Digital Signal Processing Ⅰ

4th Week EXPERIMENT

Report

(2nd report of DSP1 course)

Subject	Digital Signal Processing Ⅰ
Professor	Joon-Hyuk Chang
Submission Date	March 30th, 2021
University	Hanyang University
School	College of Engineering
Department	Department of Computer Science & Engineering
Student ID	**Name**
2019009261	최가온(CHOI GA ON)

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Ⅰ. Abstraction

1. Convolution

1-1. Definition of Convolution

Suppose there is an linear time-invariant system. h(n) is the impulse response. For an signal x(n), convolution is defined as:

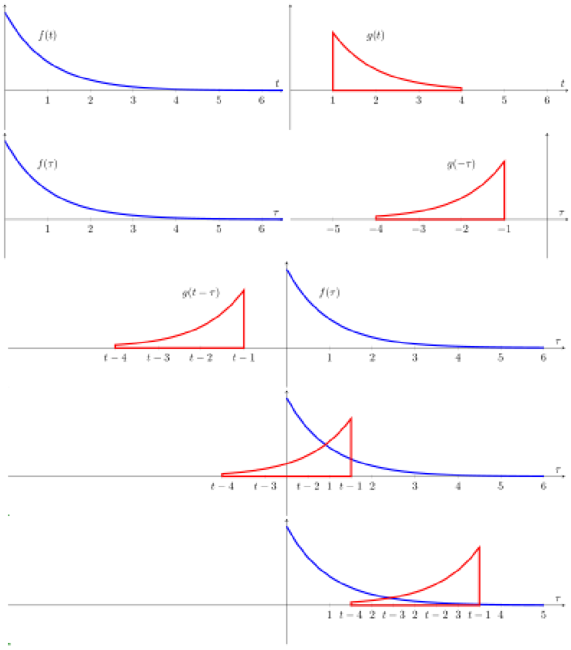
$$y(n) = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

And it is denoted by:

$$y(n) = x(n) * h(n)$$

1-2. Visual explanation of convolution

Convolution can be represented visually by fixing one signals, and shifting flipped other signal. This image[1] shows detailed process of convolution.



[1] https://en.wikipedia.org/w/index.php?title=Convolution&oldid=888199331

- Select one signal from original two signals

- Flip the selected signal

- Slide from t = -∞ to t = ∞

The selection of one signal doesn't change the convolution result.

1-3. Matlab implementation of convolution

The following code represents the implementation of convolution with Matlab.

편집기 - C:₩Users₩choig₩Documents₩MATLAB₩conv_m.m

conv_m.m

```
function [y, ny] = conv_m(x, nx, h, nh)
% Modified convolution routine for signal processing
% --------------------------------------------------
%  [y, ny] = conv_m(x, nx, h, nh)
%  [y, ny] = convolution result
%  [x, nx] = first signal
%  [h, nh] = second signal
%
nyb = nx(1) + nh(1); nye = nx(length(x)) + nh(length(h));
ny = [nyb:nye]; y = conv(x, h);
```

2. Correlation

2-1. Definition of Correlation

Correlation is an operation used in many applications in digital signal processing. It is a measure of the degree to which two sequences are similar.

$$y_{xh}(n) = \sum_{k=-\infty}^{\infty} x[k]h[k-n] = \sum_{k=-\infty}^{\infty} x[k]h[-(n-k)] = x[n] * h[-n]$$

Correlations are useful because they can indicate a predictive relationship that can be explained in practice.[2] For example, an electrical utility may produce less power on a mild day baased on the correlation between electricity demand and weather. In this example, there is a causal relationship, because extreme weather causes people to use more electricity for heating or cooling.

However, in general, the presence of a correlation is not sufficient to infer the presence of a causal relationship.

Correlation between two random variable X and Y with expected values μ_x and μ_y and standard deviations σ_x and σ_y is defined by:

$$\sigma_{X,Y} = corr(X,Y) = \frac{cov(X,Y)}{\sigma_x \sigma_y} = \frac{E\left[(X-\mu_x)(Y-\mu_y)\right]}{\sigma_x \sigma_y}$$

* cov(X, Y) : covariance(공분산)

2-2. Convolution, correlation, autocorrelation

1) Convolution

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$$

[2] https://en.wikipedia.org/wiki/Correlation_and_dependence#Definition

2) Correlation

$$r_{xh}[n] = \sum_{k=-\infty}^{\infty} x[k]h[k-n] = \sum_{k=-\infty}^{\infty} x[k]h[-(n-k)] = x[n] * h[-n]$$

3) Autocorrelation

$$r_{xx}[n] = \sum_{k=-\infty}^{\infty} x[k]x[k-n] = x[n] * x[-n]$$

Ⅱ. Exercises

In this part, there are 2 exercise questions. Each exercise consists of code and its result. All documents including Matlab code, result, and this report are uploaded in this website :

https://github.com/Gaon-Choi/ELE3076/tree/main/2_convolution

Example 2.8

Given the following two sequences,

x(n) = [3, 11, 7, 0, -1, 4, 2] $-3 \le n \le 3$;

h(n) = [2, 3, 0, -5, 2, 1] $-1 \le n \le 4$;

determine the convolution y(n) = x(n) * h(n).

(Matlab code)

편집기 - C:₩Users₩choig₩Documents₩MATLAB₩example2_8.m

example2_8.m

```
x = [3, 11, 7, 0, -1, 4, 2]; h = [2, 3, 0, -5, 2, 1];
y = conv(x, h)  % y(n) = x(n) * h(n)

```

(Result)

명령 창

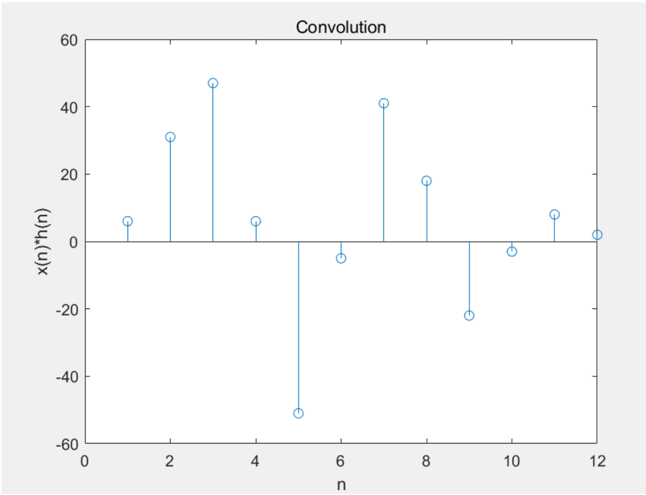
```
>> example2_8

y =

     6    31    47     6   -51    -5    41    18   -22    -3     8     2

>>
```

Additionally, y(n) can be plotted with "stem( )" function in Matlab basic function. The following shows the code and its result.

편집기 - C:₩Users₩choig₩Documents₩MATLAB₩example2_8.m

example2_8.m

```
x = [3, 11, 7, 0, -1, 4, 2]; h = [2, 3, 0, -5, 2, 1];
y = conv(x, h);  % y(n) = x(n) * h(n)
subplot(1, 1, 1);
stem(y);
title("Convolution")
xlabel('n'); ylabel('x(n)*h(n)');
```



Example 2.10

In this example we will demonstrate one application of the crosscorrelation sequence. Let

$$x(n) = [3, 11, 7, \underline{0}, -1, 4, 2]$$

be a prototype sequence, and let y(n) be its noise-corrupted-and-shifted version

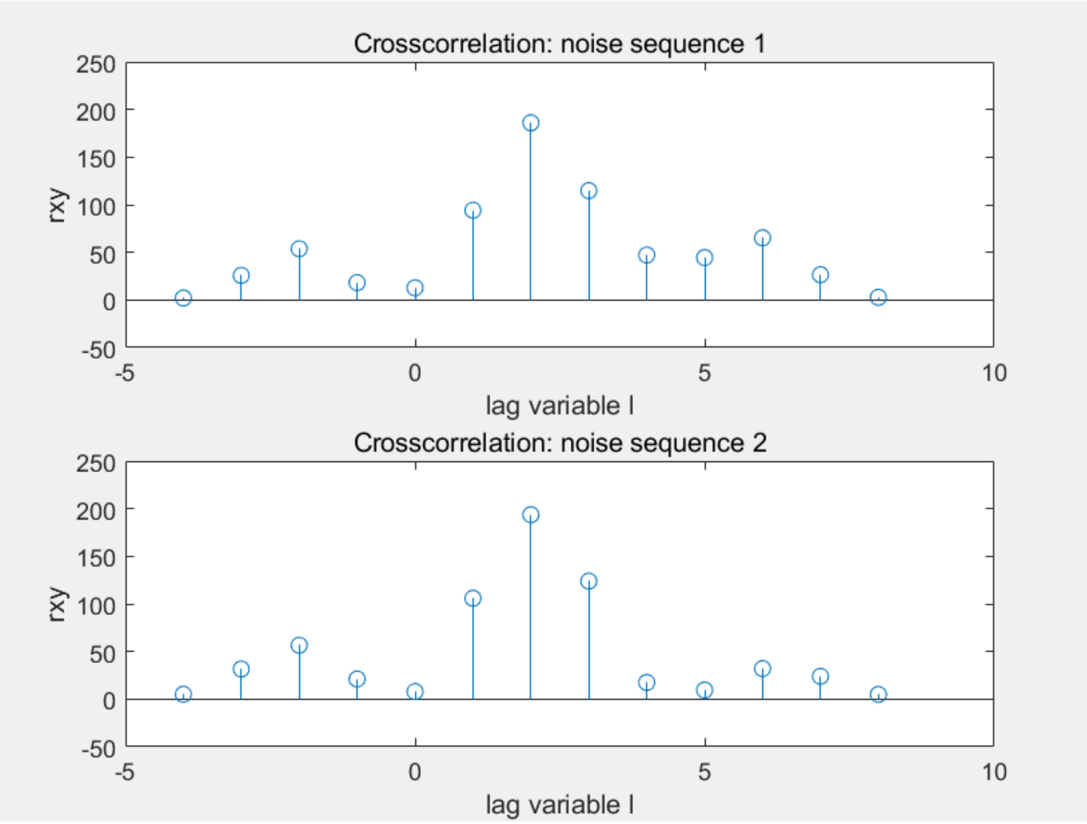
$$y(n) = x(n - 2) + w(n)$$

where w(n) is Gaussian sequence with mean 0 and variance 1. Compute the crosscorrelation between y(n) and x(n).

(Matlab code)

```
% noise sequence 1
x = [3, 11, 7, 0, -1, 4, 2];
nx = [-3:3];                           % given signal x(n)

[y, ny] = sigshift(x, nx, 2);          % obtain x(n - 2)
w = randn(1, length(y)); nw = ny;      % generate w(n)

[y, ny] = sigadd(y, ny, w, nw);        % obtain y(n) = x(n - 2) + w(n)
[x, nx] = sigfold(x, nx);              % obtain x(-n)

[rxy, nrxy] = conv_m(y, ny, x, nx);    % crosscorrelation
subplot(1, 1, 1), subplot(2, 1, 1);stem(nrxy, rxy)
axis([-5, 10, -50, 250]); xlabel('lag variable l')
ylabel('rxy'); title('Crosscorrelation: noise sequence 1')
%
% noise sequence 2
x = [3, 11, 7, 0, -1, 4, 2];
nx = [-3:3];                           % given signal x(n)

[y, ny] = sigshift(x, nx, 2);          % obtain x(n - 2)
w = randn(1, length(y)); nw = ny;      % generate w(n)

[y, ny] = sigadd(y, ny, w, nw);        % obtain y(n) = x(n - 2) + w(n)
[x, nx] = sigfold(x, nx);              % obtain x(-n)

[rxy, nrxy] = conv_m(y, ny, x, nx);    % crosscorrelation
subplot(2, 1, 2); stem(nrxy, rxy)
axis([-5, 10, -50, 250]); xlabel('lag variable l')
ylabel('rxy'); title('Crosscorrelation: noise sequence 2')
```

(Result)



Two graph always shows different result, because they are produced with randn function. We observe that the cross-correlation peaks at l=2, which implies that y[n] is similar to x[n] shifted by 2.