Digital Signal Processing Ⅱ

13^{th} EXPERIMENT

Report

(WEEK14 report of DSP2 course)

Subject	Digital Signal Processing Ⅱ
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Exercises

In this part, there are several exercise questions. Each exercise consists of code and its result. All documents including MATLAB code, result, and this report are uploaded in this website :

https://github.com/Gaon-Choi/ELE3077/tree/main/lab_experiment13

Exercise 1

exercise1-a)

Create the function 'imp_invr' that implement impulse invariance transformation from analog to digital filter.

(MATLAB Code) imp_invr.m

```
function[b, a] = imp_invr(c, d, T)
% c, d: denominator and numerator coefficient of Laplace form
% T: sampling period
% b, a: denominator and numerator coefficient after applying mapping
%       (iminv or bilinear) on z-transform

[R, p, k] = residue(c, d);
p = exp(p*T);
% residue : polynomial coefficient -> pole, zero, k(gain)
% residuez : pole, zero, k(gain) -> polynomial coefficient
[b, a] = residuez(R, p, k);
b = real(b'); a = real(a');
```

(Results)

(SKIP)

exercise1-b)

Transform

$$H_a(s) = \frac{s+1}{s^2+5s+6} = \frac{2}{s+3} - \frac{1}{s+2}$$

into a digital filter $H(z)$ by using 'imp_invr' function in which T=0.1.

(MATLAB Code) lab14_exercise1_b.m

```
b = [0, 1, 1];
a = [1, 5, 6];
[bz, az] = imp_invr(b, a, 0.1)
% imp_inva: laplace form -> impulse invariance
% we need to specify the rational form.
```

(Results)

```
명령 창
>> lab14_exercise1_b

bz =

    1.0000
   -0.8966


az =

    1.0000
   -1.5595
    0.6065

fx >>
```

The rational z-transform of the output is:

$$H(z) = \frac{2}{1-e^{-3T}z^{-1}} - \frac{1}{1-e^{-2T}z^{-1}} = \frac{1-0.8966z^{-1}}{1-1.5595z^{-1}+0.6065z^{-2}}$$

exercise1-c)

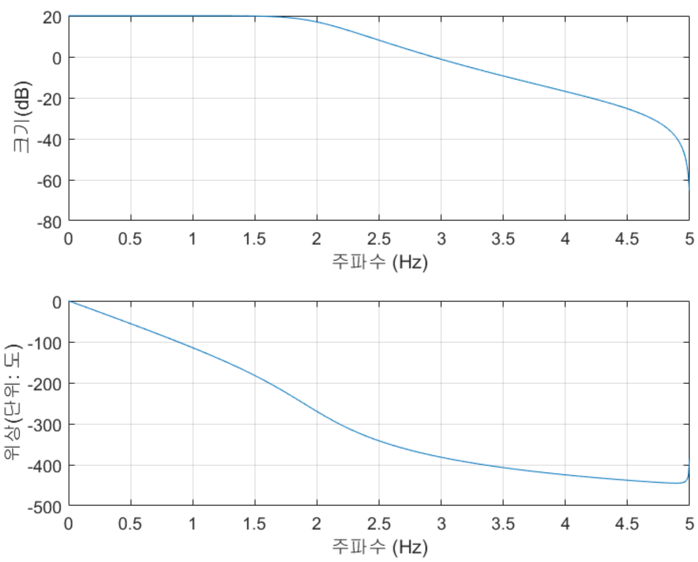
Convert a sixth-order analog Butterworth low-pass filter to a digital filter by using impulse invariance. Specify a sample rate of 10Hz and a cutoff frequency of 2 Hz. And plot its magnitude and its angle versus angular frequency.

(MATLAB Code) lab14_exercise1_c.m

```
f = 2;      % cut-off frequency
fs = 10;    % sampling frequency which is argument of our
imp_invr = 1/10 = 0.1
% [b, a] = butter(6, 2(10/2)) -> original
[b, a] = butter(6, 2*pi*f, 's');
[bz, az] = imp_invr(b, a, 0.1);

figure(1);
freqz(bz, az, 2048, fs);    % analog form graph with
specified fs
% figure(2);
% freqz(bz, az);
```

(Results)



exercise1-d)

Convert a third-order analog elliptic filter to a digital filter using impulse invariance. Specify a sample rate fs=10 Hz, a passband edge frequency of 2.5 Hz, a passband ripple of 1 dB, and a stopband attenuation of 60 dB. Plot its impulse response.

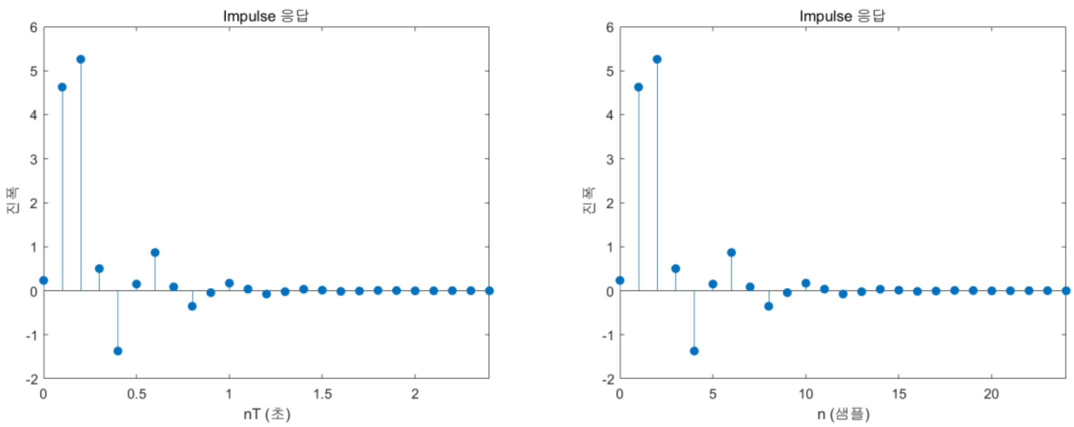
(Hint: you can use the function 'ellip' and 'impz')

(MATLAB Code) lab14_exercise1_d.m

```
% Q1-d
% requires understandings for elliptic filter
% simple API: specify specs as parameters

fs = 10;
fc = 2.5;   % WARNING: analog form -> 2pi*fc
[b, a] = ellip(3, 1, 60, 2*pi*2.5, 's');
[bz, az] = imp_invr(b, a, 1/fs);    % 3rd argument: period
figure(1)
impz(bz, az, [], fs);
% impz: returns impulse response of given transfer function
%       corresponding the coefficient of numerators
%       and denominators of given rational function.
figure(2)
impz(bz, az)
```

(Results)



Exercise 2

exercise2-a)

Transform

$$H(z) = \frac{s+1}{s^2+5s+6} = \frac{2}{s+3} - \frac{1}{s+2}$$

into a digital filter $H(z)$ by using the bilinear transformation which $T = 1$.

(MATLAB Code) lab14_exercise2_a.m

```
c = [1, 1];      % numerator
d = [1, 5, 6];  % denominator
T = 1;
Fs = 1/T;
[b, a] = bilinear(c, d, Fs)
```

(Results)

```
명령 창
>> lab14_exercise2_a

b =

    0.1500    0.1000   -0.0500


a =

    1.0000    0.2000   -0.0000

fx >>
```

The digital filter $H(z)$ is:

$$H(z) = \frac{0.15 + 0.1z^{-1} - 0.05z^{-2}}{1 + 0.2z^{-1}} = 0.15 * \frac{1 + \frac{2}{3}z^{-1} - \frac{1}{3}z^{-2}}{1 + 0.2z^{-1}}$$

Exercise 2

exercise2-b)

Design a 6th-order elliptic analog low-pass filter with 3 dB of ripple in the passband and a stopband 90 dB down. Set cutoff frequency $fc = 20\,Hz$ and sample rate $fs = 200\,Hz$. And transform it to a digital filter by using bilinear transform.

(set math frequency : $20\,Hz$)

(MATLAB Code) lab14_exercise2_b.m

```
fc = 20;
fs = 200;
[z, p, k] = ellip(6, 3, 90, 2*pi*fc, 's');
[num,den] = zp2tf(z, p, k);

[BL_num, BL_den] = bilinear(num, den, fs);
freqz(BL_num, BL_den);
```

- zp2tf function transforms zeros and poles into numerators and denominators with polynomial forms.
- bilinear(num, den, fs) converts the s-domain transfer function specified by numerator num and denominator den to a discrete equivalent.

(Results)