Numerical Analysis, 2020 Fall

Homework 5

2018533197 高世杰

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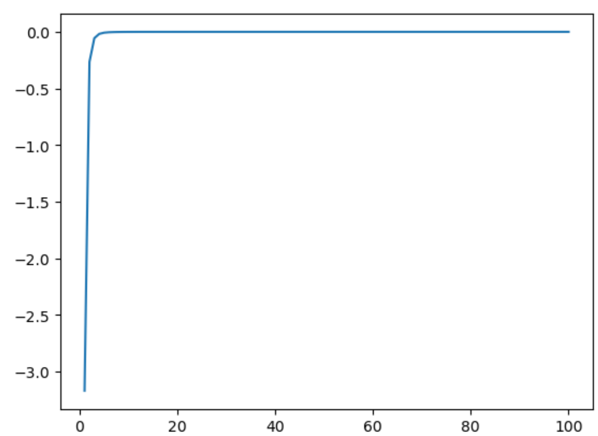
Problem1

1.a

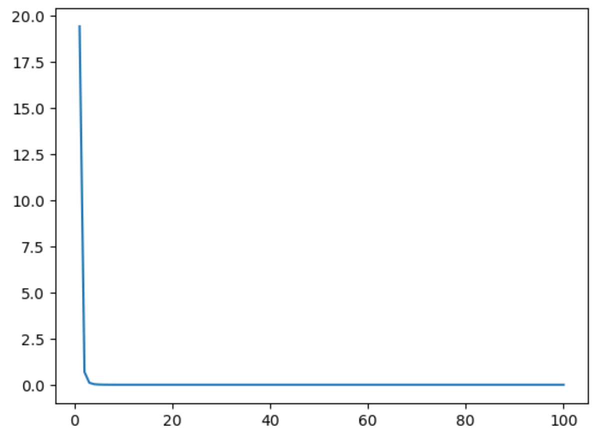


1.b

1.b $\overline{R_n(f)} = \frac{h^5}{2880}\sum_{i=1}^{n}\max_{\eta_i\in[x_i,x_{i+1}]}|f^{(4)}(\eta_i)|$ $\quad h=\frac{4}{n}\quad n=1,\dots 100$

$$= \frac{h^5}{2880}\sum_{i=0}^{n-1} e^{h(i+1)}$$

$$= \frac{16}{45n^5}\sum_{i=0}^{n-1} e^{\frac{4(i+1)}{n}}$$



Problem2

$$2.\ I_n=(b-a)\sum_{k=0}^{n} C_k^{(n)} f(x_k) \qquad x_k=a+kh,\ h=\frac{b-a}{n}\ k=1,\cdots n$$

$$I_3=\frac{\pi}{4}\left[\frac{1}{8}f(0)+\frac{3}{8}f\left(\frac{\pi}{12}\right)+\frac{3}{8}f\left(\frac{\pi}{6}\right)+\frac{1}{8}f\left(\frac{\pi}{4}\right)\right]$$

$$I_5=\frac{\pi}{4}\left[\frac{19}{288}f(0)+\frac{25}{96}f\left(\frac{\pi}{20}\right)+\frac{25}{144}f\left(\frac{\pi}{10}\right)+\frac{25}{144}f\left(\frac{3\pi}{20}\right)+\frac{25}{96}f\left(\frac{\pi}{5}\right)+\frac{19}{288}f\left(\frac{\pi}{4}\right)\right]$$

$$R_3=I_3-\left(1-\frac{\sqrt{2}}{2}\right)=1.748\times10^{-5}$$

$$R_5=I_5-\left(1-\frac{\sqrt{2}}{2}\right)=-2.0405\times10^{-8}$$

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Problem3

$$3.\ f(x)=\cos(wx)\ \ x\in\left[0,\frac{\pi}{4}\right]$$

$$y=\frac{8}{\pi}x-1,\ \ x=\frac{\pi(y+1)}{8}\ \ y\in[-1,1]$$

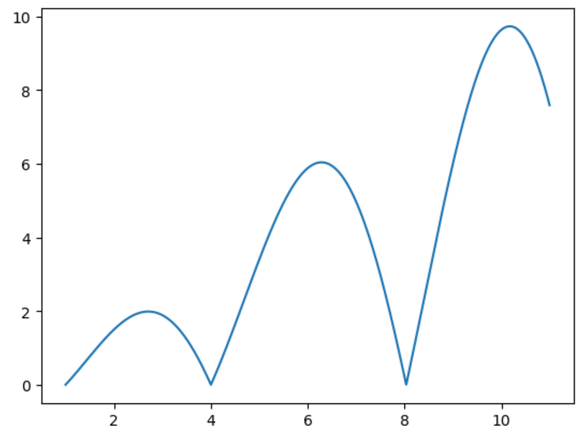
$$f(y)=\cos\left(\frac{w\pi(y+1)}{8}\right)\ \ y\in[-1,1]$$

$$\int_0^{\frac{\pi}{4}}f(x)dx=\frac{\pi}{8}\int_{-1}^{1}f(y)dy=\frac{\pi}{8}f\left(-\frac{1}{\sqrt{3}}\right)+\frac{\pi}{8}f\left(\frac{1}{\sqrt{3}}\right)$$

$$=\frac{\pi}{8}\cos\left(\frac{w\pi(1-\frac{1}{\sqrt{3}})}{8}\right)+\frac{\pi}{8}\cos\left(\frac{w\pi(1+\frac{1}{\sqrt{3}})}{8}\right)$$

$$\therefore I_1(w)=\frac{\pi}{8}\cos\left(\frac{w\pi(1-\frac{1}{\sqrt{3}})}{8}\right)+\frac{\pi}{8}\cos\left(\frac{w\pi(1+\frac{1}{\sqrt{3}})}{8}\right)$$

$$|I(1)-I_1(1)|=6.3522\times10^{-5}$$



Problem4

4. 用 $\sum_{j=0}^{n} c_j q_j(x)$ to approximate $f(x)$

$$q(x)=\begin{pmatrix} q_0(x) \\ \vdots \\ q_n(x) \end{pmatrix}$$

$$\begin{pmatrix} q_0(x_0) & \cdots & q_n(x_0) \\ \vdots & & \vdots \\ q_0(x_n) & \cdots & q_n(x_n) \end{pmatrix}\begin{pmatrix} c_0 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} f(x_0) \\ \vdots \\ f(x_n) \end{pmatrix}$$

$$\int_a^b q(x)dx = Q(b)-Q(a)$$

$$\int_a^b f(x)dx = \int_a^b c^T q(x)dx = c^T[Q(b)-Q(a)]$$

$$\begin{pmatrix} c_0 \\ \vdots \\ c_n \end{pmatrix} = \begin{pmatrix} q_0(x_0) & \cdots & q_n(x_0) \\ \vdots & & \vdots \\ q_0(x_n) & \cdots & q_n(x_n) \end{pmatrix}^{-1}\begin{pmatrix} f(x_0) \\ \vdots \\ f(x_n) \end{pmatrix}$$

$$= \left[\begin{pmatrix} q_0(x_0) & \cdots & q_n(x_0) \\ \vdots & & \vdots \\ q_0(x_n) & \cdots & q_n(x_n) \end{pmatrix}^{-1}\begin{pmatrix} f(x_0) \\ \vdots \\ f(x_n) \end{pmatrix}\right]^T [Q(b)-Q(a)]_{n\times 1}$$

$$= \alpha^T \begin{pmatrix} f(x_0) \\ \vdots \\ f(x_n) \end{pmatrix}$$

$$\therefore \alpha^T = [Q(b)-Q(a)]^T \begin{pmatrix} q_0(x_0) & \cdots & q_n(x_0) \\ \vdots & & \vdots \\ q_0(x_n) & \cdots & q_n(x_n) \end{pmatrix}^{-1}$$

$$\int_a^b 1dx = b-a$$

$$\int_a^b \sin x dx = \cos a - \cos b$$

$$\int_a^b \cos x dx = \sin b - \sin a$$

$$\therefore \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 1 & \sin a & \cos a \\ 1 & \sin\frac{a+b}{2} & \cos\frac{a+b}{2} \\ 1 & \sin b & \cos b \end{pmatrix}^{-1}\begin{pmatrix} f(a) \\ f(\frac{a+b}{2}) \\ f(b) \end{pmatrix}$$

$$(\alpha_1,\alpha_2,\alpha_3) = (b-a, \cos a-\cos b, \sin b-\sin a)\begin{pmatrix} 1 & \sin a & \cos a \\ 1 & \sin\frac{a+b}{2} & \cos\frac{a+b}{2} \\ 1 & \sin b & \cos b \end{pmatrix}^{-1}$$

the real value of function1 is:0.110614330719248

the Simpson’s formula approximate is:0.110615915219851

the new formula approximate is:0.11061395821500818

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the real value of function2 is:0.25

the Simpson’s formula approximate is:0.25

the new formula approximate is:0.251051045383168

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the real value of function3 is:0.8414709848078965

the Simpson’s formula approximate is:0.8417720922382719

the new formula approximate is:0.8414709848078968

Analysis:In function2,Simpson’ s formula approximate is better;In function1 and function3,new formula approximate is better.And we can see that if the function has sin cos,then new formula will be better.If the function has polynomial, the Simpson’s formula will be better.The reason is that Simpson’s formula is based on polynomial interpolation,and new formula is based on 1,sin and cos interpolation.