Numerical Optimization: Final Project

November 5, 2020

There are four topics for you to select for your final project. Two are about the implementation of the linear programming solver. One is on the discussion of constraint qualification. The other is on the development of an algorithm for a real assignment task.

1. Implement a primal simplex method for solving the standard form of linear programming

$$\min_{x\in\mathbb{R}^n} \quad c^T x \quad \text{s.t.} \quad Ax = b,\ x \geq 0$$

where $A \in \mathbb{R}^{m\times n}$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$. You can implement either the classic primal simplex method or the revised simplex method. You may need a first-phase to find basic feasible solution to initialize your algorithm.

2. Implement an interior point method for solving the "Big-M" form of the standard form of linear programming

$$\min_{x\in\mathbb{R}^n} \quad c^T x + Me^T s \quad \text{s.t.} \quad Ax + s = b,\ x \geq 0, s \geq 0.$$

Your code should be able to detect whether the original linear programming problem is feasible or infeasible.

3. Read the uploaded paper "A CONE-CONTINUITY CONSTRAINT QUALIFICATION AND ALGORITHMIC CONSEQUENCES". You do not need to go through every detail of the proof. Your task here is to discuss how the qualification conditions would become when considering a general convex-set constrained problem

$$\min_{x\in\mathbb{R}^n} \quad f(x) \quad \text{s.t.} \quad x \in C,$$

where C is a closed convex set.

4. Read the uploaded assignment description about a real problem from industry, and finish the tasks there.

Notice the following aspects:

- Your implementation could be in either python/matlab.
- It is understandable that your implementation is simply a prototype. However, it is your duty to make your codes "user-friendly" as much as possible.
- After successfully solving a linear problem, your code should output the "primal-dual optimal solution".
- Your final report should be neat and well written.