Digital Signal Processing Ⅱ

5^{th} EXPERIMENT

Report

(5th report of DSP2 course)

Subject	Digital Signal Processing Ⅱ
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Exercises

In this part, there are several exercise questions. Each exercise consists of code and its result. All documents including MATLAB code, result, and this report are uploaded in this website :

https://github.com/Gaon-Choi/ELE3077/tree/main/lab_experiment05

Exercise 1

1-1. Plot the pole-zero diagram (zplane) and the frequency response (magnitude (mag) & phase (angle)) of each 1st-order filter discussed in Week 5 lecture 1. Include appropriate labels along the x-axis and the y-axis.

- Low-pass filter: one with zero at z=0 and another with zero at z=-1

  to suppress high frequency components

- High-pass filter: one with zero at z=0 and another with zero at z=1

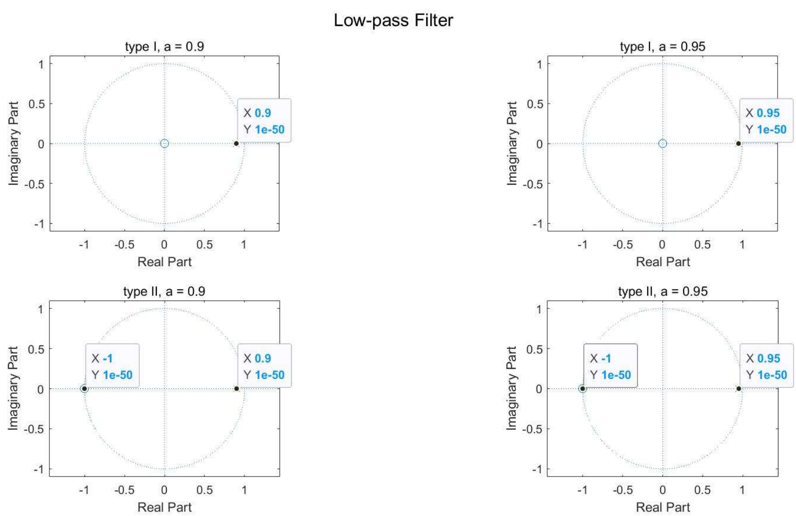
  to suppress low frequency components

(MATLAB Code) lab4_exercise1_1.m

```
subplot(2,2,1);
num = [1];   den = [1, -0.9];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("type I, a = 0.9");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,2);
num = [1];   den = [1, -0.95];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("type I, a = 0.95");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,3);
num = [1, 1];   den = [1, -0.9];
```

```
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("type II, a = 0.9");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,4);
num = [1, 1];   den = [1, -0.95];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("type II, a = 0.95");
xlabel("Real Part");    ylabel("Imaginary Part");

sgtitle("Low-pass Filter")
```

(Result)

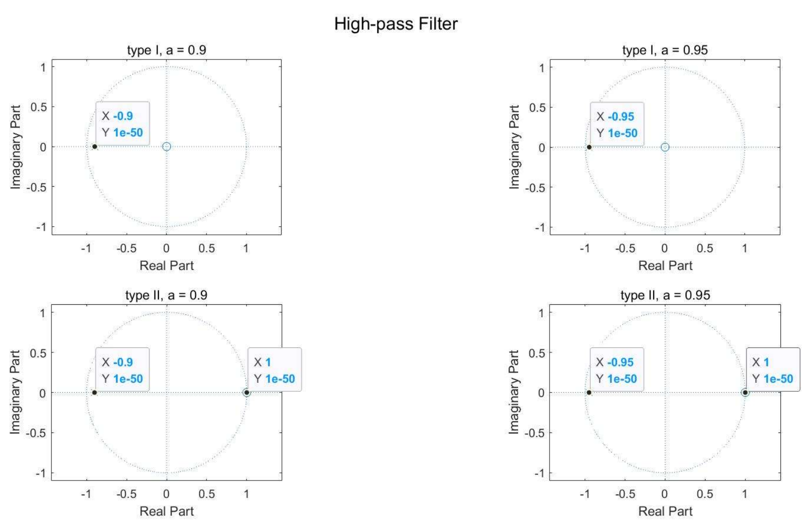


(MATLAB Code) lab4_exercise1_2.m

```
subplot(2,2,1);
num = [1];   den = [1, 0.9];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("type I, a = 0.9");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,2);
num = [1];   den = [1, 0.95];
```

```
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("type I, a = 0.95");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,3);
num = [1, -1];   den = [1, 0.9];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("type II, a = 0.9");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,4);
num = [1, -1];   den = [1, 0.95];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("type II, a = 0.95");
xlabel("Real Part");    ylabel("Imaginary Part");

sgtitle("High-pass Filter")
```

(Result)



1-2. For each of the 4 filters

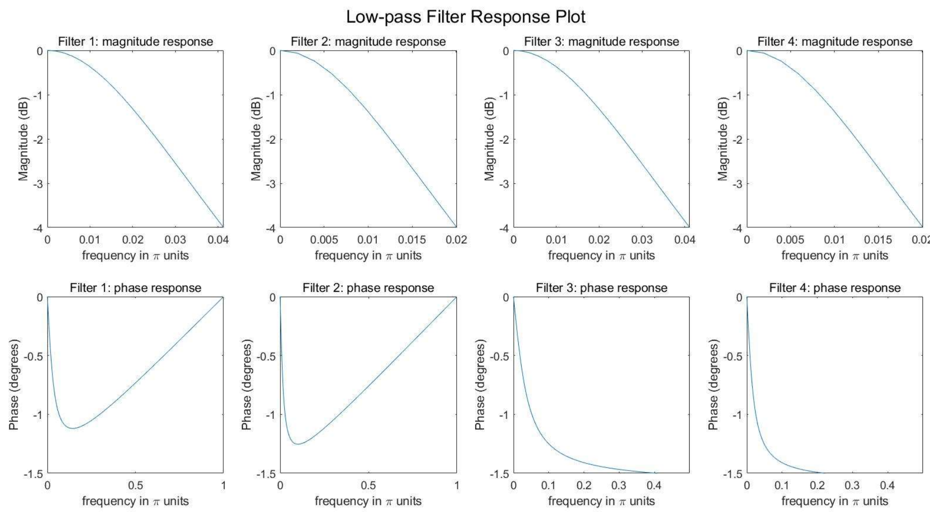
- Show the responses for different pole magnitude values $a(0.9, 0.95)$.
  - numerically check the 3dB roll-off frequency for $a(0.9, 0.95)$
- Include analytic derivations for the value of the gain G to make the filter amplitude 1 at $\Omega = 0$ for low-pass filter & $\Omega = \pi$ for high-pass filter.
  (In terms of a and b)

(MATLAB Code) lab5_exercise1_3.m

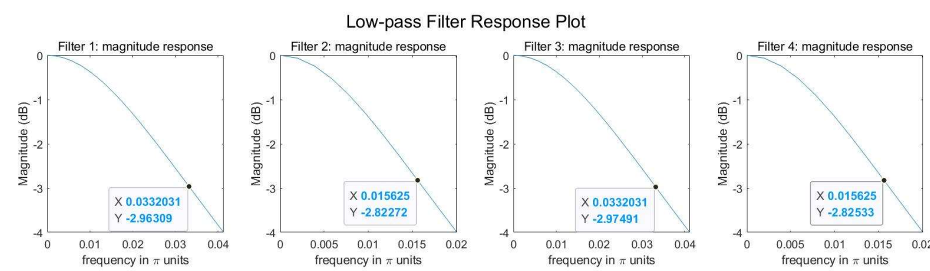
```
N = 512;

num = [1];   den = [1, -0.9];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h = angle(h);
subplot(2,4,1);
plot(w/pi, mag_h); title("Filter 1: magnitude response"); ylim([-60,40]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,5);
plot(w/pi, ang_h); title("Filter 1: phase response"); ylim([-1.5,0]);
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = [1];   den = [1, -0.95];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h = angle(h);
subplot(2,4,2);
plot(w/pi, mag_h); title("Filter 2: magnitude response"); ylim([-60,40]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,6);
plot(w/pi, ang_h); title("Filter 2: phase response"); ylim([-1.5,0]);
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = [1, 1];   den = [1, -0.9];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h = angle(h);
subplot(2,4,3);
plot(w/pi, mag_h); title("Filter 3: magnitude response"); ylim([-60,40]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,7);
plot(w/pi, ang_h); title("Filter 3: phase response"); ylim([-1.5,0]);
xlim([0, 0.5]);
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = [1, 1];   den = [1, -0.95];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h = angle(h);
subplot(2,4,4);
plot(w/pi, mag_h); title("Filter 4: magnitude response"); ylim([-60,40]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,8);
plot(w/pi, ang_h); title("Filter 4: phase response"); ylim([-1.5,0]);
xlim([0, 0.5]);
```

```
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");
sgtitle("Low-pass Filter Response Plot");
```

(Result)



(3dB cut-off frequency info.)

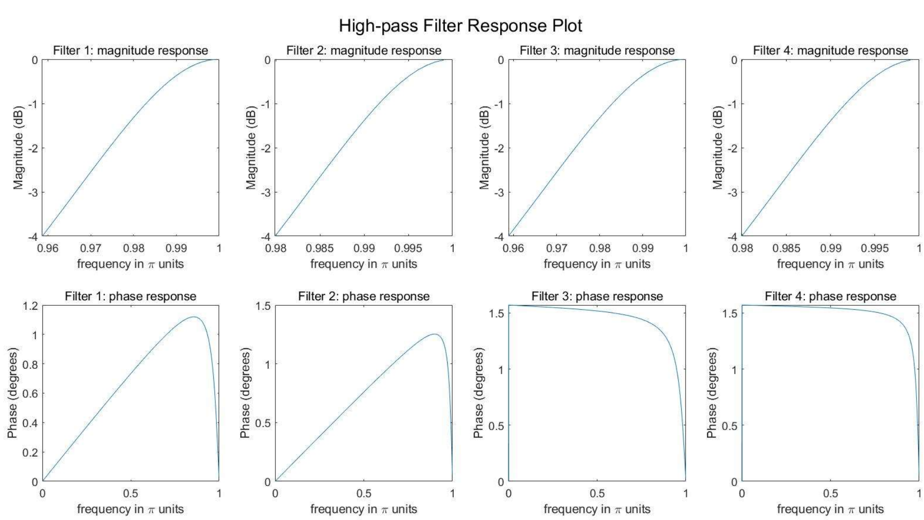


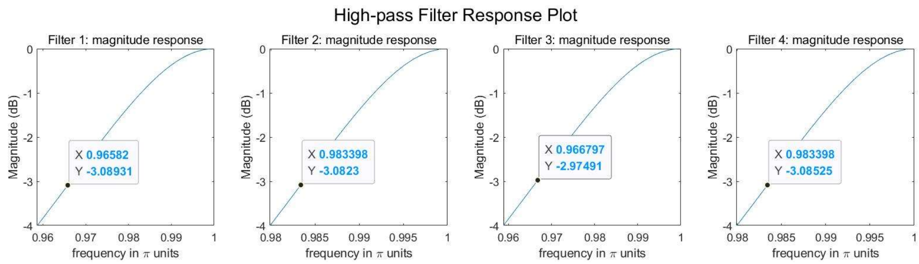
(MATLAB Code) lab5_exercise1_4.m

```
N = 1024;
G1 = 1/10; G2 = 1/20; G3 = 1/20; G4 = 1/40;

num = G1 * [1];   den = [1, 0.9];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h = angle(h);
subplot(2,4,1);
plot(w/pi, mag_h); title("Filter 1: magnitude response"); ylim([-4,0]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,5);
plot(w/pi, ang_h); title("Filter 1: phase response");
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = G2 * [1];   den = [1, 0.95];
zeros = roots(num); poles = roots(den);
```

```
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h = angle(h);
subplot(2,4,2);
plot(w/pi, mag_h); title("Filter 2: magnitude response"); ylim([-4,0]);
xlabel("frequency in \pi units");   ylabel("Magnitude (dB)");
subplot(2,4,6);
plot(w/pi, ang_h); title("Filter 2: phase response");
xlabel("frequency in \pi units");   ylabel("Phase (degrees)");

num = G3 * [1, -1];   den = [1, 0.9];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h = angle(h);
subplot(2,4,3);
plot(w/pi, mag_h); title("Filter 3: magnitude response"); ylim([-4,0]);
xlabel("frequency in \pi units");   ylabel("Magnitude (dB)");
subplot(2,4,7);
plot(w/pi, ang_h); title("Filter 3: phase response");
xlabel("frequency in \pi units");   ylabel("Phase (degrees)");

num = G4 * [1, -1];   den = [1, 0.95];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h = angle(h);
subplot(2,4,4);
plot(w/pi, mag_h); title("Filter 4: magnitude response"); ylim([-4,0]);
xlabel("frequency in \pi units");   ylabel("Magnitude (dB)");
subplot(2,4,8);
plot(w/pi, ang_h); title("Filter 4: phase response");
xlabel("frequency in \pi units");   ylabel("Phase (degrees)");

sgtitle("High-pass Filter Response Plot");
```

(Result)





Case 1: LOW-PASS FILTER

i) low pass filter with type I, $a = 0.9$

$$|H(z)| = \left|G_1 \frac{1}{1 - 0.9z^{-1}}\right| = 1$$

$$z = e^{j0} = 1 \;\rightarrow\; |H(z)| = 1 \cdot G_1 \cdot \frac{1}{|1 - 0.9|} = 1 \rightarrow G_1 = \frac{1}{10}$$

ii) low pass filter with type I, $a = 0.95$

$$|H(z)| = \left|G_2 \frac{1}{1 - 0.95z^{-1}}\right| = 1$$

$$z = e^{j0} = 1 \;\rightarrow\; |H(z)| = 1 \cdot G_2 \cdot \frac{1}{|1 - 0.95|} = 1 \rightarrow G_2 = \frac{1}{20}$$

iii) low pass filter with type II, $a = 0.9$

$$|H(z)| = \left|G_3 \frac{1 + z^{-1}}{1 - 0.9z^{-1}}\right| = 1$$

$$z = e^{j0} = 1 \;\rightarrow\; |H(z)| = 1 \cdot G_3 \cdot \frac{|1 + 1|}{|1 - 0.9|} = 1 \rightarrow G_3 = \frac{1}{20}$$

iv) low pass filter with type II, $a = 0.95$

$$|H(z)| = \left|G_4 \frac{1 + z^{-1}}{1 - 0.95z^{-1}}\right| = 1$$

$$z = e^{j0} = 1 \;\rightarrow\; |H(z)| = 1 \cdot G_4 \cdot \frac{|1 + 1|}{|1 - 0.95|} = 1 \rightarrow G_4 = \frac{1}{40}$$

Case 2: HIGH-PASS FILTER

i) high pass filter with type I, $a = 0.9$

$$|H(z)| = \left|G_1 \frac{1}{1 + 0.9z^{-1}}\right| = 1$$

$$z = e^{j\pi} = -1 \ \rightarrow \ |H(z)| = 1 \cdot G_1 \cdot \frac{|1|}{|1-0.9|} = 1 \rightarrow G_1 = \frac{1}{10}$$

ii) high pass filter with type I, $a = 0.95$

$$|H(z)| = \left| G_2 \frac{1}{1+0.95z^{-1}} \right| = 1$$

$$z = e^{j\pi} = -1 \ \rightarrow \ |H(z)| = 1 \cdot G_2 \cdot \frac{|1|}{|1-0.95|} = 1 \rightarrow G_2 = \frac{1}{20}$$

iii) high pass filter with type II, $a = 0.9$

$$|H(z)| = \left| G_3 \frac{1-z^{-1}}{1+0.9z^{-1}} \right| = 1$$

$$z = e^{j\pi} = -1 \ \rightarrow \ |H(z)| = 1 \cdot G_3 \cdot \frac{|1+1|}{|1-0.9|} = 1 \rightarrow G_3 = \frac{1}{20}$$

iv) high pass filter with type II, $a = 0.95$

$$|H(z)| = \left| G_4 \frac{1-z^{-1}}{1+0.95z^{-1}} \right| = 1$$

$$z = e^{j\pi} = -1 \ \rightarrow \ |H(z)| = 1 \cdot G_4 \cdot \frac{|1+1|}{|1-0.95|} = 1 \rightarrow G_4 = \frac{1}{40}$$

Exercise 2

2-1. Plot the pole-zero diagram (zplane) and the frequency response (magnitude (mag) & phase (angle)) of each 2nd-order filter discussed in the last lecture.

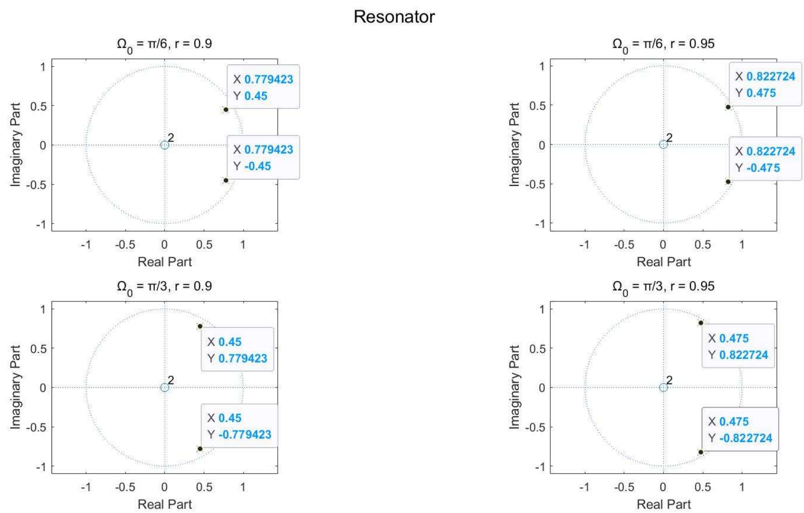
- Resonator
- Notch Filter
- Set the frequency of interest $\Omega_0 = \frac{\pi}{6}$ and $\Omega_0 = \frac{\pi}{3}$
- Set values $r = 0.9$ and $r = 0.95$

(MATLAB Code) lab4_exercise2_1.m

```
r1 = 0.9;   r2 = 0.95;
s1 = pi / 6;    s2 = pi / 3;

subplot(2,2,1);
num = [1];   den = [1, -2 * r1 * cos(s1), r1 * r1];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("¥Ø_0 = ¥ð/6, r = 0.9");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,2);
num = [1];   den = [1, -2 * r2 * cos(s1), r2 * r2];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("¥Ø_0 = ¥ð/6, r = 0.95");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,3);
num = [1];   den = [1, -2 * r1 * cos(s2), r1 * r1];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("¥Ø_0 = ¥ð/3, r = 0.9");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,4);
num = [1];   den = [1, -2 * r2 * cos(s2), r2 * r2];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("¥Ø_0 = ¥ð/3, r = 0.95");
```

```
xlabel("Real Part");    ylabel("Imaginary Part");

sgtitle("Resonator");
```

(Result)

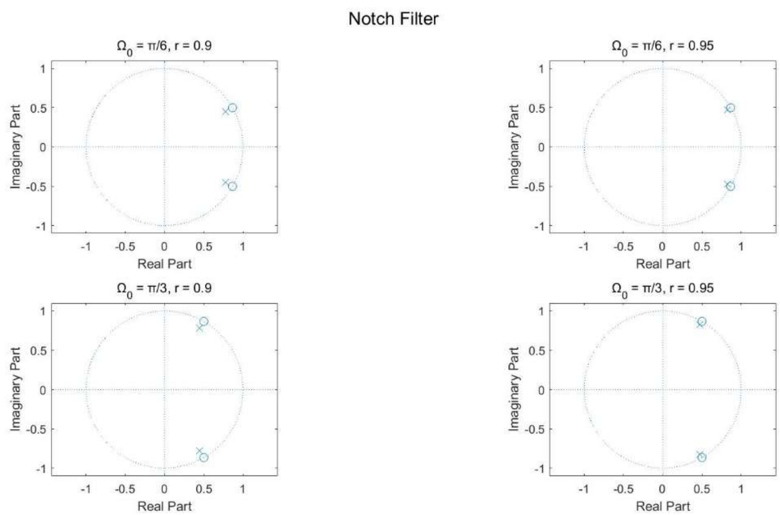


(MATLAB Code) lab4_exercise2_2.m

```
r1 = 0.9;   r2 = 0.95;
s1 = pi / 6;    s2 = pi / 3;

subplot(2,2,1);
num = [1, -2 * cos(s1), 1];   den = [1, -2 * r1 * cos(s1), r1 * r1];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("¥Ø_0 = ¥ð/6, r = 0.9");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,2);
num = [1, -2 * cos(s1), 1];   den = [1, -2 * r2 * cos(s1), r2 * r2];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("¥Ø_0 = ¥ð/6, r = 0.95");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,3);
num = [1, -2 * cos(s2), 1];   den = [1, -2 * r1 * cos(s2), r1 * r1];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
```

```
title("¥Ø_0 = ¥ð/3, r = 0.9");
xlabel("Real Part");    ylabel("Imaginary Part");

subplot(2,2,4);
num = [1, -2 * cos(s2), 1];   den = [1, -2 * r2 * cos(s2), r2 * r2];
zeros = roots(num);
poles = roots(den);
zplane(zeros, poles);   zplane(num, den);
title("¥Ø_0 = ¥ð/3, r = 0.95");
xlabel("Real Part");    ylabel("Imaginary Part");

sgtitle("Notch Filter");
```

(Result)



2-2.

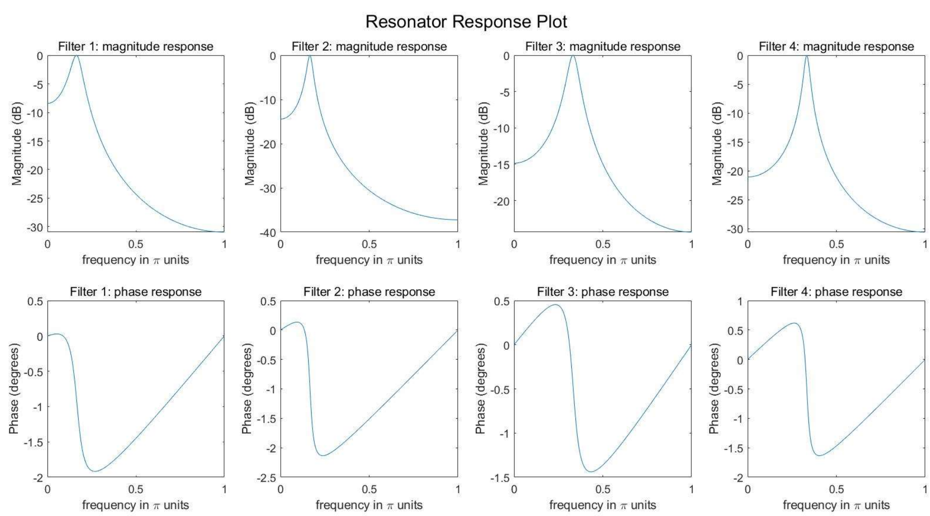
- Set the frequency of interest $\Omega_0 = \frac{\pi}{6}$ and $\Omega_0 = \frac{\pi}{3}$
- Show the responses for different pole magnitude values $r(0.9, 0.95)$
- Include analytic derivations for the value of the gain $G(r, \Omega_0)$ to make the filter amplitude 1 at $\Omega = \Omega_0$ for the resonator and $\Omega = 0$ for the notch filter. (in terms of r, Ω_0)

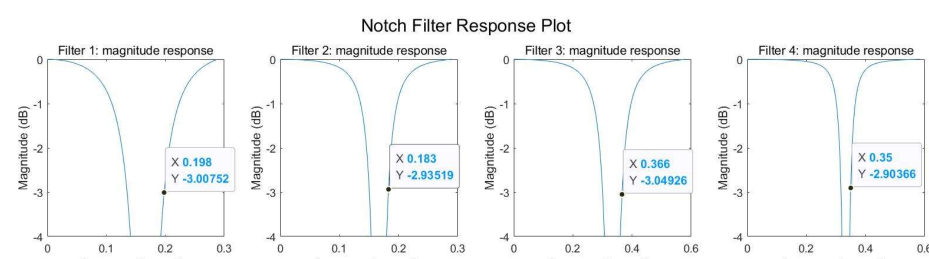
(MATLAB Code) lab4_exercise2_3.m

```
r1 = 0.9;   r2 = 0.95;
s1 = pi / 6;    s2 = pi / 3;
G1 = 0.1 * abs(1-0.9*exp(-j*pi/3));
G2 = 0.05 * abs(1-0.95*exp(-j*pi/3));
G3 = 0.1 * abs(1-0.9*exp(-j*2*pi/3));
G4 = 0.05 * abs(1-0.95*exp(-j*2*pi/3));

num = G1*[1];   den = [1, -2 * r1 * cos(s1), r1 * r1];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h =
angle(h);
subplot(2,4,1);
plot(w/pi, mag_h); title("Filter 1: magnitude response");
ylim([-4, 0]);
xlabel("frequency in \pi units"); ylabel("Magnitude (dB)");
subplot(2,4,5);
plot(w/pi, ang_h); title("Filter 1: phase response");
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = G2*[1];   den = [1, -2 * r2 * cos(s1), r2 * r2];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h =
angle(h);
subplot(2,4,2);
plot(w/pi, mag_h); title("Filter 2: magnitude response");
ylim([-4, 0]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,6);
plot(w/pi, ang_h); title("Filter 2: phase response");
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = G3*[1];   den = [1, -2 * r1 * cos(s2), r1 * r1];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h =
angle(h);
```

```
subplot(2,4,3);
plot(w/pi, mag_h); title("Filter 3: magnitude response");
ylim([-4, 0]);
xlabel("frequency in \pi units"); ylabel("Magnitude (dB)");
subplot(2,4,7);
plot(w/pi, ang_h); title("Filter 3: phase response");
xlabel("frequency in \pi units"); ylabel("Phase (degrees)");

num = G4*[1];   den = [1, -2 * r2 * cos(s2), r2 * r2];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h =
angle(h);
subplot(2,4,4);
plot(w/pi, mag_h); title("Filter 4: magnitude response");
ylim([-4, 0]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,8);
plot(w/pi, ang_h); title("Filter 4: phase response");
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

sgtitle("Resonator Response Plot");
```

(Result)



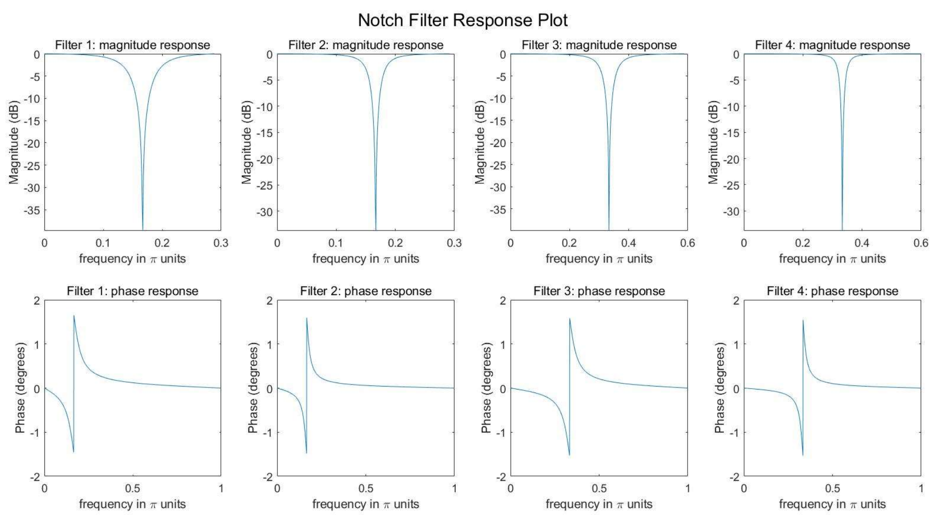


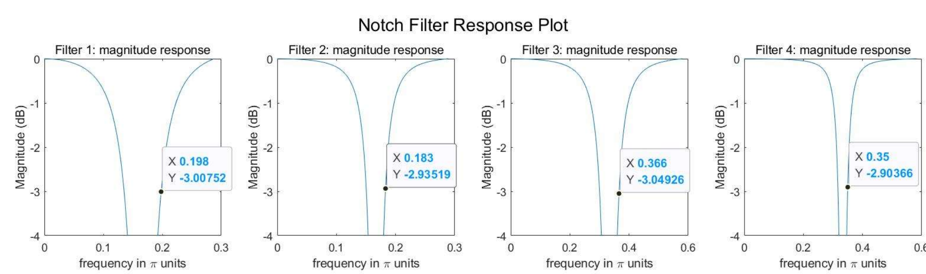
(MATLAB Code) lab4_exercise2_4.m

```
r1 = 0.9;   r2 = 0.95;
s1 = pi / 6;    s2 = pi / 3;

G1 = abs(1-r1*exp(j*s1)) * abs(1-r1*exp(-j*s1)) / (abs(1-
exp(j*s1)) * abs(1-exp(-j*s1)));
G2 = abs(1-r2*exp(j*s1)) * abs(1-r2*exp(-j*s1)) / (abs(1-
exp(j*s1)) * abs(1-exp(-j*s1)));
G3 = abs(1-r1*exp(j*s2)) * abs(1-r1*exp(-j*s2)) / (abs(1-
exp(j*s2)) * abs(1-exp(-j*s2)));
G4 = abs(1-r2*exp(j*s2)) * abs(1-r2*exp(-j*s2)) / (abs(1-
exp(j*s2)) * abs(1-exp(-j*s2)));

num = G1 * [1, -2 * cos(s1), 1];   den = [1, -2 * r1 * cos(s1), r1
* r1];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h =
angle(h);
subplot(2,4,1);
plot(w/pi, mag_h); title("Filter 1: magnitude response"); ylim([-
inf, 0]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,5);
plot(w/pi, ang_h); title("Filter 1: phase response");
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = G2*[1, -2 * cos(s1), 1];   den = [1, -2 * r2 * cos(s1), r2 *
r2];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h =
angle(h);
subplot(2,4,2);
plot(w/pi, mag_h); title("Filter 2: magnitude response");ylim([-
inf, 0]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,6);
plot(w/pi, ang_h); title("Filter 2: phase response");
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = G3*[1, -2 * cos(s2), 1];   den = [1, -2 * r1 * cos(s2), r1 *
r1];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h =
angle(h);
subplot(2,4,3);
plot(w/pi, mag_h); title("Filter 3: magnitude response");ylim([-
inf, 0]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,7);
plot(w/pi, ang_h); title("Filter 3: phase response");
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

num = G4*[1, -2 * cos(s2), 1];   den = [1, -2 * r2 * cos(s2), r2 *
```

```
r2];
zeros = roots(num); poles = roots(den);
[h, w] = freqz(num, den, N); mag_h = 20 * log10(abs(h)); ang_h =
angle(h);
subplot(2,4,4);
plot(w/pi, mag_h); title("Filter 4: magnitude response");ylim([-
inf, 0]);
xlabel("frequency in \pi units");    ylabel("Magnitude (dB)");
subplot(2,4,8);
plot(w/pi, ang_h); title("Filter 4: phase response");
xlabel("frequency in \pi units");    ylabel("Phase (degrees)");

sgtitle("Notch Filter Response Plot");
```

(Result)





I. Resonator

$$\left|H\left(e^{\Omega_0}\right)\right| = G\frac{1}{|1 - re^{j\Omega_0}e^{-j\Omega_0}||1 - re^{-j\Omega_0}e^{-j\Omega_0}|} = 1$$

$$G = |1 - r||1 - re^{-j2\Omega_0}|$$

i) $\Omega_0 = \frac{\pi}{6}$, $r = 0.9$

$$G_1 = |1-0.9|\left|1-0.9e^{-\frac{j\pi}{3}}\right| = 0.1 \cdot 0.9540 = 0.0954$$

ii) $\Omega_0 = \frac{\pi}{6}$, $r = 0.95$

$$G_2 = |1-0.95|\left|1-0.95e^{-\frac{j\pi}{3}}\right| = 0.1 \cdot 0.9759 = 0.0488$$

iii) $\Omega_0 = \frac{\pi}{3}$, $r = 0.9$

$$G_3 = |1-0.9|\left|1-0.9e^{-\frac{j2\pi}{3}}\right| = 0.1646$$

iv) $\Omega_0 = \frac{\pi}{3}$, $r = 0.95$

$$G_4 = |1-0.95|\left|1-0.95e^{-\frac{j2\pi}{3}}\right| = 0.0844$$

II. Notch Filter

$$|H(e^0)| = G\frac{|1-e^{j\Omega_0}||1-e^{-j\Omega_0}|}{|1-re^{j\Omega_0}e^{-\mathrm{j}\Omega_0}||1-re^{-j\Omega_0}e^{-j\Omega_0}|} = 1$$

$$G = \frac{|1-re^{j\Omega_0}e^{-\mathrm{j}\Omega_0}||1-re^{-j\Omega_0}e^{-j\Omega_0}|}{|1-e^{j\Omega_0}||1-e^{-j\Omega_0}|} = \frac{|1-re^{j\Omega_0}||1-re^{-j\Omega_0}|}{|1-e^{j\Omega_0}||1-e^{-j\Omega_0}|}$$

i) $\Omega_0 = \frac{\pi}{6}$, $r = 0.9$

$$G_1 = 0.9373$$

ii) $\Omega_0 = \frac{\pi}{6}$, $r = 0.95$

$$G_2 = 0.9593$$

iii) $\Omega_0 = \frac{\pi}{3}$, $r = 0.9$

$$G_3 = 0.9100$$

iv) $\Omega_0 = \frac{\pi}{3}$, $r = 0.95$

$$G_4 = 0.9525$$