COMP261 Lecture 19

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Data Compression 1: Huffman Coding

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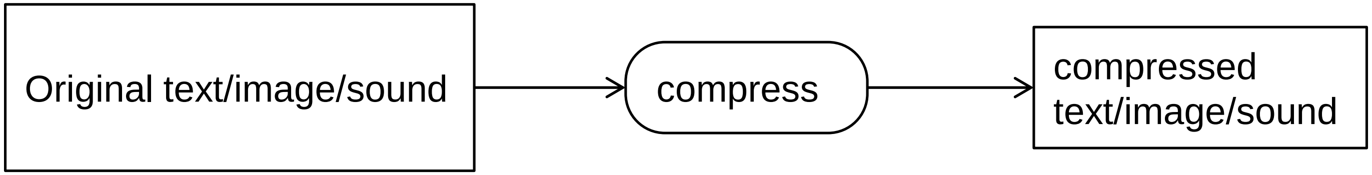
Data/Text Compression

- Files containing text, sound, video etc. can easily become huge. E.g. a blu ray movie is about 25Gb.

- Can we reduce the amount of time/space required to transmit/store them?

- E.g. text files are hugely redundant – we use 8 bits (or more) to store each character, but there is far less information than that.

- Compression is about reducing the memory required to store some information.



Lossless v. Lossy Data Compression

Data compression may be:

- Lossless:
  - No information is lost – just gets stored more compactly
  - Can retrieve the original data exactly (decompress)
  - Important for text and some numerical data
    - compress to store/transmit, decompress to use
- Lossy:
  - Information may be lost
  - Can't retrieve the original data exactly
  - Acceptable in some contexts
    - data is stored and used in compressed form
  - E.g. mp3 compresses sound files

Lossless v. Lossy Data Compression

- Lossless compression only possible if there is *redundancy* in the original.

- Compression identifies and removes some of the redundant elements.

- Eg:
  - Identify repeated patterns
  - If lots of repeated characters, replace by count and character
  - Construct a dictionary and replace words by indexes to it

Encoding : compression, one symbol at a time

- Problem:
  - Given a set of symbols (characters, numbers, …)
  - Encode them as bit strings
    - Use a separate code for each symbol
  - Try to minimise the total number of bits.
- Today: Huffman coding
  - Very clever solution, very widely used
  - Combining several great ideas!
- Note: When coding data to store/transmit, we often add extra bits (i.e. redundancy) so we can detect errors:
  - See parity bits, error-correcting codes.
  - This can still be done with compressed data.

Equal Length Codes

- Obvious approach:
  Use the same number of bits for every symbol to be encoded.

- E.g. digits:

symbol:	0	1	2	3	4	5	6	7	8	9
code:	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

- E.g. letters:

symbol:	a	b	c	d	e	f	g	…	z
code: 00001	00010	00011	00100	00101	00110	00111	…	11010	

cf: ASCII

Equal Length Codes

- How many bits are needed?

- Digits:

symbol:	0	1	2	3	4	5	6	7	8	9
code:	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

  10 symbols, 4 bits

- Letters:

symbol:		a	b	c	d	e	f	…	z
code:	00001	00010	00011	00100	00101	00110	00111	…	11010

  26 symbols, 5 bits

  Already better than 8 bits per symbol!

- Ex: How many bits for upper and lower case letters, and 0-9?

Equal Length Codes

- With N bits, we can have up to 2^N different codes.

- For N different symbols, need $\log_2 N$ bits per symbol

  10 numbers, message length = 4

  26 letters, message length = 5

- If there are many repeated symbols, can we do better?

Variable Length Codes

- **Great idea #1**:
  - Use fewer bits for more common symbols

- Eg for letters, suppose:

  a occurs 50% of the time,

  b occurs 25% of time,

  d-e each occur 5% of time,

  f-j each occur 2% of time.

Encode:
a by ‘0’
b by ‘1’
c by ‘10’
d by ‘100’
e by ‘101’
g by ‘110’
…
j by ‘1001’

Variable Length Codes

sym:	a	b	c	d	e	f	g	h	i	j
code:	0	1	11	100	101	110	111	1000	1001	1010

String: a a b a j a b a a b

Fixed: 0000 0000 0001 0000 1001 0000 0010 0000 0000 0001
(using 4 bits each as only 10 letters used)

Variable: 0 0 1 0 1001 0 10 0 0 1

Takes 14 bits, rather than 40.

Variable length encoding

- Problem: where are the boundaries?

- How can we tell if 1001 is code for i, db or baab?

- A possible approach:
  - Use 0 as a "sentinel bit" to mark the end of a code
  - But then can only use 1's for the code itself

Sym:	a	b	c	d	e	f	…	j
Code:	10	110	1110	11110	111110	1111110…		1111111110

- That's not so good – can we do better?

Prefix-free codes

- **Great idea #2**:

- Design codes so that no code is the prefix of another code!

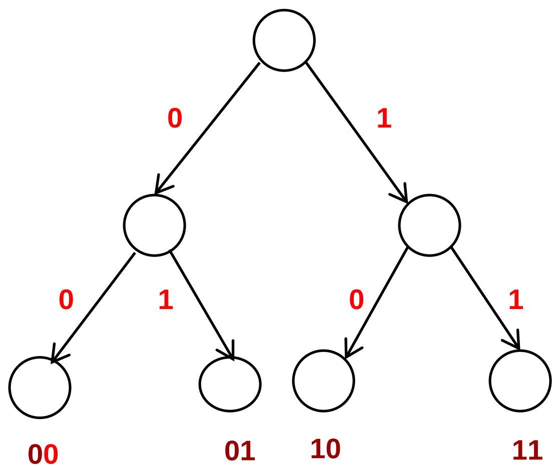
- Eg:

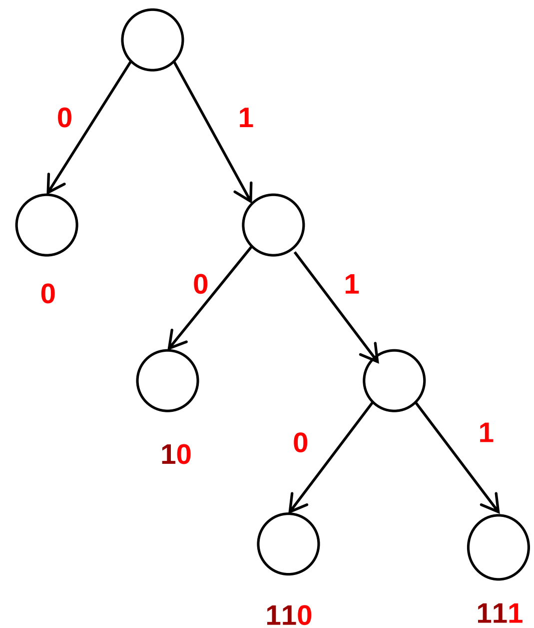
sym:	a	b	c	d	e	f	g	h	
code:	0	10	1100	11101	11100	11111	11010	110110	....

- How do we design codes that are *prefix-free*?

Prefix-free codes

- We can think of prefix-free codes as path labels to leaves in a binary tree





- Balanced tree gives equal length codes
- Linear tree is like using a sentinel bit
- What tree shape will give best codes?
- Want more frequent symbols at the top, less frequent at the bottom – but not too far away!

Designing a good prefix-free code

- **Great idea #3**:

- Build the tree from the bottom-up, combining nodes with smallest frequencies.

  – Start with a leaf for each symbol, labelled with its frequency.

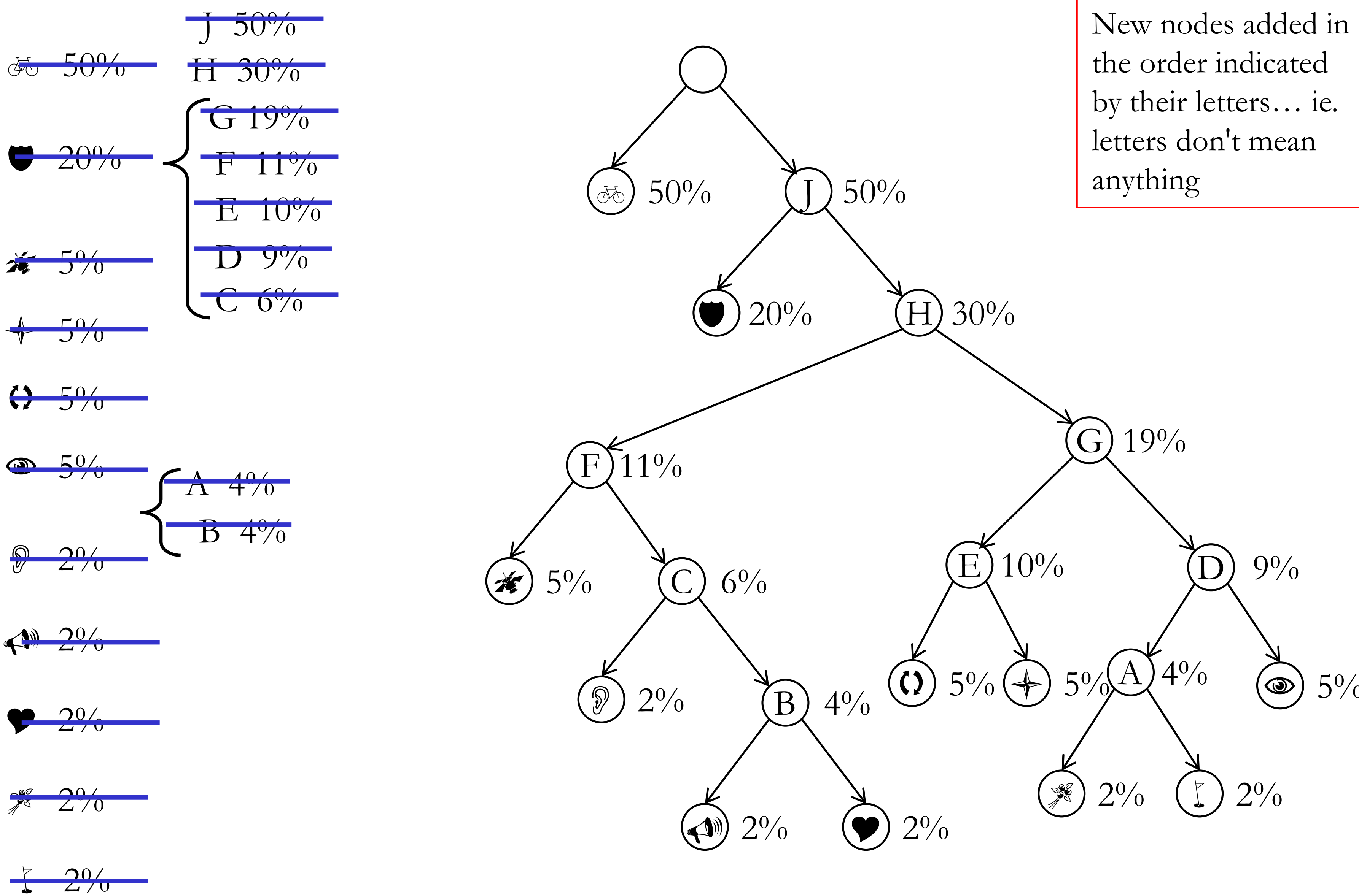
  – At each step, combine two nodes with smallest frequencies, add a new node as their parent, labelled with the sum of their frequencies.

  – Stop when all nodes are combined into a single tree.

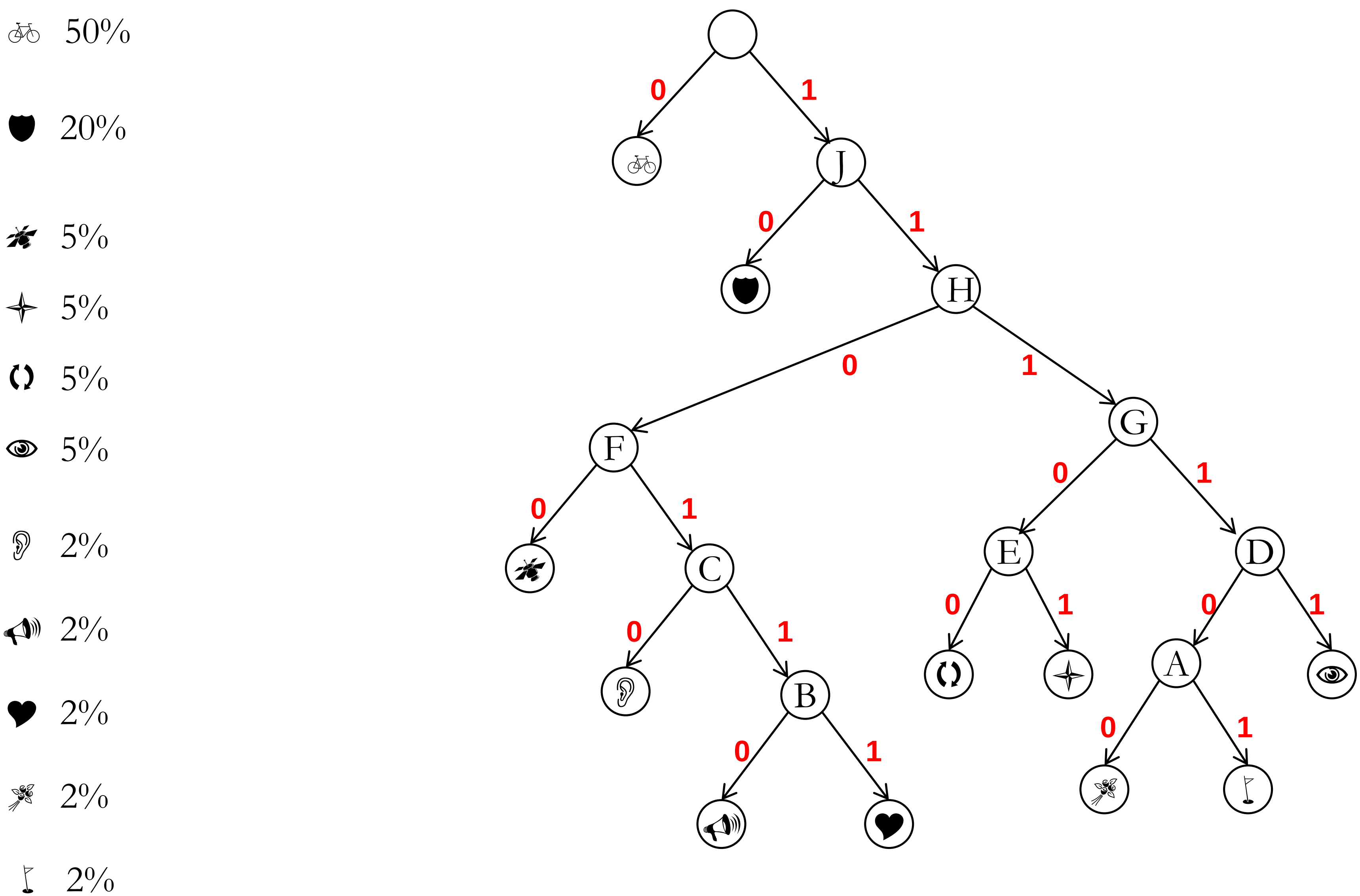
Example: Building the tree

View the powerpoint animation!

New nodes added in the order indicated by their letters… ie. letters don't mean anything



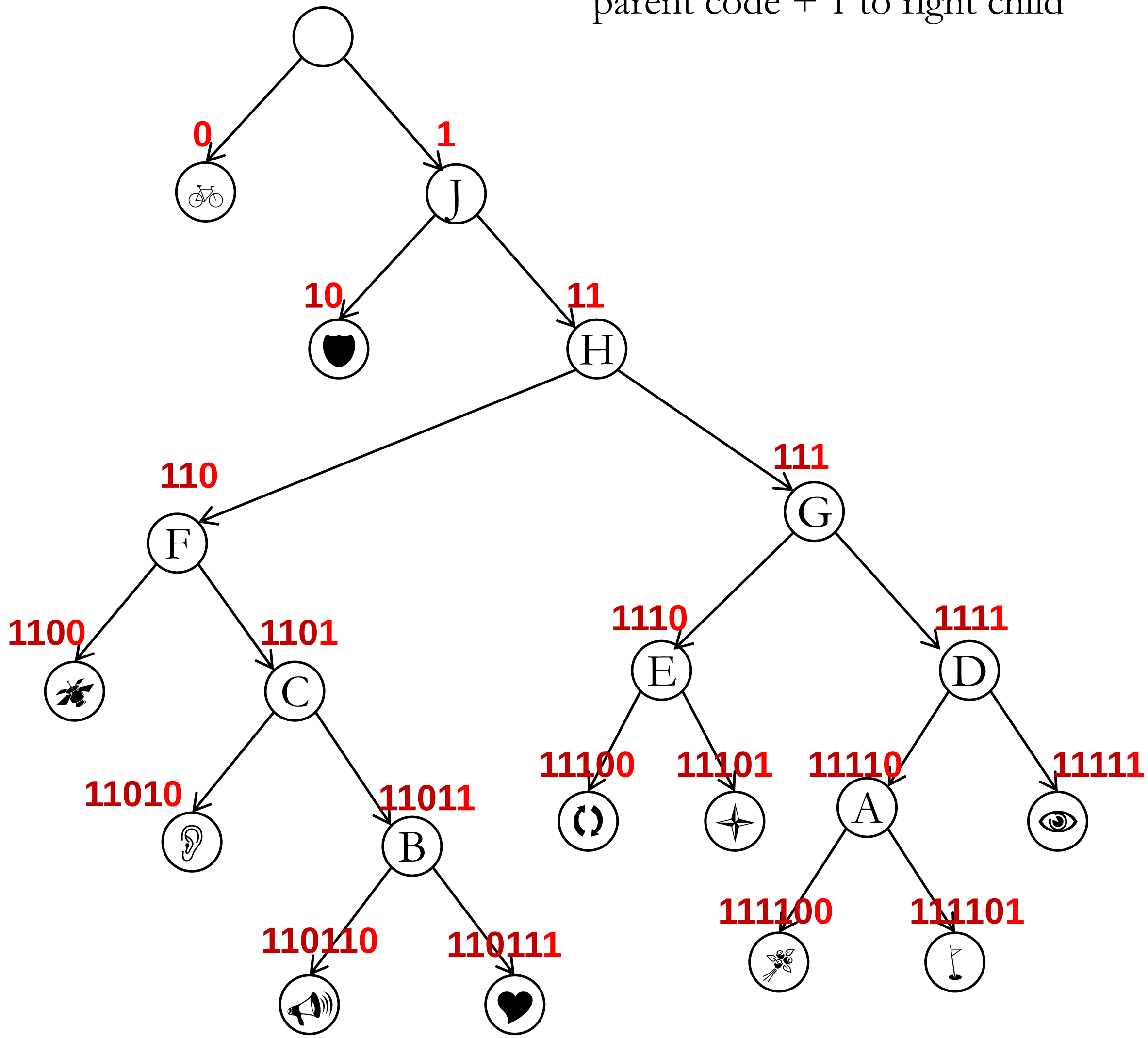
Example: assigning the codes



Example: assigning the codes

Assign
parent code + 0 to left child
parent code + 1 to right child

	50%	0
	20%	10
	5%	1100
	5%	11101
	5%	11100
	5%	11111
	2%	11010
	2%	110110
	2%	110111
	2%	111100
	2%	111101



average code length = (1*.5)+(2*.2)+(4*.05)+(5*.17)+(6*.08) = 2.43 bits

Huffman Coding

- Generates the *best* set of codes, given frequencies/probabilities on all the symbols.
- Creates a binary tree, which is used to construct the codes.

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Construct a leaf node (singleton tree) for each symbol.
Put these nodes into a priority queue, with frequency as priority.
    (lowest frequency = highest priority)

while there is more than one node in the queue:  (i.e. > 1 tree)
    remove the top two nodes
    create a new tree node with these two nodes as children.
        node frequency = sum of frequencies of the two nodes
    add new node to the queue

Final node is root of tree.
Traverse this tree to assign codes:
if node has code c, assign  c0 to left child, c1 to right child
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- See video on YouTube: 'Text compression with Huffman coding'

Huffman Coding

- To decode, we need a table of the codes used.

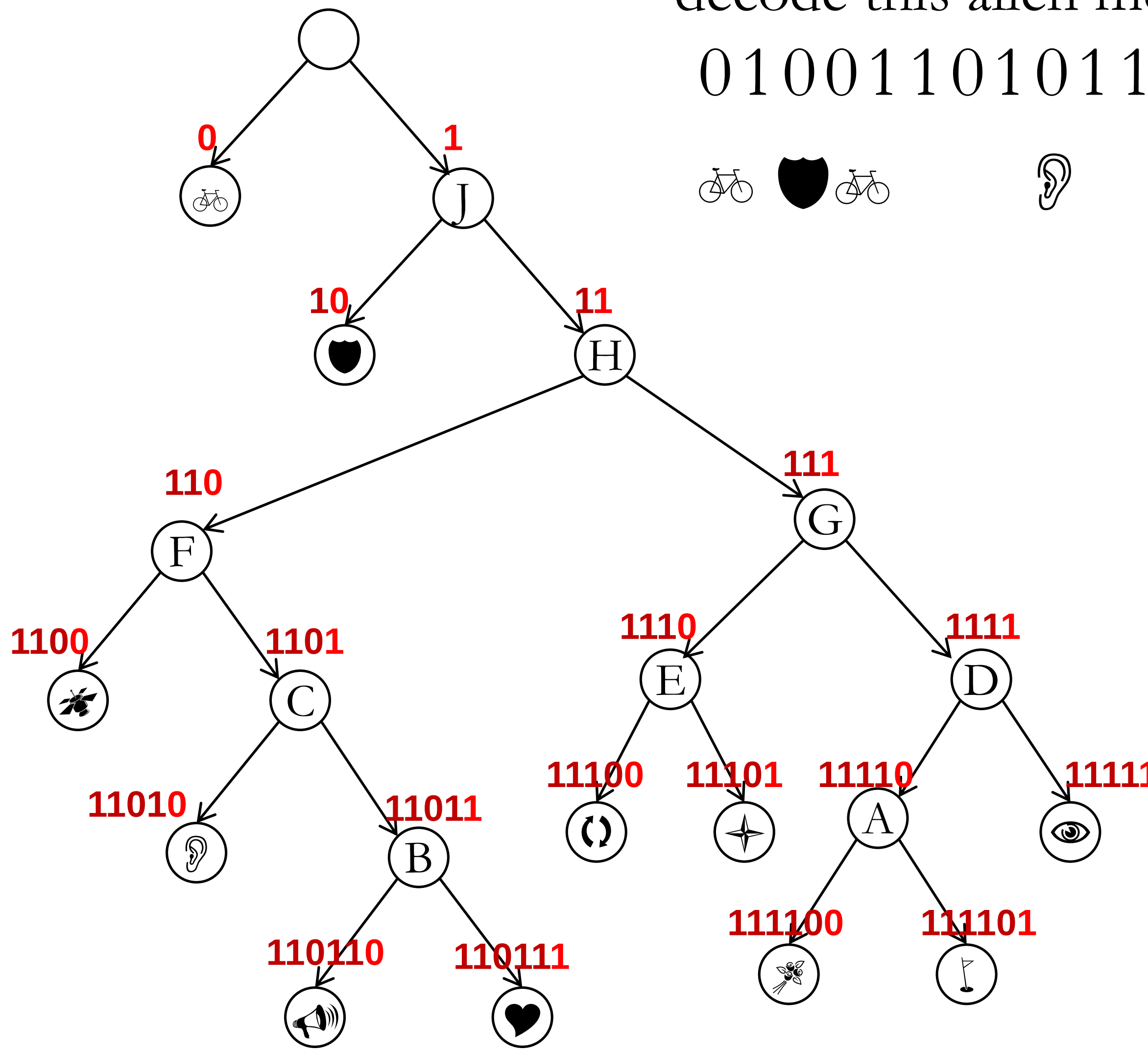
- If we label the edges of the tree with 0's and 1's, as added at each level, we get a *trie* which can be used like a scanner to split the coded string/file into separate codes to be decoded.

- Example: Use above tree to decode:

  010011010010

Example: assigning the codes

50%	0
20%	10
5%	1100
5%	11101
5%	11100
5%	11111
2%	11010
2%	110110
2%	110111
2%	111100
2%	111101



Example: Use tree to decode this alien message

0 1 0 0 1 1 0 1 0 1 0 1 1 1 1 1

Huffman Coding

- When storing/transmitting a compressed file, we need to include the tree for decompressing.
  – Can reduce efficiency of coding.

- Or, use a standard frequency table, not based on the particular file, for code.
  – E.g. use known frequencies of letters in English text.