4) Using Crank – Nicholson method, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$,

Subject to $u(x,0)=0$, $u(0,t)=0$ and $u(1,t)=t$,

taking $h=0.5$ & $k=\frac{1}{8}$.

Soln:-

Given $u_{xx} = u_t$, $h=1/2$, $k=1/8$

WKT $u_{xx} = a u_t$ $\therefore a=1$

$$\therefore \lambda = \frac{k}{ah^2} = \frac{\frac{1}{8}}{1\times(1/2)^2} = 1/2$$

Since $\lambda = 1/2$, we cannot use simplified formula.

The general formula for Crank – Nicholson difference scheme or method is

$$\lambda(u_{i+1,j+1} + u_{i-1,j+1}) - 2(\lambda+1)u_{i,j+1} = 2(\lambda-1)u_{i,j} - \lambda(u_{i+1,j} + u_{i-1,j}) \longrightarrow (*)$$

Put $\lambda = 1/2$ in (*), the formula becomes.

$$\frac{1}{2}(u_{i+1,j+1} + u_{i-1,j+1}) - 2(3/2)u_{i,j+1} = 2(-1/2)u_{i,j} - 1/2(u_{i+1,j} + u_{i-1,j}).$$

$\div 1/2$!

$$\boxed{u_{i+1,j+1} + u_{i-1,j+1} - 6u_{i,j+1} = -2u_{i,j} - (u_{i+1,j} + u_{i-1,j})} \to (1)$$

x – direction $h=1/2$

$j \backslash i$	0	$1/2$	1
0	0	0 u_{ij}	0
$\frac{1}{8}$	0	u_i $(u_{i,j+1})$	$1/8$

t – direction $t=1/8$

$u(0,t)=0$

$\leftarrow j^{th}$

$\leftarrow j+1$ th.

$\uparrow$ $u(0,t)=0$ $\qquad$ $\uparrow$ $u(1,t)=t$

by using (1), $\frac{1}{8}+0-6u_1 = -2(0)-(0+0)$

$$u_1 = \frac{1}{48} = 0.0208333$$

$$\boxed{\therefore\ u_1 = 0.0208333}$$

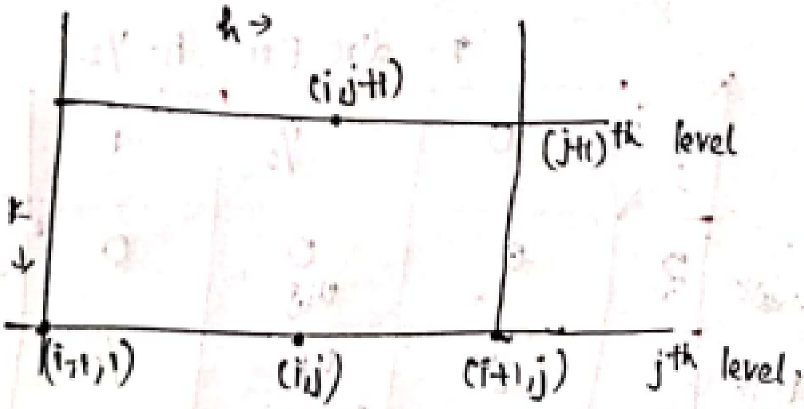
Note: ① Write an explicit formula to solve numerically the heat eqn. (parabolic eqn) $u_{xx} - au_t = 0$.

Soln: Bender - Schmidt method

$$\boxed{u_{i,j+1} = \lambda u_{i+1,j} + (1-2\lambda)u_{i,j} + \lambda u_{i-1,j}} \quad \text{where } \lambda = \frac{k}{ah^2}$$

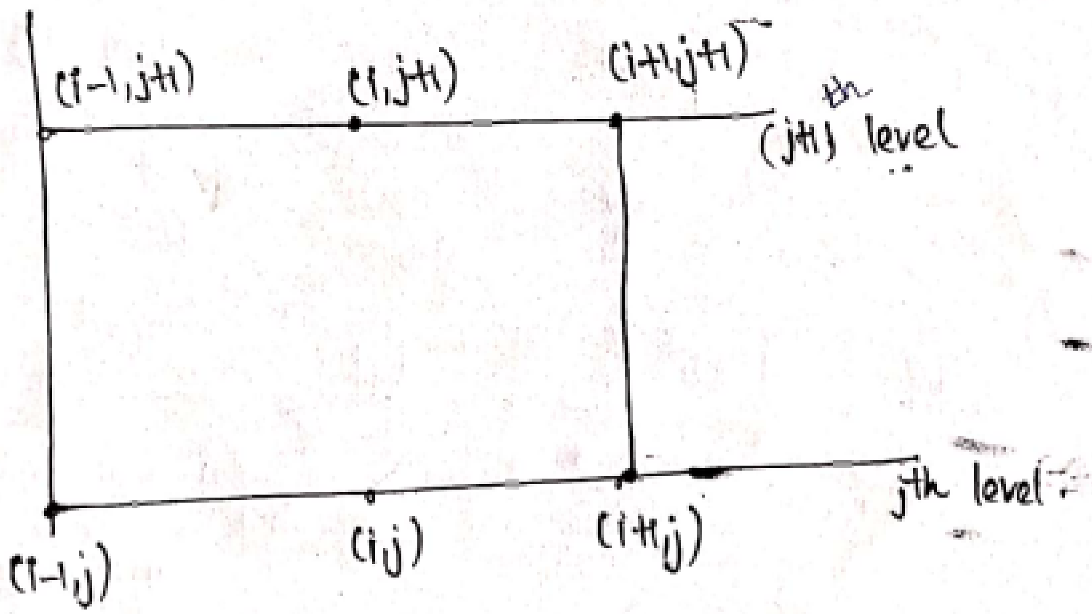
(h is the space for the variable x & k is the space in the time direction)

The above formula is a relation between the function values at the two levels $j+1$ and j and is called a two level formula. The solution value at any point $(i,j+1)$ on the $(j+1)$th level is expressed in terms of the solution values at the points $(i-1,j)$, (i,j) & $(i+1,j)$ on the j^{th} level. Such a method is called explicit formula. The formula is geometrically represented below.



2) Why is Crank - Nicholson's scheme called an implicit scheme?

Sol:- The schematic representation of Crank - Nicholson method is shown below.



The solution value at any point (i,j+1) on th (j+1)th level is dependent on the solution value the neighbouring points on the same level and three values on the jth level. Hence it is an implicit method.

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