Algorithms and Data Structures

COMP261

Articulation Points 2: Implementation

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Outline

- Idea revisited
- Implementation of the AP algorithm
  - Recursive version
  - Iterative version

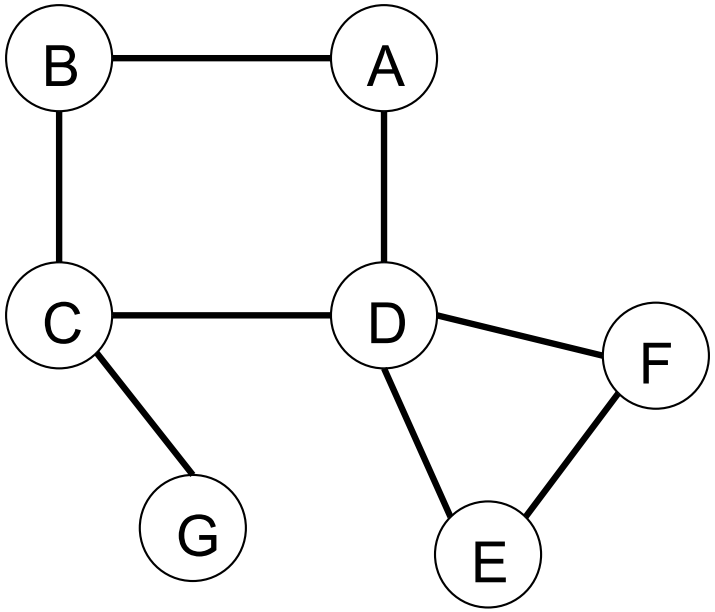
Idea Revisited

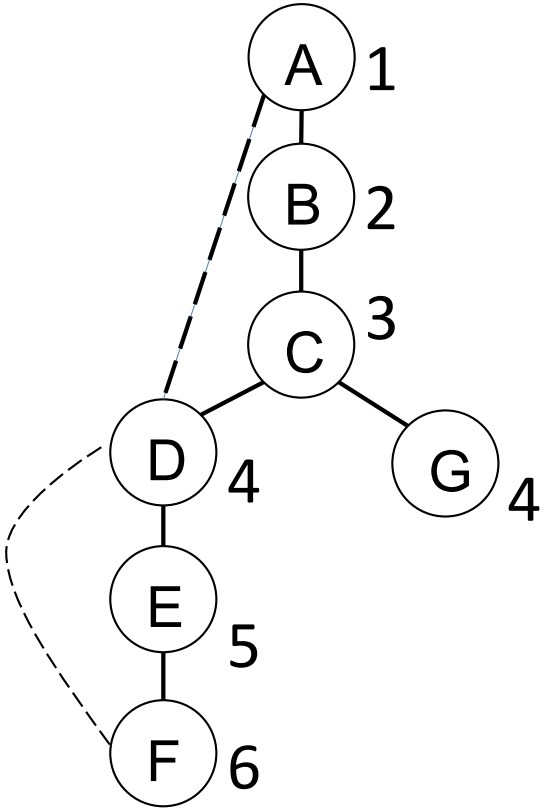
- Articulation point: a node whose removal will disconnect the graph
- Brute force search
  - For each node, run a DFS starting from one neighbour (one sub-tree)
  - If all the nodes are visited (one sub-tree), then the node is not an articulation point. Otherwise, it is an articulation point.
  - Complexity: O(n*e): n is number of nodes, e is number of edges
  - Very inefficient, many checks are unnecessarily repeated

- A new efficient algorithm with a single DFS
  - Assign depth to each node during DFS (larger depths are children, smaller are parents)
  - Check for root node: number of sub-trees
  - Check for other nodes: alternative path from children to parents

Example of Idea

- A single DFS rooted from node A
  - One single DFS to assign depth, record the edges that are in the DFS and not in the DFS: (A, D), and (D, F)
  - A is not articulation point: root node, one sub-tree
  - B is not articulation point: all the children can reach parents via (A, D)
  - C is articulation point: no alternative path from G to parents
  - D is articulation point: no alternative path from E, F to parents
  - E, F, G?





Implementation

- DFS can be implemented by recursion and iteration
  - **Recursion**: easy to design/write, hard to debug
  - **Iteration**: easy to debug, hard to design/write
- We also design these two versions of the AP algorithm

```
RecDFS(node) {
    if (node is unvisited) {
        set node as visited;
        for (each neighbour of node) {
            RecDFS(neighbour);
        }
    }
}
```

```
IterDFS(node) {
    Initialise fringe as an empty stack;
    Add node into fringe;
    while (fringe is not empty) {
        get&remove n* from the fringe;
        set n* as visited;
        add all unvisited neighbour of n* into fringe;
    }
}
```

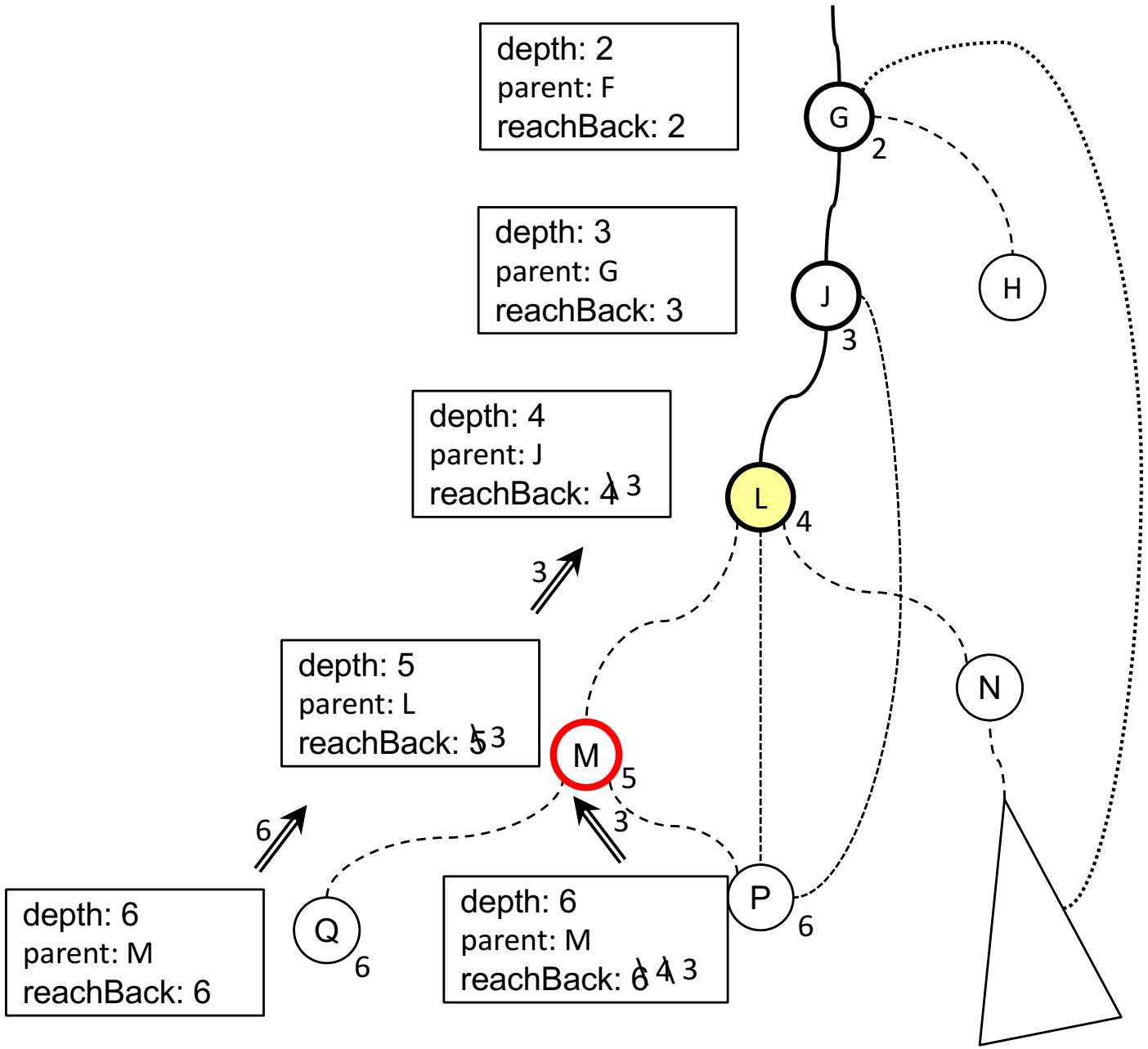
Recursive AP Algorithm

```
Initialise depth of all nodes as depth(node) = ∞, meaning all nodes are unvisited;
Initially, APs = {}, that is, no articulation point is found;
Randomly select a node as the root node, set depth(root) = 0, numSubTrees = 0;

for (each neighbour of root) {
   if (depth (neighbour) = ∞) {
      recArtPts(neighbour, 1, root);  // recursive DFS for the neighbour
      numSubTrees ++;
   }

   if (numSubTrees > 1) then add root into APs;
}

-------------------------------------------------------------------------------------------------------

recArtPts(node, depth, parent) {
   …
}
```

Recursive AP Algorithm

```
recArtPts(node, depth, parent) {
    depth(node) = depth;
    // store the minimum depth the node can reach back via alternative path
    reachBack = depth;
    for (each neighbour of node other than parent) {
        // case 1: direct alternative path: neighbour is visited before
        if (depth(neighbour) < ∞)
            reachBack = min(depth(neighbour), reachBack);
        // case 2: indirect alternative path: neighbour is an unvisited child in the same sub-tree
        else {
            // calculate alternative paths of the child, which can also be reached by itself
            childReach = recArtPts(neighbour, depth+1, node);
            reachBack = min(childReach, reachBack);
            // no alternative path from neighbour to any parent
            if (childReach >= depth) then add node into APs;
        }
    }
    return reachBack;
}
```

Example of Recursive AP Algorithm



Iterative AP Algorithm

```
Initialise depth(node) = ∞, APs = {};
Randomly select a node as the root node, set depth(root) = 0, numSubTrees = 0;

for (each neighbour of root) {
   if (depth(neighbour) = ∞) {
      iterArtPts(neighbour, 1, root);
      numSubTrees ++;
   }

   if (numSubTrees > 1) then add root into APs;
}

-------------------------------------------------------------------------------------------------------

iterArtPts(node, depth, parent) {
   …
}
```

The only difference from the recursive version

Iterative AP Algorithm

- **Goal**: set/update reachBack of each node in the sub-tree
- For each node, reachBack is set/updated in the following cases:
  - When first visited: reachBack = depth
  - When one of its children's reachBack is updated: reachBack = min(childReach, reachBack)
  - When all the children have been calculated, update the reachBack of its parent
- Need to update the node information in different stages of DFS

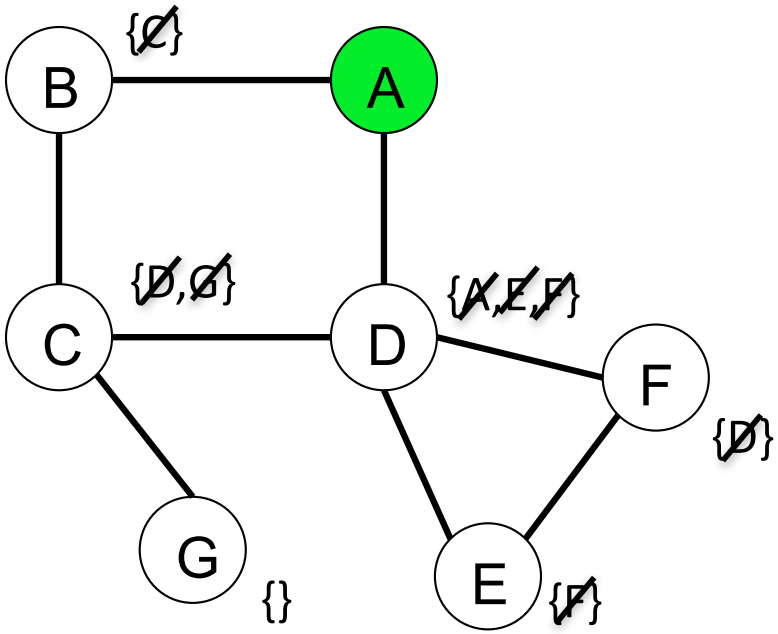
- However, traditional DFS updates node information only once (when visiting the node), and never revisits the node again
- Need a new data structure (modified stack)
  - Last-In-First-Out
  - Traditional stack: pop = get and remove
  - Modified stack: peek = get but not remove

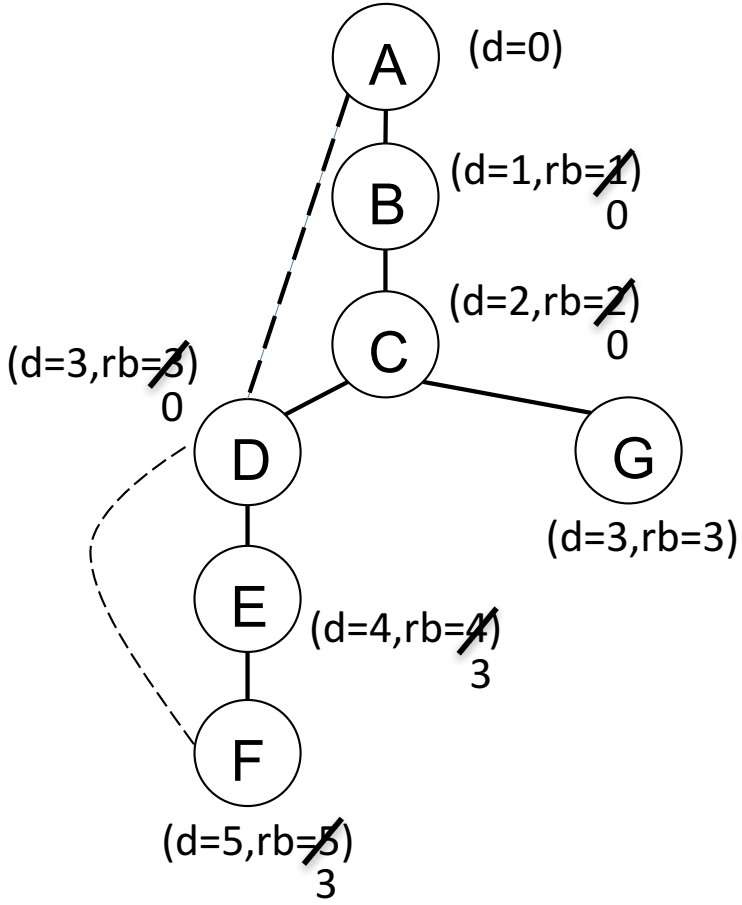
Iterative AP Algorithm

```
iterArtPts(firstNode, depth, root) {
   Initialise stack as a single element <firstNode, depth, root>;
   repeat until (stack is empty) {
      peek <n*, depth*, parent*> from stack;
      if (depth(n*) = ∞) {
         depth(n*) = depth*, reachBack(n*) = depth*;
         children(n*) = all the neighbours of n* except parent*;
      }
      else if (children(n*) is not empty) {
         get a child from children(n*) and remove it from children(n*);
         if (depth(child) < ∞) then reachBack(n*) = min(depth(child), reachBack(n*));
         else push <child, depth*+1, n*> into stack;
      }
      else {
         if (n* is not firstNode) {
            reachBack(parent*) = min(reachBack(n*), reachBack(parent*));
            if (reachBack(n*) >= depth(parent*) then add parent* into APs;
         }
         remove <n*, depth*, parent*> from stack;
}}}
```

Example of Iterative AP Algorithm

<B,1,A>
<C,2,B>
<D,3,C>
<E,4,D>
<F,5,E>
<G,3,C>





APs				
	D	**C**		

Summary

- Two versions of implementing the AP algorithm
  - Recursive
  - Iterative

- DFS can be implemented in two ways
- Recursive is more straightforward to design
- Iterative is more tricky
  - Need to process nodes in different stages
  - A new operation to the fringe: **peek -** process without removing