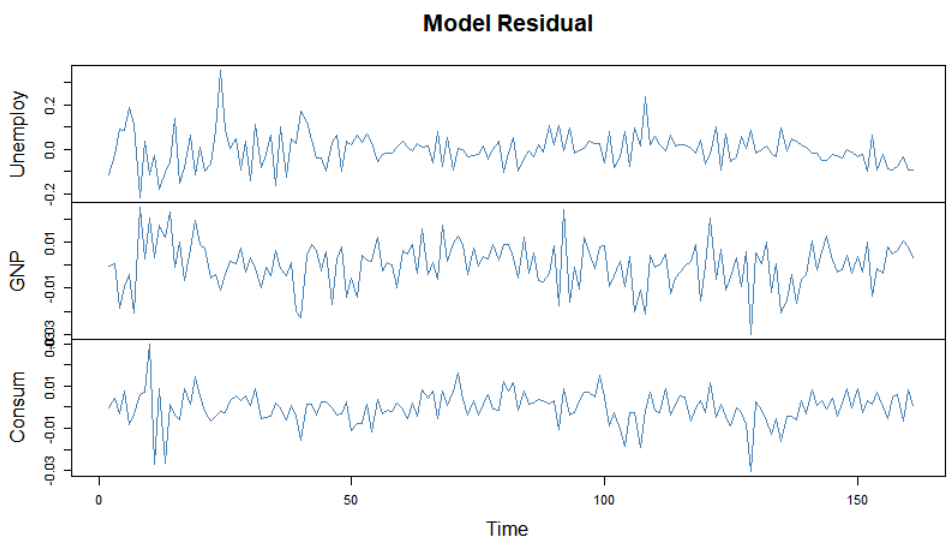

Fig 19. The diagnostics on the residuals of U_t, G_t and C_t

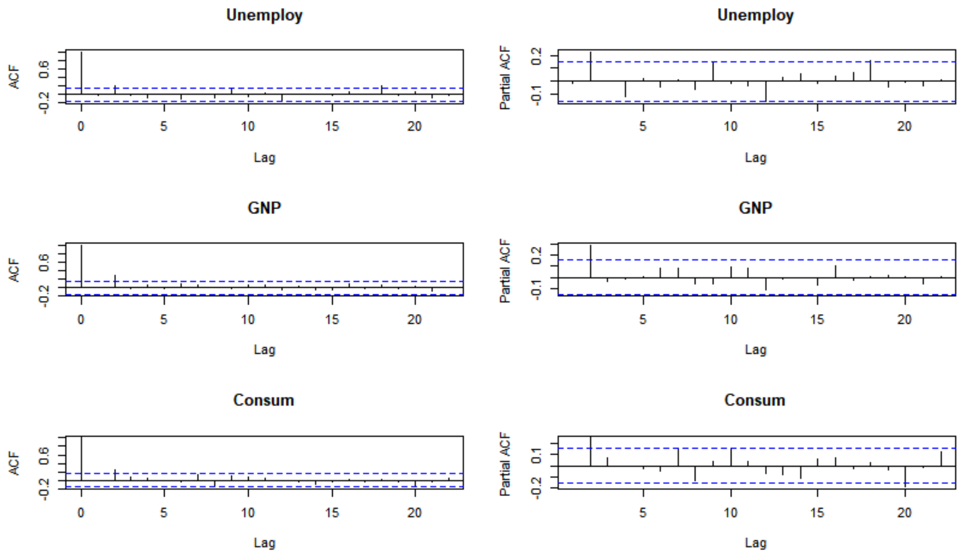

Fig 20. The diagnostics on the residuals of U_t, G_t and C_t; Plot of ACF and PACF

Rcode

Set up

```
# Loading required packages
library(astsa)
library(fracdiff)
library(xtable)
library(latex2exp)
library(stats)
library(fGarch)
library(marima)
library(vars)
```

5.1

```
# Plot the time-line of arf
plot.ts(arf,
        col='steelblue')
```

```
par(mfrow=c(1,2))
# ACF and PACF analysis of arf
acf(arf,100,
    col='steelblue')
pacf(arf,100,
    col='steelblue')
# Begian ARFIMA model fit
arf.mean <- arf-mean(arf)
arf.fd <- fracdiff(arf.mean,nar=1,
                   ar=1,h=0,M=500)
arf.fd.sum <- summary(arf.fd)
# Output LaTex code
xtable(arf.fd.sum$coef,
       digits = 4)
# Display model coefficients and Plot it
p <- rep(1,50)
for (k in 1:50){ p[k+1]=(k-arf.fd$d)*p[k]/(k+1) }
plot(1:50, p[-1],
     ylab=expression(pi(d)),
     xlab="Index",
     type="h")
# Plot the first difference of arf
plot.ts(diff(arf),
        col='steelblue',
        ylab=TeX('First Difference of x_t'))
par(mfrow=c(1,2))
# ACF and PACF of first difference of arf
acf(diff(arf),100,
    col='steelblue')
pacf(diff(arf),100,
    col='steelblue')
# ARMA model fit to arf
arf.arma.110 <- arima(arf,order=c(1,1,1))
arf.arma.predicted <- arf.arma.110$residuals + arf
# Compared the true and the prediction of time series
plot(arf,col='steelblue',lwd=1)
```

```
lines(arf.arma.predicted,col='orange',lwd=1)
legend('topleft',
       legend = c('Real',
                  'Predicted'),
       col=c('steelblue',
             'orange'),
       lwd=c(2,2),
       lty=c(1,1))
# Output LaTex Code
arf.arma.ps <- rbind(arf.arma.110$coef,
      arf.arma.110$sigma2)
xtable(arf.arma.110$var.coef,digits = 4)
```

5.7

```
# Plot of the European Stock Markets
plot.ts(EuStockMarkets,
        col='steelblue')
# For example
DAX <- EuStockMarkets[,1]
# Compute the return of the time series
DAX.r <- diff(log(DAX))
plot(DAX.r,type='l')
# fit GARCH(1,1) model for the return
DAX.garch <- garchFit(~garch(1,1),
         data=DAX.r,
         cond.dist = 'norm')
# Output the LaTex Code
xtable(DAX.garch@fit$matcoef,
       digits=4)
# Plot the estimated results
plot.ts(DAX.r,
     type='l',
     ylab='',
     col='darkgrey',
```

```
      col.axis='darkgrey')
par(new=T)
DAX.h <- ts(DAX.garch@fit$series$h,
    start = start(DAX),
    end = end(DAX),
    frequency = frequency(DAX))
plot.ts(DAX.h,
      type='l',
      col='steelblue',
      lwd=1,axes=F,
      ylab='')
axis(4,col="steelblue",
      col.ticks="steelblue",
      col.axis="steelblue")
```

5.10

```
# Plot all the climate time series
plot.ts(climhyd,
        col='steelblue')
# Plot the Precipitation and the prediction by SARIMA model
Precip.season <- sarima(sqrt(climhyd$Precip),
        p=0,d=0,q=0,
        P=0,D=1,Q=1,S=12)
Precip.season.predict <- sqrt(climhyd$Precip)+Precip.season$fit$
    residuals
plot.ts(sqrt(climhyd$Precip),
        col='steelblue',
        ylab='Precip',ylim=c(-10,50))
lines(Precip.season.predict,
      col='orange')
# Plot the inflow series and prediction by SARIMA model
inflow.season <- sarima(log(climhyd$Inflow),
                        p=0,d=0,q=0,
                        P=0,D=1,Q=1,S=12)
```

```
inflow.season.predict <- log(climhyd$Inflow)+inflow.season$fit$
   residuals
plot.ts(log(climhyd$Inflow),
        col='steelblue',
        ylab='Inflow',
        ylim=c(3,9))
lines(Precip.season.inflow.predict,
      col='orange')

# Prewhittened version of inflow and Plot
inflow.pw <- inflow.season$fit$residuals
plot(inflow.pw ,
     col='steelblue',
     ylab='Prewhitened flow residuals')
smal <- Precip.season.inflow$ttable[1]
# Construct the filter parameters
season_AR <- sapply(0:10,function(x)0.9815^x)*-0.0185
int <- rep(0,11)
AR_par <- c(1,int)
for(i in season_AR)
{
  AR_par <- c(AR_par,i,int)
}
pre.fil <- filter(sqrt(climhyd$Precip),
                  filter = AR_par,
                  sides=1)
# Plot the filtered transformed precipitation series
plot(pre.fil,
     col='steelblue',
     ylab='Filtered Precipitation series')
par(fig = c(0,0.35,.5,1),new = TRUE)
plot(AR_par,
     col='orange',
     type='l',xaxt='n',
     yaxt='n',
     ylab='',
```

```
        xlab='')
# Compute the CCF of the prewhitten version of inflow and filted
    transformed precipitation
ccf(inflow.pw,pre.fil,
     na.action=na.omit,
     panel.first=grid())
```

5.12

```
# Plot the original time series
matplot(econ5[,1:3],
        type='l',
        col= c('steelblue','orange','green'),
        ylab='Unemploy&GNP&Consum',
        lwd=2)
# Log transformation of econ5
econ5.log <- log(econ5)
m <- dim(econ5.log)[1]
time <- 1:m
# Least-square detrended time series
econ5.log.detrend <- apply(econ5.log,2,function(x)
  {
  fit <- lm(x~time)
  return(fit$residuals)
})
# Plot the detrended time series
matplot(econ5.log.detrend[,1:3],
        type='l',
        col= c('steelblue','orange','green'),
        ylab='Detrended Unemploy&GNP&Consum',
        lwd=2)
econ5.log.detrend.3 <- econ5.log.detrend[,1:3]

# VAR(econ5.log.detrend.3,lag.max = 100,ic='AIC')
# Define VARMA(1,1) model
```

```r
model <- define.model(kvar = 3,ar=c(1),ma=c(1))
arp <- model$ar.pattern
map <- model$ma.pattern
model.fit <- marima(econ5.log.detrend.3,
         ar.pattern = arp,
         ma.pattern = map)
# Obtain the fitted residuals
model.residual <- t(resid(model.fit))
colnames(model.residual) <- c('Unemploy',
                              'GNP',
                              'Consum')
# Plot the model Residuals
plot.ts(model.residual,
         col='steelblue',
         main='Model Residual')
# Output the LaTex Code
xtable(model.fit$ar.estimates[,,2],digits = 4)
xtable(model.fit$ma.estimates[,,2],digits=4)
model.fit$Constant
# Run ACF and PACF test of model residuals
par(mfrow=c(3,2))
acf(model.residual[,1][-1],
    main='Unemploy')
pacf(model.residual[,1][-1],
    main='Unemploy')
acf(model.residual[,2][-1],
    main='GNP')
pacf(model.residual[,2][-1],
     main='GNP')
acf(model.residual[,3][-1],
    main='Consum')
pacf(model.residual[,3][-1],
     main='Consum')
```