

# Algorithm assignment II

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## Question 1: Heap Sort

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assume the heap is Max heap

### When $a[1] < a[2] < a[3] < \dots < a[n]$

Build the heap: from  $n/2$  to  $n$  assume  $n = 2^k, k = \log_2(n)$

Then the Tree has  $k+1$  layers. We will have to exchange the elements for  $1 + 2 + 3 + \dots + k$  times to build the initial Max-heap.

Generally, we will need to exchange for  $\Theta(\log_2(n)^2)$  times to build the initial Heap.

Then we have to maintain the Max-heap.

final result is  $\Theta(n \times \log_2(n))$

### When $a[1] > a[2] > a[3] > \dots > a[n]$

It is already a MAX-heap, so it takes no time to build the MAX-heap

Then it will take  $O(n \log_2(n))$  to maintain the Max\_heap

## Question 2: quick sort

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### (a)

assume the  $H$  is the max-height of the Tree and  $h$  is the minimize height of the Tree.

We have following relationships:

$$1 \div \alpha^h = n$$

$$a \div (1 - \alpha)^H = n$$

The result is  $h = -\lg n \div \lg \alpha$

$$H = -\lg n \div \lg(1 - \alpha)$$

### (b)

假设产生两个不同的1划分 $\alpha, 1 - \alpha$ 和 $\theta, 1 - \theta$ 的概率相同

那么比 $\alpha, 1 - \alpha$ 更好的划分必须更加接近中心 $1/2$

$$\text{概率为 } (1 \div 2 - \alpha) \div (1 \div 2) = (1 - 2 \times \alpha)$$