芯片阱离子输运调研：输运操作

目标势场→电极电压：

目标函数

$$\Phi(\mathbf{r}) = \phi_{\mathrm{ps}}(\mathbf{r}) + \sum_{n=1}^{N} V_n \tilde{\phi}_n(\mathbf{r}).$$

V_n：输运求解对象

N：DC电极数量

目标势场(实验人员提供)

各DC电极的单位势场：BEM

$$\phi_{\mathrm{ps}}(\mathbf{r}) = \frac{q}{4m\Omega_{\mathrm{rf}}^2}(V_{\mathrm{rf}}\nabla\tilde{\phi}_{\mathrm{rf}}(\mathbf{r}))^2$$

$$\Phi_{\mathrm{rf}}(\vec{r},t) = V_{\mathrm{rf}}\,\tilde{\phi}_{\mathrm{rf}}(\vec{r})$$

约束条件：在期望位置产生局部谐振势，约束3个主轴频率和主轴旋转(依赖于阱)；

目标势场的矩阵表示形式

$$\mathbf{C}_1[\mathcal{P}\otimes\boxed{\Psi^{\mathrm{T}}(\mathbf{r}_0)]\mathbf{V}} \doteq \mathbf{C}_2$$

单位势场：$\Psi(\mathbf{r}) \equiv [\,\phi_{\mathrm{ps}}(\mathbf{r})\ \tilde{\phi}_1(\mathbf{r})\ \tilde{\phi}_2(\mathbf{r})\cdots\tilde{\phi}_N(\mathbf{r})\,]^{\mathrm{T}}$,

求解的个电极电压波形：$\mathbf{V} \equiv [\,1\ V_1\ V_2\cdots V_N\,]^{\mathrm{T}}$

C1：j×12 矩阵； C2：j×1 矩阵；

定义矩阵C_1，挑选出期望的约束元素

成本函数：$\min_{|V_i|\leqslant V_{\max}} \left|\mathbf{C}_1[\mathcal{P}\otimes\Psi^{\mathrm{T}}(\mathbf{r}_0)]\mathbf{V} - \mathbf{C}_2\right|^2$

10V

$$\nabla\Phi(\mathbf{r}_0) \doteq 0, \qquad \mathcal{H}(\mathbf{r}_0) \equiv q\begin{bmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x\partial y} & \frac{\partial^2}{\partial x\partial z} \\ \frac{\partial^2}{\partial y\partial x} & \frac{\partial^2}{\partial y^2} & \frac{\partial^2}{\partial y\partial z} \\ \frac{\partial^2}{\partial z\partial x} & \frac{\partial^2}{\partial z\partial y} & \frac{\partial^2}{\partial z^2} \end{bmatrix}\Phi(\mathbf{r}_0) \doteq m\begin{bmatrix} \omega_{x'}^2 & 0 & 0 \\ 0 & \omega_{y'}^2 & 0 \\ 0 & 0 & \omega_{z'}^2 \end{bmatrix}$$

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$$\mathbf{C}_1[\mathcal{P} \otimes \Psi^{\mathrm{T}}(\mathbf{r}_0)]\mathbf{V} \doteq \mathbf{C}_2$$

$$\mathbf{C}_1 = \begin{bmatrix} 1\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,1\,0 & 0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,1 & 0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0 & 1\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0 & 0\,0\,0\,0\,1\,0\,0\,0\,0 \\ 0\,0\,0 & 0\,0\,0\,0\,0\,0\,0\,0\,1 \\ 0\,0\,0 & 0\,1\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0 & 0\,0\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0 & 0\,0\,0\,0\,0\,1\,0\,0\,0 \end{bmatrix} \quad \mathbf{C}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ (m/q)\omega_x^2 \\ (m/q)\omega_y^2 \\ (m/q)\omega_z^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{P} \equiv \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \\ \frac{\partial^2}{\partial x^2} \\ \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial x \partial z} \\ \frac{\partial^2}{\partial y \partial x} \\ \frac{\partial^2}{\partial y^2} \\ \frac{\partial^2}{\partial y \partial z} \\ \frac{\partial^2}{\partial z \partial x} \\ \frac{\partial^2}{\partial z \partial y} \\ \frac{\partial^2}{\partial z^2} \end{bmatrix}$$

芯片阱离子输运调研：输运操作

过约束（约束个数 > 控制电极个数）：可得到 最佳拟合 V；　等约束和轻微过约束 可实现很好的拟合
欠约束（约束个数 < 控制电极个数）：如大电极阵列case, 可以得到多个独立的解；

上述C2是9个约束条件：For QIP in a linear trap array, constraining the axial frequency and orientation is often sufficient；

其他的隐式的物理/几何约束(不能与 C2的约束冲突)：如 3个久期频率wx,wy,wz，由总势场（RF+DC）决定；

RF和DC对频率的贡献 可数学分解为 $\omega_{\mathrm{rf},i}$ and $\tilde{\omega}_i$

$\omega_i^2 = \tilde{\omega}_i{}^2 + \omega_{\mathrm{rf},i}^2$. (An imaginary frequency component would imply antitrapping, while a real component yields trapping.)

$$\sum_{i=1}^{3} \tilde{\omega}_i^2 = 0$$

frequencies. For a *linear* Paul trap, where $\omega_{\mathrm{rf},z} = 0$, the secular frequencies must obey

$$\omega_x^2 + \omega_y^2 + \omega_z^2 = 2\omega_{\mathrm{rf}}^2, \qquad (13)$$

where ω_{rf} is the pseudopotential radial trapping frequency.

几何约束(不能与 C2的约束冲突)：阱的对称性，则势阱的对称性

芯片阱离子输运调研：输运操作

约束：
（1）势最小值的位置**r0** 在三个方向上的约束；
（2）主轴之一 约束为 沿阱轴（包含主轴旋转的2个约束）；
（3）轴向阱频的约束（某理论值）；
（4）供电范围约束（最小二乘求解时的不等约束）： ±10V；

多区域输运，求解的波形，**各步 V 是离散的**，尤其在两组控制电极之间转换时，未约束问题（有许多独立解）会产生 jumps，算法从当前步至下一步 返回一个不同的波形解：输运中，某些地方 突然 利用新组合电极 很容易产生期望势场。

最小二乘——新的成本函数：

$$\min_{|V_i|\leqslant V_{\max},\ |V_i-V_{i,\mathrm{last}}|\leqslant\alpha} \left|\mathbf{C}_1[\mathcal{P}\otimes\Psi^{\mathrm{T}}(\mathbf{r}_0)]\mathbf{V}-\mathbf{C}_2\right|^2, \qquad (14)$$

for a positive constant α. This removes the need for iteratively

new combination of electrodes. In principle, such jumps shoul
not have adverse effects on the potential at the ion, as th
potentials on both side of the jump fulfill the same constrain
and should transition smoothly. However, since the potentia
on the electrodes are filtered, we would expect the potential
the ion to experience a transitory jump during the transition.

These solution jumps can be handled by various mean
We used the constrained least-squares method to seed eac
new solution with the solution of the previous step whi
introducing a cost for deviating from the previous solutio
by replacing Eq. (12) with

$$\min_{|V_i|\leqslant V_{\max},\ |V_i-V_{i,\mathrm{last}}|\leqslant\alpha} \left|\mathbf{C}_1[\mathcal{P}\otimes\Psi^{\mathrm{T}}(\mathbf{r}_0)]\mathbf{V}-\mathbf{C}_2\right|^2, \qquad (14$$

for a positive constant α. This removes the need for iterativel
choosing weights to keep the voltages within bounds, a
suggested in Ref. [25]. This forced the jump transition t
be extended over multiple steps, rather than allowing
discontinuous jump. Another approach is to average the tw
V's on each side of the discontinuity, taking advantage of th
linearity of the equations, to produce an intermediate solutio
that still satisfies the constraints [28]. Performing several step
of such averaging will smooth the jump. Alternately, trial an
error can often be used to determine a set of constraints tha
does not produce a jump, but this can require significant effor

芯片阱离子输运调研：输运操作

抛物势阱最小值每移动1步，执行1次积分，即用最小二乘法计算残余势1次（式53）

最小二乘求解

成本函数:

优化对象

$$\min_{a_m,\varphi_0} \int_{q_0(t)-\delta q}^{q_0(t)+\delta q} |\phi_{\mathrm{res}}(q,t)|^2\, dq \qquad \forall\, t \tag{53}$$

对电极宽度W：典型地优化范围 $2\delta q = 0.5W$
即每步求解的范围

while $\phi(q,t)$ from Eq. (52) enters here through Eq. (4). For any time t we want to find a set a_m, φ_0 for which

输运形式：双曲正切 $p(t) = \tanh\left(N\frac{t-T}{T}\right)$ ；t 瞬时输运时间，T 总输运时间；N 参数

输运形式：d输运距离

$$q_0(t) = \frac{d}{2}\,\frac{\tanh\left(N\,\frac{2t-T}{T} + \tanh N\right)}{\tanh N}$$

可控输运条件:

In the following we consider an idealized situation for the transport, i.e. we assume that we could produce arbitrarily shaped external potentials in the experiment while locally maintaining parabolic potentials around q_0, i.e. $1 \gg |a'_{\mathrm{res}}(q_0)|/\omega_0^2$ and $|\ddot{q}_0| \gg |a_{\mathrm{res}}(q_0)|$ for all positions q_0 or times $q_0(t)$. Deviations from these

$$|a'_{\mathrm{res}}(q_0)|/\omega_0^2 \ll 1 \qquad |a_{\mathrm{res}}(q_0)| \ll |\ddot{q}_0|$$

输运势场:

$$\phi(q,t) = \phi_{\mathrm{local}}(q - q_0(t)) + \varphi_0(t),$$

a *transport function* $q_0(t)$.

where $\phi_{\mathrm{local}}(q) \sim m\omega_0^2 q^2/2Q$ is purely quadratic

残余势场:

$$\phi_{\mathrm{res}}(q,t) = \phi(q,t) - \varphi_0(t) - \frac{m\omega_0^2(q - q_0(t))^2}{2Q} \quad \text{and} \quad a_{\mathrm{res}}(q) = -\frac{Q}{m}\frac{\partial \phi_{\mathrm{res}}(q)}{\partial q}, \tag{4}$$

理想势函数

输运时ion运动方程:

$$\ddot{u} + \omega_0^2 u = -\ddot{q}_0(t) + a_{\mathrm{res}}(u + q_0(t))$$

围绕 势阱最小点 对式5右边展开：忽略高次项

$$\ddot{u} + \omega_0^2\left(1 - a'_{\mathrm{res}}[q_0(t)]/\omega_0^2\right)u - a''_{\mathrm{res}}[q_0(t)]/2\, u^2 + \ldots = -\ddot{q}_0(t) + a_{\mathrm{res}}[q_0($$

$\omega^2(t)$ $f(t)$

抛物势阱最小值每移动1步，执行1次积分，即用最小二乘法计算残余势1次（式53）

最小二乘求解成本函数:

$$\min_{a_m,\varphi_0} \int_{q_0(t)-\delta q}^{q_0(t)+\delta q} |\phi_{\mathrm{res}}(q,t)|^2\, dq \qquad \forall\, t \tag{53}$$

优化对象

对电极宽度W：典型地优化范围 $2\delta q = 0.5W$
即每步求解的范围

while $\phi(q,t)$ from Eq. (52) enters here through Eq. (4). For any time t we want to find a set a_m, φ_0 for which

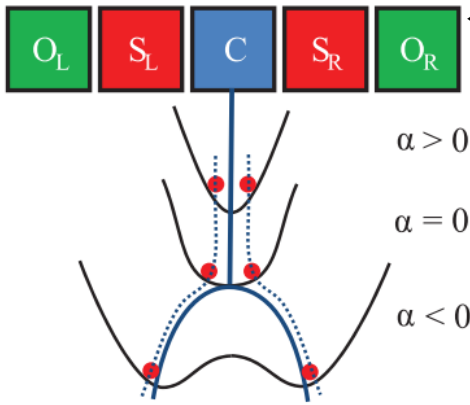
修正成本函数

$$\underline{a}_{+0,\nu} = \arg\min_{\underline{a},\varphi_0} \left\{ \|\mathbf{S}_{+0} \cdot \underline{a}_{+0} - \eta \underline{K}\|^2 + \underline{\nu}^2 \|\mathbf{L}(\underline{a}_{+0} - \underline{a}^*_{+0})\|^2 \right\}$$

能量观测时长：典型地，2倍的输运时间

其值的选择：限制am为实际的实验值

芯片阱离子输运调研：分裂操作



分裂势：四次+二次+一次（寄生电场，实验时已补偿，该项可忽略）只要阱沿阱轴方向足够对称则无3次项；

$$\Psi(x,t) \approx \beta x^4 + \alpha x^2 + \gamma x \qquad (1)$$

α≥0, Eq1的近似成立；α≤0,只要两个势阱间距 < 中心电极C宽度则Eq1的近似成立；
当ions距电极C的距离 与 C宽度相当时，＞4的非谐项主导总势场，本文忽略此case，因分裂结果围绕α=0确定；

临界点α=0之外，平衡距离主要依赖于势阱的高阶项，但是分裂阱间的距离随α的减小而单调增加（只要β的变化足够小），对应的阱频单调增加，因此，临界点之外的运动对应于1个stiff阱的正常输运。

Including Coulomb repulsion, the total electrostatic potential of a two-ion crystal at a center-of-mass position x_0 and distance d is given by

x0：两ions的质心(阱中心)；d:两ions间距；

$$\Phi_{tot}(x_0, d) = \Phi(x_0 + d/2) + \Phi(x_0 - d/2) + \frac{\kappa}{d}, \qquad \kappa = e/4\pi\epsilon_0 \qquad (2)$$

对 基于BEM计算的单位势$\phi_n(x)$ Taylor展开： 只要ion距阱中心足够近， Taylor approximation有效

$$\phi_n(x) = \phi_n|x_0{}^{(m)} + \phi_n'|x_0{}^{(m)}\,\delta x + \frac{1}{2}\phi_n''|x_0{}^{(m)}\,\delta x^2 + \frac{1}{24}\phi_n^{(4)}|x_0{}^{(m)}\,\delta x^4 + \mathcal{O}(\delta x^6)$$

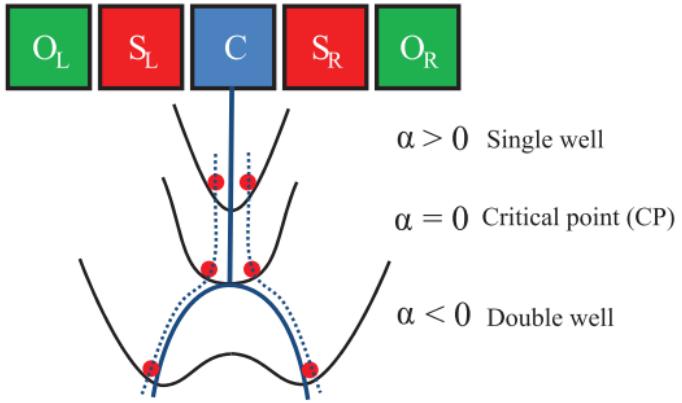
$\delta x = x - x_0{}^{(m)}$, 即围绕中心电极$m$的中心$x_0{}^{(m)}$展开

$$\alpha_n = \frac{1}{2} f_n \phi''_{n,C}(0) \qquad \beta_n = \frac{1}{24} f_n \phi^{(4)}_{n,C}(0) \qquad \gamma_n = f_n \phi'_{n,C}(0)$$

f_n：电极标号，该阱模型 f_C =1， $f_{S,O}$ = 2 $m = C$; $n = C,S,O$ 取$\gamma_C = 0$

$$\alpha = U_C\alpha_C + U_S\alpha_S + U_O\alpha_O$$
$$\beta = U_C\beta_C + U_S\beta_S + U_O\beta_O$$
$$\gamma = \Delta U_S\gamma_S + \Delta U_O\gamma_O + \gamma'.$$

芯片阱离子输运调研：分裂操作



2个ion：质量m，电荷*e*

平衡位置

$$x_{L,R} = x_0 \pm d/2$$

局部阱频：

$$\omega = \sqrt{\frac{e}{m}\Phi''(x_{L,R})}.$$

$\Psi(x,t) \approx \beta x^4 + \alpha x^2 + \gamma x$的极点

$$x_0^{(0)} = \frac{\alpha}{3^{1/3}\zeta} - \frac{\zeta}{2\cdot 3^{2/3}\beta}$$

$$x_0^{(\pm)} = \frac{(i\sqrt{3}\ \pm 1)\alpha}{2\cdot 3^{1/3}\zeta} + \frac{(1\mp i\sqrt{3})\zeta}{4\cdot 3^{2/3}\beta}$$

$$\zeta(\alpha,\beta,\gamma) = \left(9\beta^2\gamma + \sqrt{3}\sqrt{8\alpha^3\beta^3 + 27\beta^4\gamma^2}\right)^{1/3}$$

初始状态：$\alpha=\alpha_i$，由2次谐振势+库伦作用主导,忽略4次项，阱频w、ions距离d

$$\omega^2 = 2\alpha e/m \qquad d = (\kappa/\alpha)^{1/3}$$

CP(α=0)：4次势+库伦作用主导(2次势=0)，阱频w_{cp}(即w_{min})、ions距离d_{cp}

$$\omega_{CP} = \beta^{3/10}(3e/m)^{1/2}(2\kappa)^{1/5}. \qquad d_{CP} = (2\kappa/\beta)^{1/5}.$$

CP(α=0)附近，ions的平衡距离d(α)：用2阶扰动表达式表示；

$$d(\alpha) \approx d_{CP} - \frac{1}{5}\left(\frac{16}{\beta^4\kappa}\right)^{1/5}\alpha + \frac{2}{25}\left(\frac{4}{\beta^7\kappa^3}\right)^{1/5}\alpha^2, \quad \text{for } |\alpha| \ll \beta d_{CP}^2 \text{ and } |\alpha| \ll \kappa d_{CP}^{-3}.$$

CP(α=0)附近，ions晶体的质心位置(同种ions，即中心)：倾斜参数γ的1阶

$$x_0(\alpha,\gamma) \approx \gamma\left(-\frac{1}{3\cdot 2^{2/5}\beta^{3/5}\kappa^{2/5}} - \frac{2^{1/5}}{45\cdot\beta^{6/5}\kappa^{4/5}}\alpha + \frac{26\cdot 2^{4/5}}{675\beta^{9/5}\kappa^{6/5}}\alpha^2\right)$$

若ions分离足够远(α<<0)，库伦力忽略，ions的平衡位置(≈外部势的极值)：

$$d_f = \sqrt{-2\alpha_f/\beta} \qquad \omega_f^2 = -4\alpha_f e/m$$

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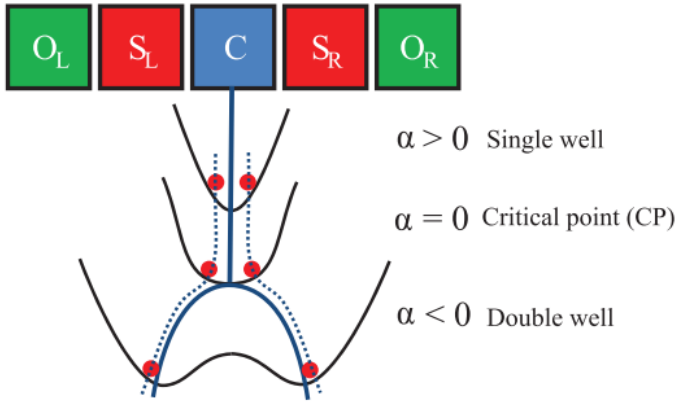


$$\Psi(z,t) \approx \beta z^4 + \alpha z^2 + \gamma z$$

为实现 CP处β最大化，分裂初始，ramp外电极O至+U_{lim},在CP附近保持恒定bias，分裂完成后再ramp至0

通过设置合适的电压Uc,s,使得α=0，剩余的1各自由度Uo可通过最大化βcp消除

$$\alpha = U_C\alpha_C + U_S\alpha_S + U_O\alpha_O$$

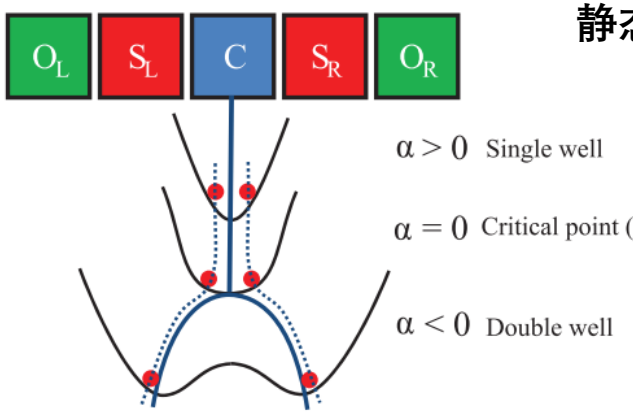
$$\beta = U_C\beta_C + U_S\beta_S + U_O\beta_O$$

$$\gamma = \Delta U_S\gamma_S + \Delta U_O\gamma_O + \gamma'.$$

$$U_C = \frac{1}{\alpha_C}\left(\alpha - \alpha_O U_O - \alpha_S U_S\right).$$

$$U_O^{(\mathrm{CP})} = +U_{\mathrm{lim}},\ U_S^{(CP)} = -U_{\mathrm{lim}}$$

$$\max_{U_C,U_S}\beta_{\mathrm{CP}} = \left(\beta_O + \frac{\beta_C}{\alpha_C}\alpha_S - \beta_S - \frac{\beta_C}{\alpha_C}\alpha_O\right)U_{\mathrm{lim}}$$



如果选择 $U_S^{(CP)} < U_{lim}$，
则β_{CP}小，ions间距大
（即β_{CP}受限于U_{lim}）

$\gamma = 0$

静态电压集 的计算：通过 固定初始、CP、分裂末端电压配置 及各区间插值 获得，过程如下4步：

（1）确定 初始 $\alpha_i > 0$: 设置初始电压$U_C^{(i)} < 0$ V, $U_S^{(i)} = U_O^{(i)} = 0$ V，由 $\alpha = U_C\alpha_C + U_S\alpha_S + U_O\alpha_O$

（2）选择电压 使其 β_{CP} 最大：通过设置$U_O^{(CP)} = +U_{lim}$, $U_S^{(CP)} = -U_{lim}$, $U_C^{(CP)}(\alpha = 0)$；一个可变偏置 $\Delta U_C^{(CP)}$ 被叠加至$U_C^{(CP)}$，以保证 CP电压set实际对应于 α=0，因此可调参数实现 缺陷补偿；

$$\max_{U_C,U_S}\beta_{CP} = \left(\beta_O + \frac{\beta_C}{\alpha_C}\alpha_S - \beta_S - \frac{\beta_C}{\alpha_C}\alpha_O\right)U_{\lim}$$

如果 几何参数 使得$U_C^{(CP)}$超出$\pm U_{lim}$，则设置中心电极$U_C^{(CP)} = -U_{lim}$ 、通过上述方程求$U_S^{(CP)}$ 而非$U_C^{(CP)}$；

（3）确定**最终电压**：设置$U_C^{(f)} = 0$V，$U_S^{(f)} = U_S^{(CP)} = -U_{lim}$，$U_O^{(f)} = 0$ V，当$U_C^{(i)} \approx -U_{lim}$ 时，这个电压设置比较方便，并且可确保ions最终保持再S电极的各自中心(类似初始状态)，从上式确定 α_f；

（4）for 接近CP，$\alpha_i \geq \alpha > 0$，设置 $U_S(\alpha) = \left(1 - \frac{\alpha}{\alpha_i}\right)U_S^{(CP)}$

$$U_O(\alpha) = \begin{cases} 2\left(1 - \frac{\alpha}{\alpha_i}\right)U_{\lim} & \alpha > \frac{\alpha_i}{2} \\ U_{\lim} & \alpha \leqslant \frac{\alpha_i}{2} \end{cases}$$

结合上式 获得 $U_C(\alpha)$

（5）for 超过CP，$0 \geq \alpha > \alpha_f$，设置 $U_S(\alpha) = -U_{\lim}$

$$U_O(\alpha) = \begin{cases} U_{\lim} & \alpha > \frac{\alpha_f}{2} \\ 2\left(1 - \frac{\alpha}{\alpha_f}\right)U_{\lim} & \alpha \leqslant \frac{\alpha_f}{2} \end{cases}$$

结合上式 获得 $U_C(\alpha)$

时间域-电压集 $U_n(t)$： CP点的加速度$\ddot{d}_{CP}$要小，能量传递小，保证了分裂过程中距离$d(t)$的可控变化；

ions的平衡距离$d(\alpha)$：**$d(\alpha)$ 随α 的减小而单调变化 → 反函数 $\alpha(d)$ → $U_n(\alpha(d(t)))$**

Possible choices for $d(t)$ are a sine-squared ramp

$$d(t) = d_i + (d_f - d_i)\sin^2\left(\frac{\pi t}{2T}\right)$$

or a polynomial ramp

$$d(t) = d_i + (d_i - d_f)\left(-10\frac{t^3}{T^3} + 15\frac{t^4}{T^4} - 6\frac{t^5}{T^5}\right)$$

$$d(0) = d_i,\ d(T) = d_f,\ \dot{d}(0) = \dot{d}(T) = 0$$

优选多项式d(t)：在始末点的二阶导为0, $\ddot{d}(0) = \ddot{d}(T) = 0$

sine方的二阶导不连续，因数字电压和滤波，可能实验中也无影响

the equations of motion for the ion positions $x_1 < x_2$. Employing the Taylor approximation potential equation (1), these read

ion运动方程：基于分裂势的泰勒近似，电压ramps确定系数α、β、γ

$$-m\ddot{x}_{1,2} = 4\beta(t)x_{1,2}^3 + 2\alpha(t)x_{1,2} + \gamma \pm \frac{\kappa}{(x_2 - x_1)^2}, \tag{40}$$

where the coefficients are given by using the voltage ramps in equations (3), (4), (5). For realistic trap potentials, we obtain

ion运动方程：基于真实势or仿真势（BEM）

$$-m\ddot{x}_{1,2} = \sum_{n=C,S,O} U_n(t)\left.\frac{\mathrm{d}\phi_n}{\mathrm{d}x}\right|_{x_{1,2}} \pm \frac{\kappa}{(x_2 - x_1)^2}. \tag{41}$$

两种方法验证电压ramps的性能

The possibility to perform the simulations with approximate and realistic potentials serves the purpose of verifying the performance of the voltage ramps. These are determined purely by trap properties around the CP, which are conveniently accessible by measurements. More precisely,

Ions平衡距离$d(\alpha)$ 的计算方法:

- ✓ 通过仿真数据中真实势阱计算，利用关于给定α的电压配置（如通过静态电压集$U_n(\alpha)$确定的）， 该方法要求仿真势与实际势阱高精度匹配;
- ✓ 利用α_n, β_n的校准测试值，计算平衡距离：不需要仿真，且计入参数漂移；只对小于电极宽度的距离产生有效值;
- ✓ 减小α值的电压配置被施加，通过CCD测量离子距离：从可分辨的边带谱中测量阱频，确定成像像素中的离子距离；放大倍数由阱频决定$d = \left(\frac{2e\kappa}{m\omega^2}\right)^{1/3}$;该方法得益于可分辨边带谱的精度（典型10 kHz ~100 Hz）

势场基已知，泰勒近似确定系数$\alpha_i,\beta_i,\gamma_i$ →β_{CP}; → **运动方程求 ions平衡位置**

↑

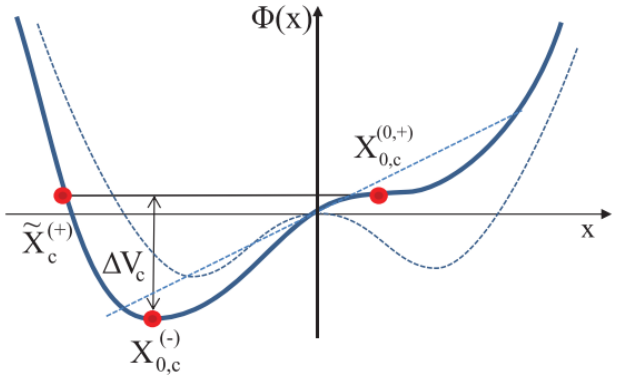
静态电压集 $U_n(\alpha)$:
由分裂初始/CP/末端状态顶点确定并插值

$d(\alpha)$ → **反函数 $\alpha(d)$** → **$U_n(\alpha(d(t))$** → **叠加得 分裂势**

↑ **静态电压集 $U_n(\alpha)$:**
由分裂初始/CP/末端状态顶点确定并插值

↑ **势场基已知，泰勒近似确定系数$\alpha_i,\beta_i,\gamma_i$**

芯片阱离子输运调研：分裂操作



γ临界值$\tilde{\gamma}$： 可利用 分裂势的两个边界极值$x_0^{(\pm)}$估计$\tilde{\gamma}$， 如果寄生势场超过$\tilde{\gamma}$， 则分裂无法工作;

$$x_0^{(\pm)} = \frac{(i\sqrt{3} \pm 1)\alpha}{2 \cdot 3^{1/3}\zeta} + \frac{(1 \mp i\sqrt{3})\zeta}{4 \cdot 3^{2/3}\beta}$$

$$\zeta(\alpha, \beta, \gamma) = \left(9\beta^2\gamma + \sqrt{3}\sqrt{8\alpha^3\beta^3 + 27\beta^4\gamma^2}\right)^{1/3}$$

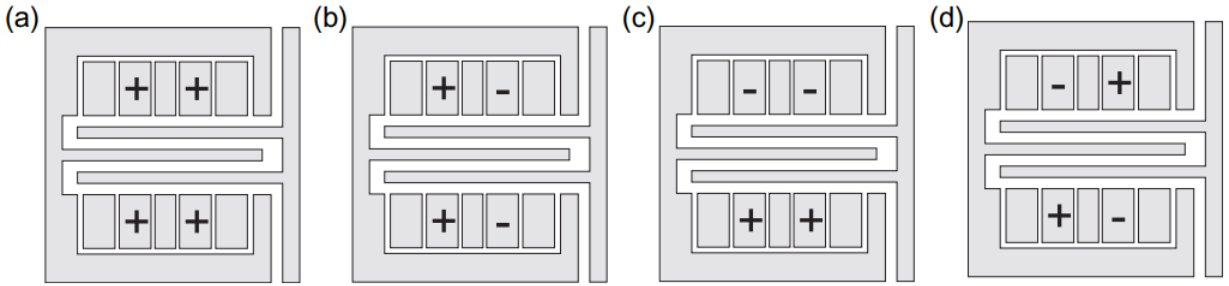
$$\tilde{\gamma} < \pm C_\gamma \left(\kappa^3\beta^2\right)^{1/5}$$ 数值计算确定 $C_\gamma = 1.06$

尽管实际中不能发生上图情况，因为寄生电场已被补偿，则在鞍点处外力消失（不去平衡库仑力），但通过上式$\tilde{\gamma}$的计算 （仅与$\beta_{C,S,O}$相关，且$\propto \beta^{2/5}$），可确定背景轴向势场需要修正的精度（对应电极电压的精度，如ΔU_O）;

芯片阱离子输运调研：SWAP操作

垂直于阱面方向z的囚禁强度最强（保证在旋转过程中ions平面旋转）：

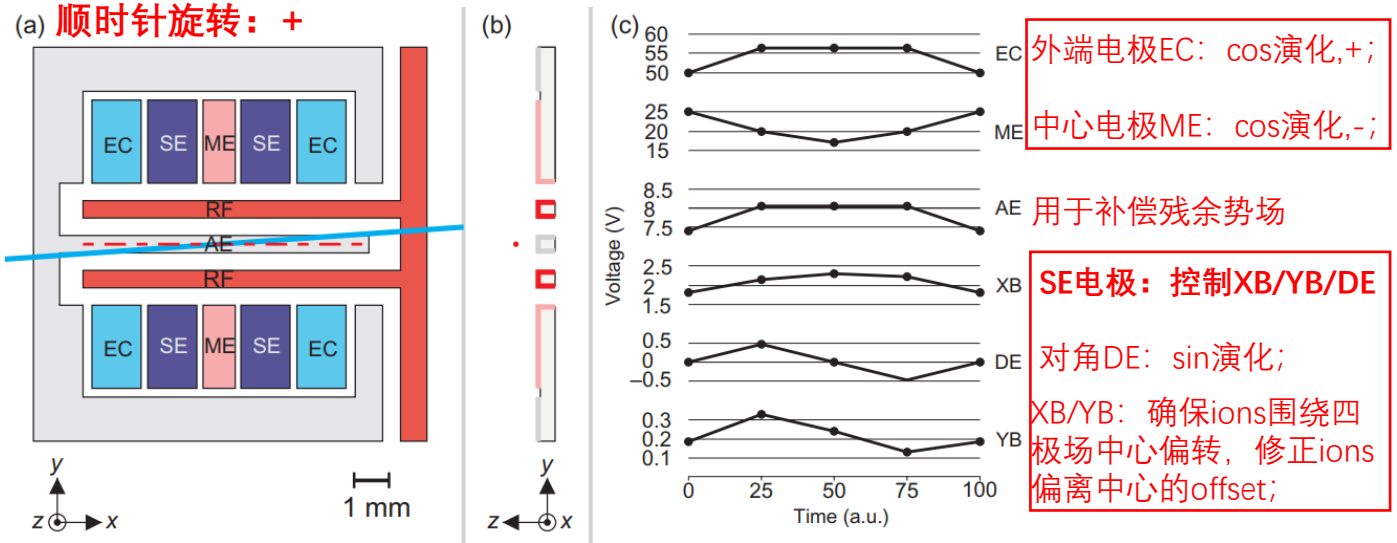
- ✓ 轴向囚禁x增加，直至＞最弱的径向囚禁y，使得ions旋转90°；
- ✓ 当轴向囚禁势relax，对操作电极施加反相电压，则可垂直对齐转换为水平对齐；



SE电极：控制offset/XB/YB/DE　XB：x-balance　YB：y-balance　DE：对角场

Figure 2. Four possible ways in which the 'steering electrode' voltages can be varied. These are termed (a) offset, (b) x-balance, (c) y-balance and (d) diagonal.

效应：z轴移动ions　平面移动ions，对角势是为了旋转光滑过渡



外端电极EC：cos演化,+；

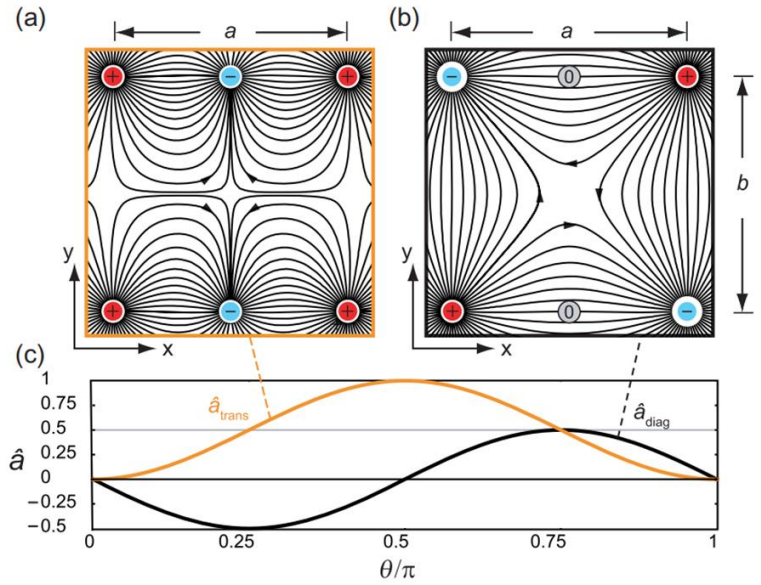
中心电极ME：cos演化,-；

用于补偿残余势场

SE电极：控制XB/YB/DE

对角DE：sin演化；

XB/YB：确保ions围绕四极场中心偏转，修正ions偏离中心的offset；



约束：ions的中心位置、旋转角度；所设置电压保持ions处于线性而非Zigzag；

SWAP所需的电压ramps：**首先确定 关键点0°、45°、90°、135°**等处的电压设置，然后通过线性插值，确定SWAP所需的电压ramps；

旋转过程中，ions间距会变化，轴向阱频也会变化；通过初步的旋转电压ramps确定后，优化电压ramps，使得旋转过程中阱频恒定

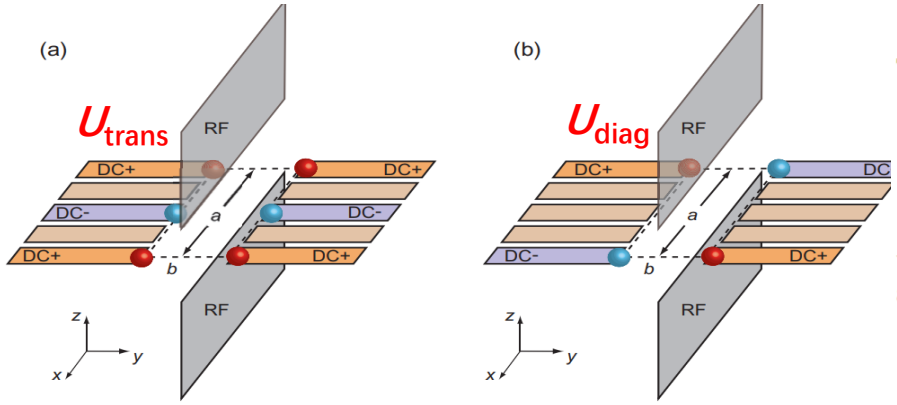
旋转的速度(时间)受限于电极的LPF(ms)，如果LPF更快及阱轴的旋转可以更光滑的执行，则ions旋转更快(us)

旋转过程中，ions间距会变化，**轴向阱频**也会变化；通过初步旋转电压ramps确定后，优化电压ramps，使得旋转过程中**阱频**恒定

中心长电极AE：阻止ions垂直于阱面的运动，源于ME和EC的推or拉；调节AE，保持ions晶体化

芯片阱离子输运调研：SWAP操作

3D阱：阱形式不同（ions初始囚禁在中心轴上）；控制电极与前面平面阱略不同；

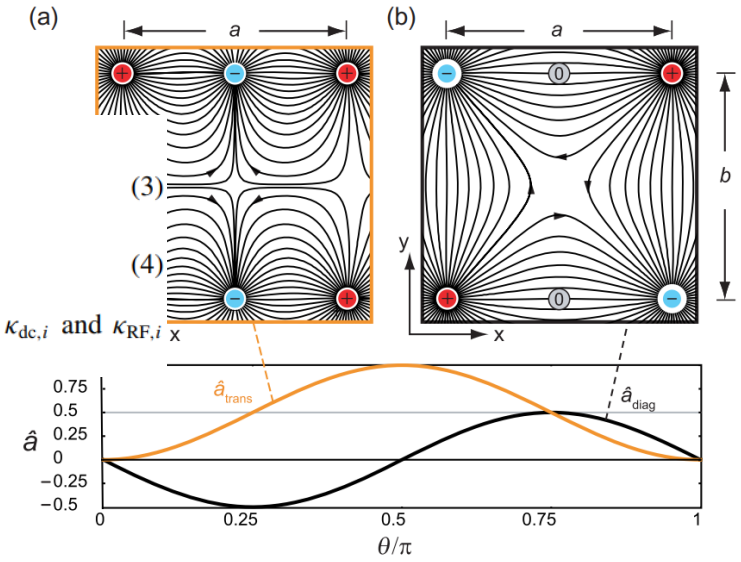


The initial effective potential, U_0, is given by

$$U_0 = \tfrac{1}{2}(\kappa_{\mathrm{dc},x})x^2 + \tfrac{1}{2}(\kappa_{\mathrm{RF}} + \kappa_{\mathrm{dc}},\, y)y^2 + \tfrac{1}{2}\left(\kappa_{\mathrm{RF}} - (\kappa_{\mathrm{dc},x} + \kappa_{\mathrm{dc},y})\right)z^2$$

$$\equiv \tfrac{1}{2}\kappa_x\, x^2 + \tfrac{1}{2}\kappa_y\, y^2 + \tfrac{1}{2}\kappa_z\, z^2,$$

with, in general, three different curvatures, κ_i, that are the sum of curvatures $\kappa_{\mathrm{dc},i}$ and $\kappa_{\mathrm{RF},i}$ arising from the static potential and the RF pseudopotential, respectively.



ions旋转$\theta(t)$所需的势场$\Delta U(t)$：由截面势场U_{trans}和对角势场U_{diag}组合实现

旋转所需的场 → $\Delta U(t) = U(t) - U_0.$

目标场（旋转后）　初始场（旋转前）

$$\Delta U(t) = \hat{a}_{\mathrm{trans}}(t)\,\Delta U_{\mathrm{trans}} + \hat{a}_{\mathrm{diag}}(t)\,\Delta U_{\mathrm{diag}}.$$

ions沿x轴→y轴对齐的势场转换，xy各自的势曲率交换

$$\Delta U_{\mathrm{trans}} = \tfrac{1}{2}\Delta\kappa\,(x^2 - y^2),$$

$$\Delta\kappa = \kappa_y - \kappa_x.$$

矩阵形式表示

$$Q(\theta) = Q_{\mathrm{trans}}\underbrace{\tfrac{1}{2}(1-\cos(2\theta))}_{\hat{a}_{\mathrm{trans}}} - Q_{\mathrm{diag}}\underbrace{\tfrac{1}{2}\sin(2\theta)}_{\hat{a}_{\mathrm{diag}}}.$$

$$Q_{\mathrm{trans}} = \Delta\kappa\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \quad Q_{\mathrm{diag}} = R(\pi/4)\,Q_{\mathrm{trans}}\,R(-\pi/4) = \Delta\kappa\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

截面势场U_{trans}的电压配置（如图6个电极）：施加电压依赖于几何a,b，从而确定囚禁强度 $V_{\mathrm{middle}}/V_{\mathrm{corner}} = -2(1+(a/b)^2)^{-3/2}$

对角势场U_{diag}的电压配置（如图4个电极）：施加电压独立于几何a,b，对称±