Tuna Can Sensitivity Analysis – Runs Design

The design of the computational runs needed to perform the sensitivity analysis on the Tuna Can Case has been carried out according to Pavelic and Saxena (1969). They propose a factorial design approach which is very useful when the results of a computation are affected by a large number of variables and, then, studying the effect of each variable at a time would require too many, expensive and time consuming, runs. According to the factorial design approach a fixed number of levels for each variable is chosen and computations are run at all possible combinations of those levels.

In our case the attention is focused on the sensitivity of computations on three main variables: pool fire diameter, D_{PF}, wind speed, v_W, and position, P[1]. For D_{PF} and v_W, we chose two levels of variation, denoted by subscripts min and max, while P was determined both on an absolute and relative (to D_{PF}) basis, i.e. subscripts min, min_rel, max and max_rel respectively. Then we have:

- Pool fire diameter: $D_{PF,\,min} = 0.5$ m and $D_{PF,\,max} = 1$ m;
- Wind speed: $v_{W,\,min} = 0$ m/s and $v_{W,\,max} = 4$ m/s;
- Position: $P_{min} = (0, 1)$, $P_{min\_rel} = (0, D_{PF})$, $P_{max} = (R_{PF}, 0.5)$ and $P_{max\_rel} = (R_{PF}, 0.5\ D_{PF})$.

The number of runs to be performed is then equal to $\prod_i n_{l_i}^{\mathrm{var}_i}$, where n_{l_i} is the number of levels chosen for each set i of variables, var_i. In our case n_{l_i} equals 2 for D_{PF} and v_W ($\mathrm{var}_i = 2$) and 4 for P ($\mathrm{var}_i = 1$). The total number of tests is then $2^2 \cdot 4^1 = 16$. However, being the maximum diameter, D_{PF}, equal to 1 m, the levels (P_{min}, P_{min_rel}) and (P_{max}, P_{max_rel}) are exactly the same, thus reducing the total number of runs to 12.

We can then write down all the possible combinations for two levels of three variables as reported in Table 1.

To simplify writing all the possible combinations of the variables levels we can use a coding system so that the max and min conditions for each variable can be represented by +1 and -1 respectively (the max_rel and min_rel cases will be denoted by +1_rel and -1_rel). If x_1 represents the coded value of the pool fire diameter, D_{PF}, the relation between z_1 and D_{PF} is given by:

$$x_1 = \frac{D_{PF} - \langle D_{PF} \rangle}{\frac{D_{PF,\max} - D_{PF,\min}}{2}} = \frac{(D_{PF} - 0.75)}{0.25}$$

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[1] To reduce the number of variables and, then, the number of required runs, the position of the container in the fire can be described by a point P defined by a radius, R, and a height, H. The position can be varied only along the direction identified by $P_{min} = (R_{min}, H_{min})$ and $P_{max} = (R_{max}, H_{max})$.

which equals +1 if $D_{PF} = D_{PF,max}$, -1 if $D_{PF} = D_{PF,min}$ and 0 if $D_{PF} = \langle D_{PF} \rangle$. We can proceed similarly for the other variables. Then x_2 and x_3 will represent the coded values of v_W and P respectively.

Test No.	D_{PF}	v_W	P
1	0.5	0	P_{min}
2	0.5	0	P_{min_rel}
3	1	0	P_{min}, P_{min_rel}
4	0.5	4	P_{min}
5	0.5	4	P_{min_rel}
6	1	4	P_{min}, P_{min_rel}
7	0.5	0	P_{max}
8	0.5	0	P_{max_rel}
9	1	0	P_{max}, P_{max_rel}
10	0.5	4	P_{max}
11	0.5	4	P_{max_rel}
12	1	4	P_{max}, P_{max_rel}

Table 1 – Factorial design for Tuna Can Sensitivity Analysis.

All the possible combinations for two levels of three variables in term of coded values are reported in Table 2

Test No.	x_1	x_2	x_3
1	-1	-1	-1
2	-1	-1	-1_rel
3	+1	-1	-1
4	-1	+1	-1
5	-1	+1	-1_rel
6	+1	+1	-1
7	-1	-1	+1
8	-1	-1	+1_rel
9	+1	-1	+1
10	-1	+1	+1
11	-1	+1	+1_rel
12	+1	+1	+1

Table 2 - Factorial design for Tuna Can Sensitivity Analysis in terms of coded values.

If we consider our three coded variables as three mutually perpendicular coordinate axes, the factorial deign can be represented by a cube (Figure 1). The eight corner points of the cube represent the eight test conditions listed in coded values in Table 2, for the case obtained by varying the position of the container on an absolute basis. The same representation is obtained for the relative position case, by replacing the third indexes of the coordinate points from -1 to -1_rel and

+1 to +1_rel, respectively. The center of the cube represents physically the midvalue conditions for the three variables, i.e. D_{PF} = 0.75 m, v_W = 2 m/s and P = (0.5, 0.75).

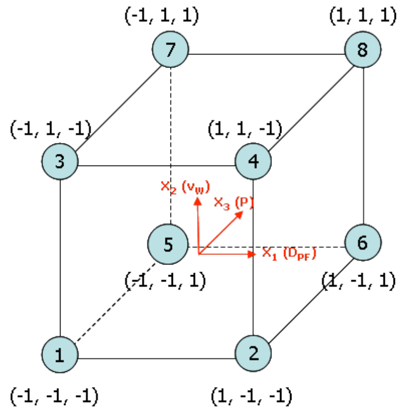

Figure 1 - Geometrical representation of factorial design. The Figure refers to the absolute position case.

References

Pavelic, V. and Saxena, U., “Statistical Experiment Design”, *Chemical Engineering*, October 6. 175-180, (1969)