Digital Signal Processing Ⅰ

6th Week EXPERIMENT

Report

(4th report of DSP1 course)

Subject	Digital Signal Processing Ⅰ
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Ⅰ. Abstraction

1. Discrete-time Fourier Transform

1-1. Definition of DTFT

If x[n] is absolutely summable, that is $\sum_{-\infty}^{\infty} |x[n]| < \infty$, then its discrete time Fourier transform is given by:

$$X(e^{jw}) = \mathcal{F}[x[n]] = \sum_{-\infty}^{\infty} x[n]e^{-jwn}$$

The inverse discrete-time Fourier transform(IDTFT) of $X(e^{jw})$ is given by:

$$x[n] = \mathcal{F}^{-1}[X(e^{jw})] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{jw})e^{jwn}dw$$

e.g. $x[n] = (0.5)^n u[n]$

Since the sequence x[n] is absolutely summable, its DTFT exists.

$$\begin{aligned} X(e^{jw}) &= \sum_{n=-\infty}^{\infty} x(n)e^{-jwn} = \sum_{n=0}^{\infty}(0.5)^n e^{-jwn} \\ &= \sum_{n=0}^{\infty}\left(0.5e^{-jw}\right)^n = \frac{1}{1-0.5e^{-jw}} = \frac{e^{jw}}{e^{jw}-0.5} \end{aligned}$$

1-2. Two important properties

1-2-1. Periodicity

The discrete-time Fourier transform $X(e^{jw})$ is periodic in ω with period 2π.

$$X(e^{jw}) = X(e^{j[w+2\pi]})$$

Therefore, only one period of $X(e^{jw})$ is needed. (i.e. $w \in [0, 2\pi]$, or $[-\pi, \pi]$, etc.) for analysis, not the whole domain.

1-2-2. Symmetry

For real-valued x[n], $X(e^{jw})$ is conjugate symmetric.

$$X\left(e^{-jw}\right) = X^*(e^{jw})$$

$$Re\left[X\left(e^{-jw}\right)\right] = Re\left[X\left(e^{jw}\right)\right] \quad (even\ symmetry)$$

$$Im\left[X\left(e^{-jw}\right)\right] = -Im\left[X\left(e^{jw}\right)\right] \quad (odd\ symmetry)$$

$$\left|X\left(e^{-jw}\right)\right| = \left|X\left(e^{jw}\right)\right| \quad (even\ symmetry)$$

$$\angle X\left(e^{-jw}\right) = -\angle X\left(e^{jw}\right) \quad (odd\ symmetry)$$

It implies that only a half period of $X(e^{jw})$ is needed to plot $X(e^{jw})$. Generally, in practice this period is chosen to be $\omega \in [0, \pi]$.

1-3. MATLAB implementation

Since x[n] has infinite duration, MATLAB cannot be used directly to compute $X(e^{jw})$ from x[n]. Instead, we can evaluate the expression $X(e^{jw})$ over [0, 2π] frequencies an then plot its magnitude and angle.

In addition, if we evaluate $X(e^{jw})$ at equispaced frequencies between [0, π], then can be implemented as a matrix-vector multiplication operation.

Let us assume that the sequence x[n] has N samples between $n_1 \leq n \leq n_N$ and that we want to evaluate $X(e^{jw})$ at

$$w_k = \frac{\pi}{M} k, \quad k = 0, 1, \dots, M$$

which are (M + 1) equispaced frequencies between [0, π].

$$X\left(e^{jw_k}\right) = \sum_{l=1}^{N} e^{-j\left(\frac{\pi}{M}\right)kn_l}\, x(n_l), \quad k = 0, 1, \ldots \ , M$$

Ⅱ. Exercises

In this part, there are four exercise questions. Each exercise consists of code and its result. All documents including Matlab code, result, and this report are uploaded in this website :

https://github.com/Gaon-Choi/ELE3076/tree/main/4_DTFT

Example 3.3

Evaluate $X(e^{jw})$ in Example 3.1 at 501 equispaced points between [0, π] and plot its magnitude, angle, real, and imaginary parts.

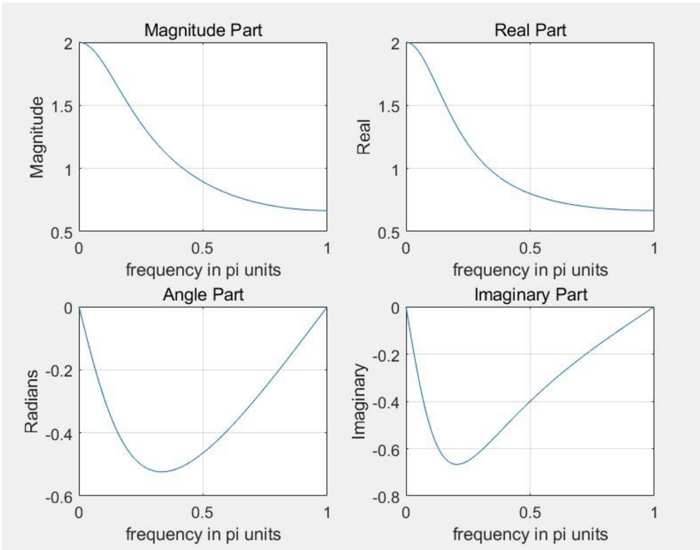
$$x[n] = (0.5)^n u[n] \;\rightarrow\; X(e^{jw}) = e\hat{}jw/(e\hat{}jw - 0.5)$$

(Matlab code)

```
w = [0:1:500] * pi / 500; % [0, pi] axis divided into 501
points.
X = exp(j * w) ./ (exp(j * w) - 0.5 * ones(1, 501));
magX = abs(X); angX = angle(X); % magX: magnitude, angX:
angle
realX = real(X); imagX = imag(X);   % realX: real part,
imagX: imaginary part

subplot(2, 2, 1); plot(w / pi, magX); grid
xlabel("frequency in pi units"); title("Magnitude Part");
ylabel("Magnitude");

subplot(2, 2, 3); plot(w / pi, angX); grid
xlabel("frequency in pi units"); title("Angle Part");
ylabel("Radians");

subplot(2, 2, 2); plot(w / pi, realX); grid
xlabel("frequency in pi units"); title("Real Part");
ylabel("Real");
```

```
subplot(2, 2, 4); plot(w / pi, imagX); grid
xlabel("frequency in pi units"); title("Imaginary Part");
ylabel("Imaginary");
```

(Result)



Example 3.4

Numerically compute the discrete-time Fourier transform of the sequence x(n) given in Example 3.2 at 501 equispaced frequencies between [0, π].

$$x[n] = \{1, 2, 3, 4, 5\}$$

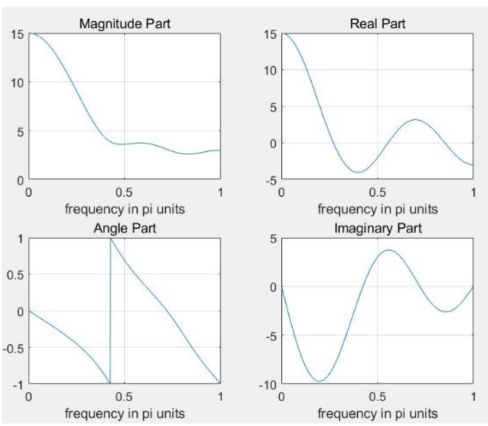
Then, the Fourier transform of x[n] is:

$$X(e^{jw}) = \sum_{-\infty}^{\infty} x(n)e^{-jwn} = e^{jw} + 2 + 3e^{-jw} + 4e^{-2jw} + 5e^{-3jw}$$

(Matlab code)

```
n = -1:3; x = 1:5; k = 0:500; w = (pi / 500) * k;
X = x * (exp(-j * pi / 500)) .^ (n'*k);
magX = abs(X); angX = angle(X);
realX = real(X); imagX = imag(X);

subplot(2, 2, 1); plot(k / 500, magX); grid
xlabel("frequency in pi units"); title("Magnitude Part");

subplot(2, 2, 3); plot(k / 500, angX / pi); grid
xlabel("frequency in pi units"); title("Angle Part");

subplot(2, 2, 2); plot(k / 500, realX); grid
xlabel("frequency in pi units"); title("Real Part");

subplot(2, 2, 4); plot(k / 500, imagX); grid
xlabel("frequency in pi units"); title("Imaginary Part");
```

(Result)



Example 3.5

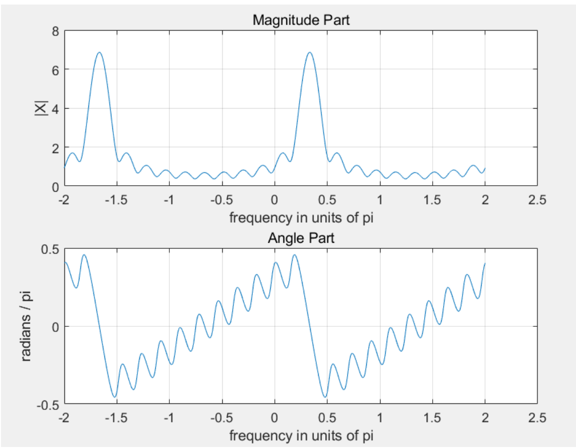
Let $x(n) = \left(0.9\,exp\left(\frac{j\pi}{3}\right)\right)^n$, 0 ≤ n ≤ 10. Determine $X(e^{jw})$ and investigate its periodicity.

Since x(n) is complex-valued, $X(e^{jw})$ satisfies only the periodicity property. Therefore it is uniquely defined over one period of 2π. However, we will evaluate and plot it at 401 frequencies over two periods between [-2π, 2π] to observe its periodicity.

(Matlab code)

```
n = 0:10; x = (0.9 * exp(j * pi / 3)) ^. n;
k = -200:200; w = (pi / 100) * k;

X = x * (exp(-j * pi / 100)) .^ (n'* k);
magX = abs(X); angX = angle(X);

% Magnitude Part
subplot(2, 1, 1); plot(w / pi, magX); grid
xlabel("frequency in units of pi"); ylabel("|X|")
title("Magnitude Part")

% Angle Part
subplot(2, 1, 2); plot(w / pi, angX / pi); grid
xlabel("frequency in units of pi"); ylabel("radians / pi")
title("Angle Part");
```

(Result)



Example 3.6

Let $x(n) = (0.9)^n$, -10 ≤ n ≤ 10. Investigate the conjugate-symmetry property of its discrete-time Fourier transform.

(Matlab code)

```
n = -10:10; x = (0.9).^n;
k = -200:200; w = (pi/100) * k; X = x * (exp(-
j*pi/100)).^(n'*k);
magX = abs(X); angX = angle(X);
subplot(2, 1, 1); plot(w/pi, magX); grid; axis([-2, 2, 0, 30])
xlabel("frequency in units of pi"); ylabel("|X|")
title("Magnitude Part")
subplot(2, 1, 2); plot(w/pi, angX/pi); grid; axis([-2, 2, -1,
1])
xlabel("frequency in units of pi"); ylabel("radians/pi")
title("Angle Part")
```

(Result)