Machine Learning, 2021 Spring
Homework 2

Due on 23:59 MAR 28, 2021

Problem 1

Prove that $f : \mathbb{R}^n \to \mathbb{R}$ is *affine* if and only if f is both convex and concave. [2pts]

Problem 2

Suppose A and B are both convex sets, prove that $C = A \cap B$ is also convex. [1pts]

Problem 3

Suppose your algorithm for solving the problem:

$$\min_{\boldsymbol{x} \in \mathbb{R}^n} \ f(\boldsymbol{x}) \tag{1}$$

takes iteration:

$$\boldsymbol{x}^{k+1} = \boldsymbol{x}^k + \alpha_k \boldsymbol{p}^k \tag{2}$$

where $\boldsymbol{p}^k = \boldsymbol{H}^k \nabla f(\boldsymbol{x}^k)$. What kind of $\boldsymbol{H}^k$ can guarantee that $\boldsymbol{p}^k$ is a descent direction ? [2pts]

Problem 4

Suppose $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable. For a given $\boldsymbol{x} \in \mathbb{R}^n$, show that moving along $-\nabla f(\boldsymbol{x}) \neq 0$ with sufficiently small stepsize causes decrease on f, that is,

$$f(\boldsymbol{x} - \alpha \nabla f(\boldsymbol{x})) < f(\boldsymbol{x}) \tag{3}$$

for sufficiently small $\alpha > 0$. [2pts]

Problem 5

Use gradient descent to solve the *underdetermined* linear system:

$$\min_{\boldsymbol{x} \in \mathbb{R}^n} \ \frac{1}{2} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\|_2^2 \tag{4}$$

with stepsize chosen as *exact line search*, initial point $\boldsymbol{x}^0 = 0$ and maximum iteration 1000. Plot :

1. The objective value against the iteration.(Use log scale for y-axis)
2. The ℓ_2 norm of gradient against the iteration.(Use log scale for y-axis)
3. The stepsize against the iteration.

The data $\boldsymbol{A} \in \mathbb{R}^{500 \times 1000}, \boldsymbol{b} \in \mathbb{R}^{500 \times 1}$ is attached in data/A.csv and data/b.csv with comma-separated (delimiter=','). [*Hint:* what is the solution to the exact line search for quadratic function?][3pts]