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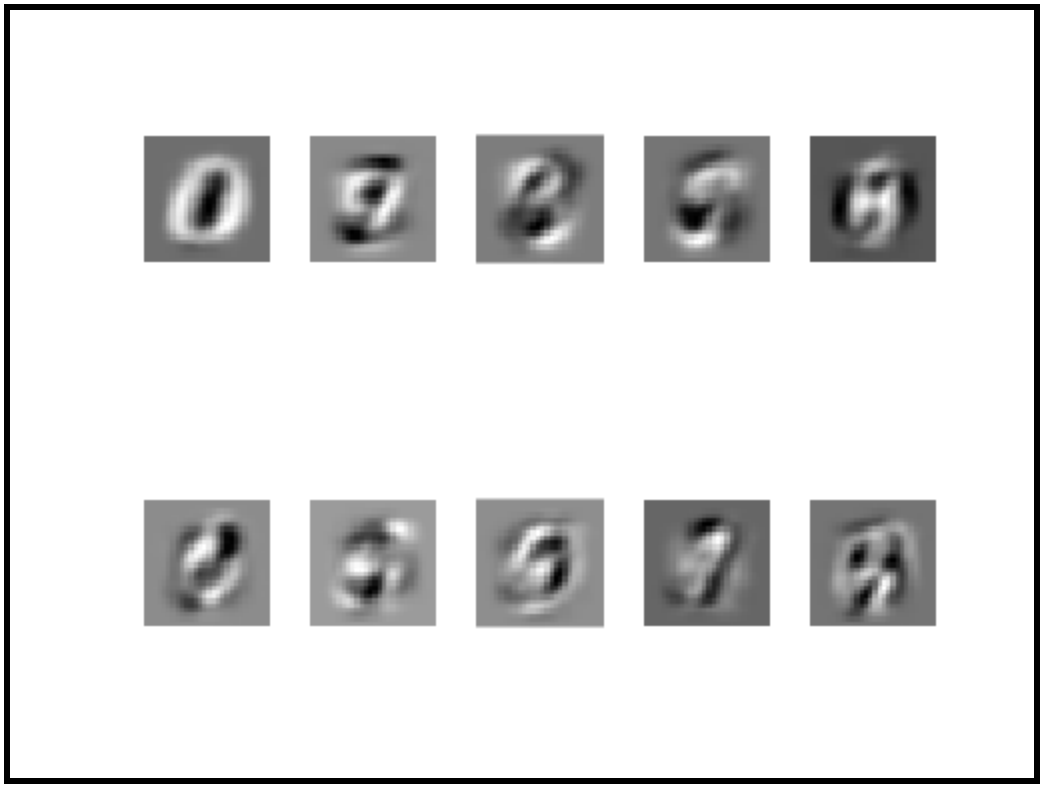
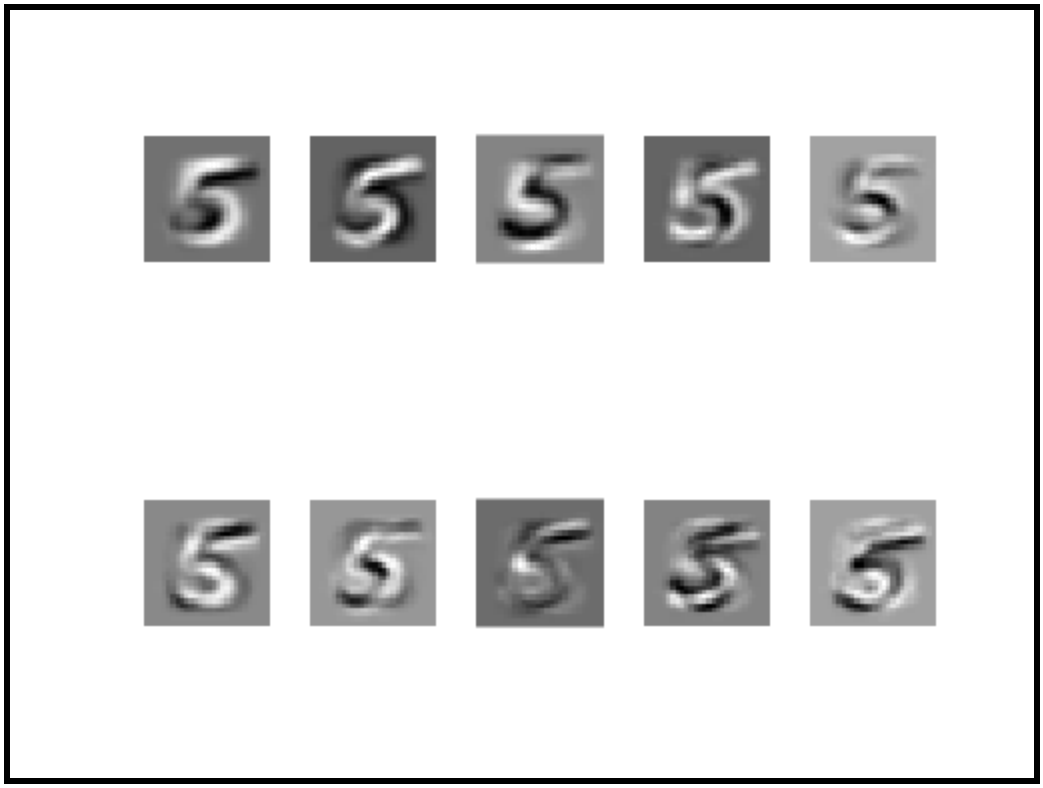
HW6

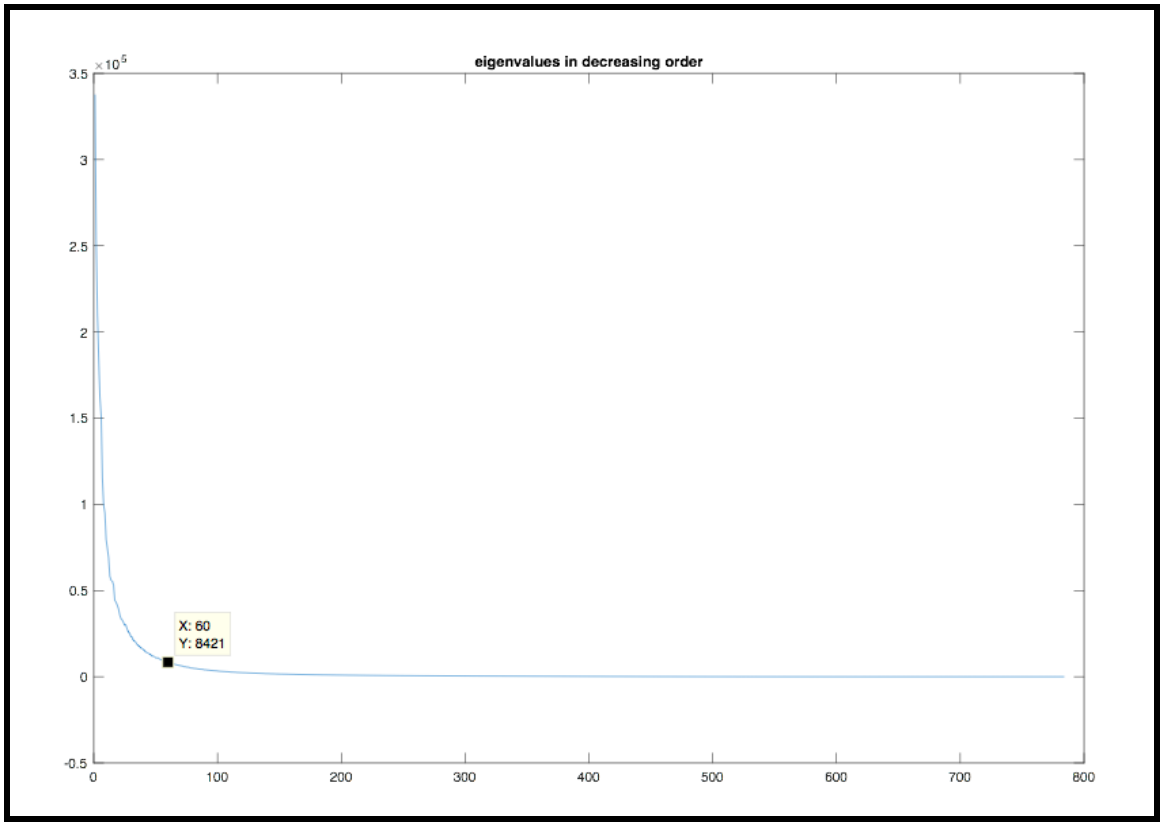
(1)



Top 10 principal components for all training images



Top 10 principal components for all training images digit 5



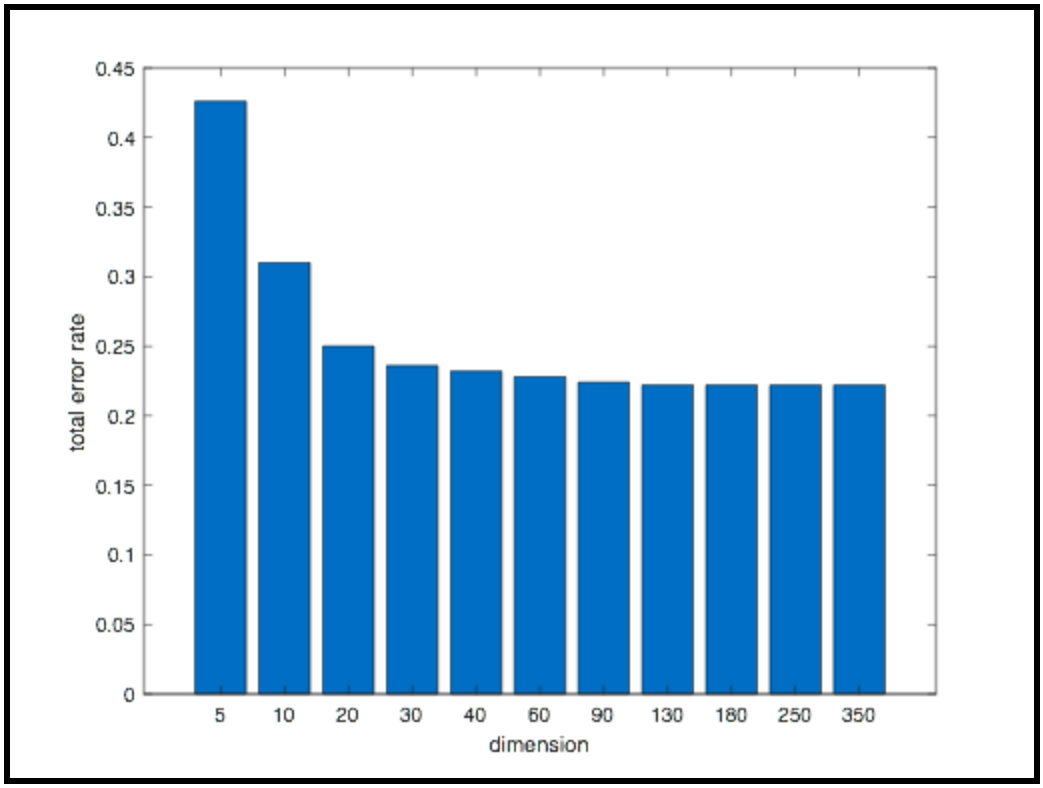
Plot of eigenvalues for the covariance matrix

(2)

(a)

From the plot of eigenvalues for the covariance matrix above, we can see that the best subspace dimension is around 60, which is **at the knee** of the plot. The magnitudes of the eigenvalues after the 60th one are at a relatively flat level, which means the eigenvectors after the 6th one are not principal components.

(b)



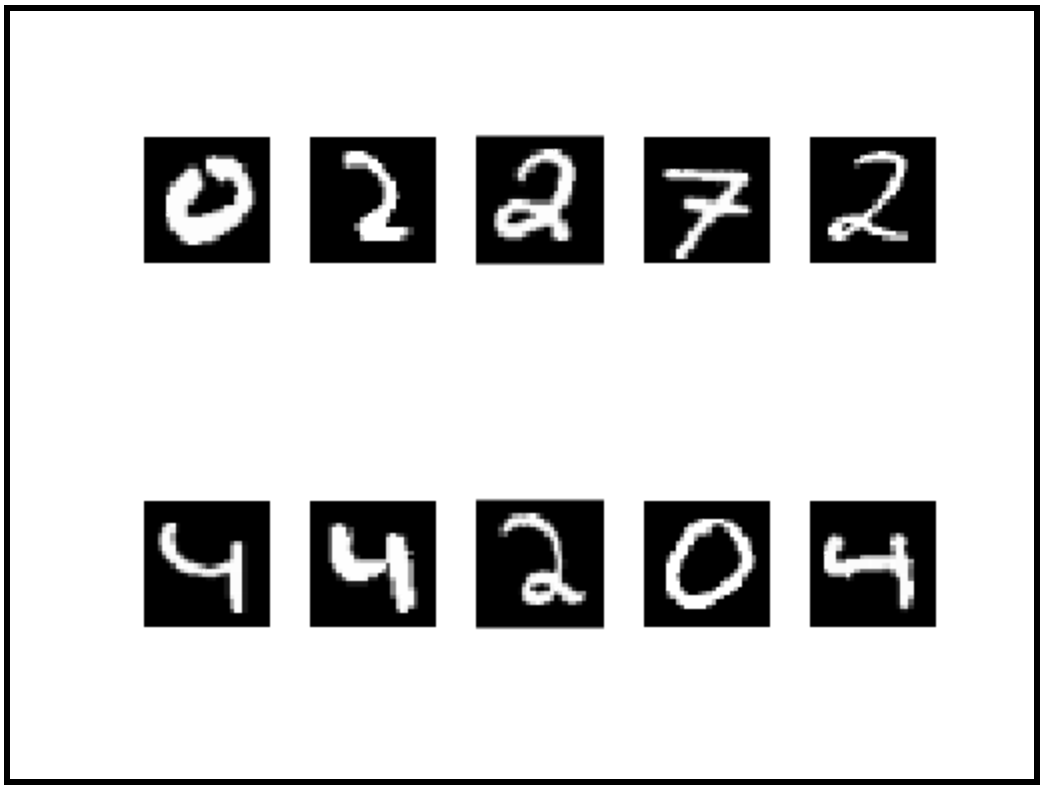

```
total_error_rate =

    0.4260    0.3100    0.2500    0.2360    0.2320    0.2280    0.2240    0.2220    0.2220    0.2220    0.2220
```

(c)

When the dimension is less than 60 (ie. 5,10,20,30,40), the error rate is larger than the one I got from HW3 (0.2220); when the dimension is 5, the error rate is largest: 0.426; as the dimension increases, the error rate decreases. This is because when the dimension is small, we do not have enough principal components to discriminate features, which leads to high error rates; as dimension increases, more and more principal components are included and there will be enough components to discriminate features, resulting in lower error rates. When the dimension is equal or greater than 60, the error rate approaches the one I got from HW3 and remains relatively unchanged. This is because components after the 60th component are not discriminant and redundant; adding them into the dimension does not affect the error rates too much.

(3)



Top 10 least 5 like digits

(1)

```
%get the mean of the trainning set
mean_total=zeros(28,28);
sum=zeros(28,28);
for i=1:5000
    sum=sum+imageTrain(:,:,i);
end
mean_total=sum/5000;
mean_total=reshape(mean_total,[784,1]);
%get the covariance matrix of the trainning set
reshapeTrain=zeros(784,5000);
for i=1:5000
    reshapeTrain(:,i)=reshape(imageTrain(:,:,i),[784,1]);
end
cov=zeros(784,784);
sum=zeros(784,784);
for i=1:5000
    sum=sum+(reshapeTrain(:,i)-mean_total)*(reshapeTrain(:,i)-
mean_total)';
end
cov=sum/5000;
%get the eigenvalue and eigenvector of the covariance matrix
figure;
[eigenvector,eigenvalue]=eig(cov);
eigenvalue=fliplr(eigenvalue);
eigenvalue=flip(eigenvalue);
lamda=zeros(784,1);
for i=1:784
    lamda(i)=eigenvalue(i,i);
end
plot(lamda); %plot the eigenvalue in decreasing order
title('eigenvalues in decreasing order');
eigenvector=fliplr(eigenvector);
eigenimage=zeros(28,28,10);
figure;
for j=1:10
eigenimage(:,:,j)=reshape(eigenvector(:,j),[28,28]);
subplot(2,5,j);
imshow(eigenimage(:,:,j),[]);
end
% repeat above for digit 5
 index5=find(labelTrain==5);
 mean5=zeros(28,28);
 sum5=zeros(28,28);
for i=1:size(index5,1)
```

```
    sum5=sum5+imageTrain(:,:,index5(i));
end
mean5=sum5/size(index5,1);
mean5=reshape(mean5,[784,1]);
cov5=zeros(784,784);
sum=zeros(784,784);
for i=1:size(index5,1)
    sum=sum+(reshapeTrain(:,index5(i))-mean5)*(reshapeTrain(:,index5(i))-
mean5)';
end
cov5=sum/size(index5,1);
[eigenvector5,eigenvalue5]=eig(cov5);
eigenvector5=fliplr(eigenvector5);
eigenimage5=zeros(28,28,10);
figure;
for j=1:10
eigenimage5(:,:,j)=reshape(eigenvector5(:,j),[28,28]);
subplot(2,5,j);
imshow(eigenimage5(:,:,j),[]);
end
```

(2)

```
%2(b)
dimension=[5,10,20,30,40,60,90,130,180,250,350];
reshapeTest=zeros(784,500);
total_error_rate=zeros(1,size(dimension,2));
for i=1:500
   reshapeTest(:,i)=reshape(imageTest(:,:,i),[784,1]);
end
test_subtract=zeros(784,500);
train_subtract=zeros(784,5000);
for i=1:500
 test_subtract(:,i)=reshapeTest(:,i)-mean_total;
end
for i=1:5000
 train_subtract(:,i)=reshapeTrain(:,i)-mean_total;
end
for n=1:size(dimension,2)
eigenvector_reduce=zeros(784,dimension(n));
for i=1:dimension(n)
  eigenvector_reduce(:,i)=eigenvector(:,i);
end
test_image_reduce=eigenvector_reduce'*test_subtract;
train_image_reduce=eigenvector_reduce'*train_subtract;
mean_reduce=zeros(dimension(n),10);
for i=0:9
[a,b]=find(labelTrain==i);
c=size(a);
```

```
sum=0;
 for j=1:c(1)
    sum=sum+train_image_reduce(:,a(j));
 end
m=sum/c(1);
mean_reduce(:,i+1)=mean_reduce(:,i+1)+m;
end
dis=zeros(10,500);
for i=1:10
   for j=1:500
        dis(i,j)=-0.5*(test_image_reduce(:,j)-mean_reduce(:,i))'*
(test_image_reduce(:,j)-mean_reduce(:,i));
   end
end
I=zeros(500,1);
class=zeros(500,1);
for i=1:500
 [M,I]=max(dis(:,i));
 class(i)=I-1;
end
errorrate=zeros(1,10);
error=zeros(1,10);
for c=0:9
x=find(labelTest==c);
for j=1:length(x)
    if class(x(j))~=labelTest(x(j))
    error(c+1)=error(c+1)+1;
    end
end
 errorrate(c+1)=error(c+1)/length(x);
end
 total_error=0;
for i=1:10
    total_error=error(i)+total_error;
end
total_error_rate(n)=total_error/500;
end
```

(3)

```
for i=1:500
  imageTest5(:,i)=reshape(imageTest(:,:,i),[784,1]);
end
imageTest5_subtract=zeros(784,500);
for i=1:500
    imageTest5_subtract(:,i)=imageTest5(:,i)-mean5;
end
imageTest5_reduced=eigenvector5(:,41:end)'*imageTest5_subtract;
norm5=zeros(1,500);
```

```
for i=1:500
    norm5(i)=norm(imageTest5_reduced(:,i));
end
[a,b]= sort(norm5,'descend');
figure;
for i=1:10
   subplot(2,5,i)
   imshow(imageTest(:,:,b(i)),[]);
end
```