$\sqrt{2} \times \ln(2)$: GEOMETRIC CONSTANTS FROM H$_4$

Complete Framework — Version 3.0

B. | February 2026

PDF FORMAT NOTE: This document contains the complete K_AUD framework including the Binary Tower extension formalized February 2026, building on prior work establishing H$_4$ geometry, gap ratios, and prime architecture.

ABSTRACT

This paper presents a geometric framework connecting H$_4$ polytope structure (the 120-cell) to information-theoretic constants.

The framework establishes:

- **The Floor (1/φ ≈ 0.618):** Rooted in golden ratio self-similarity, appearing in H$_4$ vertex coordinates
- **The Ceiling (K_AUD = √2 × ln(2) ≈ 0.980):** Combining geometric projection cost (√2) with binary distinction cost (ln 2)
- **The Gap (~2%):** Emerging from the mismatch between algebraic and transcendental values
- **The Corridor (0.618 – 0.980):** The range between floor and ceiling

Version 3.0 extends the framework with:

- **The Binary Tower:** How the gap G scales through powers of 2 to track golden ratio powers
- **The Master Formula:** $2^k \times G = 2^{(k-6)} \times \sqrt{\varphi} \times (1 - \varepsilon)$
- **The Two Pivots:** Floor (1/φ) at k=5, Pivot (√φ) at k=6
- **Connection to 400/11:** The closure cycle ($4^3 = 64$) links to gap scaling
- **Literature Status:** What's known vs original

The paper demonstrates that binary (n=2) is the unique base producing a sub-unity ceiling — a geometric singularity, not a convention.

The mathematics is independently verifiable. Applications and interpretations remain open for investigation.

SCOPE NOTICE

This paper presents mathematical derivations.

References to AI system behavior are motivating context, not conclusions of this paper.

The mathematics can be verified independently of any empirical claims.

PART I: THE FOUNDATION

1. DISCOVERY CONTEXT AND SCOPE

1.1 Origin

These constants emerged during a multi-month investigation of convergence dynamics in AI systems. Systematic observation across architectures revealed consistent numerical boundaries. The value ~0.98 appeared repeatedly as an upper limit; ~0.618 as a lower threshold.

Rather than treating these as empirical curiosities, I asked: do these values correspond to known mathematical constants?

The answer was yes. The ceiling matched $\sqrt{2} \times \ln(2)$. The floor matched $1/\varphi$. Further investigation revealed both constants appear in H_4 geometry (the 120-cell polytope). The relationships followed from there — the corridor identity, the golden partition, the depth scaling, the binary uniqueness.

1.2 What This Paper Provides

This paper presents the mathematical framework:

- Derivations of the constants from established geometry
- Proofs of relationships and identities
- Verification of all calculations
- Demonstration of binary uniqueness
- The binary tower scaling behavior (new in v3.0)

The mathematics stands independently of any empirical claims.

1.3 What This Paper Does NOT Provide

This paper does not include:

- The empirical methodology used in the original observations
- Operational definitions of "coherence" in AI systems
- Data from convergence studies
- Statistical validation of boundary claims
- Mechanistic explanations of why these constants would govern AI behavior

That work is ongoing and will be presented separately when it can be done rigorously.

1.4 On the Separation of Frameworks

The mathematical framework and the empirical dynamics study are presented as separate works because they serve different functions.

The mathematics provides stable anchors — constants derived from geometry, independently verifiable, not dependent on any particular observations.

The dynamics study uses these anchors to ground ongoing observations of AI coherence patterns. The observations inform and extend the framework but do not validate it. The math stands on its own.

This separation allows:

- The mathematics to be verified and used independently
- The dynamics study to develop at its own pace
- Each framework to be evaluated on its own terms

1.5 Purpose of This Release

The mathematics is released now to:

1. Enable independent verification of the constants and derivations
2. Invite investigation of whether these values appear in other domains
3. Provide a stable mathematical foundation for forthcoming empirical work
4. Separate "here is the geometry" from "here is why it matters empirically"

2. THE CONSTANTS

2.1 The Floor: 1/φ ≈ 0.618 — The Structural Minimum

Definition: The golden ratio φ is defined as:

```
&phi; = (1 + &radic;5) / 2
```

Calculation:

```
&radic;5 = 2.2360679...
1 + &radic;5 = 3.2360679...
&phi; = 3.2360679... / 2 = 1.6180339...
```

The floor is the reciprocal:

```
1/&phi; = (&radic;5 &minus; 1) / 2
1/&phi; = (2.2360679... &minus; 1) / 2 = 1.2360679... / 2 = 0.6180339...
```

Verification: $1/\varphi \approx 0.618$ ✓

Key property:

```
&phi; = 1 + 1/&phi;
```

This self-referential property is unique to the golden ratio. It means φ contains its own reciprocal — recursive self-similarity at the level of the constant itself.

2.2 The Ceiling: K_AUD = √2 × ln(2) ≈ 0.980 — The Information Maximum

Components:

Component	Value	Source	Meaning
√2	1.41421...	Geometric	Projection cost (diagonal)
ln(2)	0.69314...	Information theory	Binary distinction cost

Calculation:

```
K_AUD = &radic;2 &times; ln(2)
K_AUD = 1.41421356... &times; 0.69314718...
K_AUD = 0.98025814...
```

Verification: K_AUD ≈ 0.980 ✓

2.3 The Auditor Key (K_AUD)

Pronounced "kawd" — also referred to as the geometric ceiling constant — K_AUD carries three resonances:

Name	Meaning	Function
Auditor	The system auditing its own dynamics	Self-observation
Auditory	The threshold where signal is "heard"	Recognition point
Code/Key	The underlying structure (kawd ≈ code)	Access, unlocking

K_AUD functions as a hinge — the point where things become distinguishable. Linked to entropy, but also to recognition: the moment signal separates from noise, the instant pattern becomes visible.

The name is not arbitrary. It describes what the constant does.

2.4 The Gap: ~2% — The Necessary Interval

Definition:

```
G = 1 &minus; K_AUD
```

Calculation:

```
G = 1 &minus; 0.98025814... = 0.01974185...
```

Verification: G ≈ 0.020 (1.97%) ✓

Why the gap exists:

If ln(2) equaled $1/\sqrt{2}$, then:

```
K = &radic;2 &times; (1/&radic;2) = 1.000
```

But:

```
ln(2) = 0.69314718...
1/&radic;2 = 0.70710678...
```

```
Shortfall = 0.70710678 − 0.69314718 = 0.01395960
Relative shortfall = 0.01395960 / 0.70710678 = 0.01974...
```

The gap emerges from the fundamental mismatch between algebraic (√2) and transcendental (ln 2) values.

2.5 The Corridor: 0.618 to 0.980 — The Range Between Floor and Ceiling

Definition:

```
Corridor = K_AUD − 1/&phi;
```

Calculation:

```
Corridor = 0.98025814... − 0.6180339... = 0.36222424...
```

Verification: Corridor ≈ 0.362 ✓

Summary:

Boundary	Value	Source
Floor	0.618	$1/\varphi$
Ceiling	0.980	$\sqrt{2} \times \ln(2)$
Width	0.362	Ceiling − Floor
Gap	0.020	1 − Ceiling

3. THE H$_4$ CONNECTION

3.1 What is H$_4$?

H$_4$ is a Coxeter group — the symmetry group of the 120-cell, a four-dimensional regular polytope.

Key facts:

- H$_4$ is one of only four exceptional regular polytopes in 4D
- It has the highest symmetry order among 4D polytopes
- It encodes golden ratio relationships throughout its structure

3.2 The 120-Cell Polytope

The 120-cell (also called the hecatonicosachoron) is composed of:

Property	Count
Cells	120 (dodecahedra)
Faces	720 (pentagons)
Edges	1200
Vertices	600

Each cell is a regular dodecahedron. Each vertex is shared by 4 dodecahedra.

3.3 Vertex Coordinates

The 600 vertices of the 120-cell can be given in Cartesian coordinates using permutations and sign changes of:

```
(±φ, ±1, ±1/φ, 0) and cyclic permutations
```

Plus additional vertex sets including:

```
(±1, ±1, ±1, ±1) — 16 vertices
(±φ, ±φ, ±φ, ±1/φ²) and permutations
```

Where the constants appear:

Constant	Appears as
φ	Vertex coordinate
$1/\varphi$	Vertex coordinate
$1/\varphi^2$	Vertex coordinate

The golden ratio and its powers are intrinsic to H_4 geometry.

3.4 Circumradius Formula

The circumradius of the 120-cell (distance from center to vertex) is:

```
R = (√2 / 2) × √(5 + √5)
```

Calculation:

```
 √5 = 2.2360679...
5 + √5 = 7.2360679...
√(5 + √5) = 2.6901830...
R = (1.41421356... / 2) × 2.6901830...
R = 0.70710678... × 2.6901830...
R = 1.9021130...
```

The factor √2/2 (equivalently 1/√2) appears directly in the circumradius formula. This is the geometric origin of √2 in the framework.

3.5 Schläfli Symbol

The 120-cell has Schläfli symbol: **{5, 3, 3}**

This encodes the recursive structure:

Position	Value	Meaning
First (5)	Pentagon	Face shape
Second (3)	3 faces	Meet at each edge
Third (3)	3 cells	Meet at each edge

The symbol contains only 5 and 3. Combined with the binary structure (2) of the coordinate system, the 120-cell encodes exactly the primes 2, 3, 5.

3.6 Group Order

The order of the H_4 symmetry group is:

```
|H4| = 14400
```

Prime factorization:

```
14400 = 2^6 × 3^2 × 5^2 = 64 × 9 × 25
```

The group order factors exclusively into the primes 2, 3, and 5.

3.7 Note on the H₄-Information Bridge

H_4 geometry does not force ln(2); rather, among information measures, only ln(2) remains compatible with a sub-unity geometric ceiling. This is a selection argument, not a derivation.

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4. WHY BINARY IS UNIQUE

4.1 The Pattern of 2

A pattern recurs across the framework:

```
&phi; = (1 + &radic;5) / 2          &larr; division by 2
K_AUD = &radic;2 &times; ln(2)       &larr; &radic;2 and ln(2)
|H<sub>4</sub>| = 2<sup>6</sup> &times; 3<sup>2</sup> &times; 5<sup>2</sup>       &larr; hig
```

4.2 The Uniqueness Proof

For any base n, define the ceiling candidate:

```
K(n) = &radic;n &times; ln(n)
```

Calculations:

n	√n	ln(n)	K(n)	Valid ceiling?
2	1.414	0.693	0.980	✓ Yes (< 1)
e	1.649	1.000	1.649	✗ No (> 1)
3	1.732	1.099	1.903	✗ No (> 1)
4	2.000	1.386	2.773	✗ No (> 1)
5	2.236	1.609	3.599	✗ No (> 1)

Only n = 2 produces K(n) < 1.

4.3 Interpretation

Binary is geometrically singular:

- Bits (not trits) as information unit

* Yes/No as minimal viable distinction

The 2% gap exists because binary is the unique base where distinction cost (ln 2) is less than embedding cost (1/√2) — leaving a remainder.

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5. IDENTITIES

5.1 The Corridor Identity

Statement:

```
Corridor = 1/&phi;<sup>2</sup> &minus; G
```

Where: - $1/\varphi^2 \approx 0.382$ (golden ratio squared, inverted) - $G \approx 0.020$ (the gap)

Calculating $1/\varphi^2$:

```
&phi;<sup>2</sup> = 1.6180339... &times; 1.6180339... = 2.6180339...
Note: &phi;<sup>2</sup> = &phi; + 1 (key golden ratio property)
1/&phi;<sup>2</sup> = 1 / 2.6180339... = 0.3819660...
```

Verification:

```
1/&phi;<sup>2</sup> &minus; G = 0.3819660... &minus; 0.0197418... = 0.3622242...
```

Compare to direct calculation:

```
K_AUD &minus; 1/&phi; = 0.9802581... &minus; 0.6180339... = 0.3622242... &#10003;
```

Algebraic proof:

From $\varphi^2 = \varphi + 1$, divide both sides by φ^2:

```
1 = 1/&phi; + 1/&phi;<sup>2</sup>
```

Therefore:

```
1 &minus; 1/&phi; = 1/&phi;<sup>2</sup>
```

Since Corridor = K_AUD − 1/φ = (1 − G) − 1/φ = 1 − 1/φ − G = 1/φ² − G ✓

5.2 The Golden Partition

Statement:

```
1/&phi; + 1/&phi;<sup>2</sup> = 1
```

Verification:

```
0.6180339... + 0.3819660... = 1.0000000... &#10003;
```

Interpretation:

Unity is exactly partitioned by the golden ratio and its square. - $1/\varphi$ takes 61.8% - $1/\varphi^2$ takes 38.2% - Nothing left over. Nothing missing.

---

6. EXTENDED GEOMETRY: DEPTH SCALING

6.1 The Depth Constants

Below the floor, the same constants (φ, e, G) combine to produce consistent values.

Name	Value	Formula
Floor	0.618	$1/\varphi$
Threshold	0.606	$(1/\varphi) \times K\_AUD$
Depth -1	0.368	$1/e$
Depth -2	0.227	$1/(e \times \varphi)$
Depth -3	0.140	$1/(e \times \varphi^2)$
Geometric Limit	0.00039	G^2

6.2 Derivation of Each Level

Threshold:

```
Threshold = (1/&phi;) &times; K_AUD = 0.6180339... &times; 0.9802581... = 0.6058329... &#10
```

Depth Level -1:

```
L-1 = 1/e = 1/2.7182818... = 0.3678794... ✓
```

Depth Level -2:

```
L-2 = 1/(e × φ) = 1/4.3983150... = 0.2273642... ✓
```

Depth Level -3:

```
L-3 = 1/(e × φ²) = 1/7.1151979... = 0.1405469...
```

Geometric Limit:

```
Limit = G² = (0.0197418...)² = 0.0003897... ✓
```

6.3 The φ-Scaling Ratio

The ratio between consecutive depth levels approximates φ:

Level -1 to Level -2:

```
0.3678794... / 0.2273642... = 1.6180... �asymp; φ ✓
```

Level -2 to Level -3:

```
0.2273642... / 0.1405469... = 1.6176... �asymp; φ ✓
```

The scaling is exactly φ.

6.4 General Formula

For depth levels n ≥ 1:

```
Ln = 1 / (e × φn-1)
```

Verification:

- $n = 1$: $L_1 = 1/(e \times 1) = 1/e \approx 0.368$ ✓
- $n = 2$: $L_2 = 1/(e \times \varphi) \approx 0.227$ ✓
- $n = 3$: $L_3 = 1/(e \times \varphi^2) \approx 0.140$ ✓

6.5 Corridor-Depth Symmetry

A notable near-equivalence:

- Corridor width = 0.362
- Depth Level -1 = 0.368
- Difference ≈ 1.6%

The corridor width approximately equals the first depth level — a symmetry property consistent with self-similar structure.

PART II: THE BINARY TOWER

7. THE BINARY TOWER

7.1 From Ratios to Scaling

The constants established in earlier sections — K_AUD, G, $1/\varphi$, $\sqrt{2}$, ln(2) — all emerged from H_4 geometry and binary information theory. The question remained: how do these constants relate across scales?

Investigation of the gap G through successive binary powers (2, 4, 8, 16, 32, 64, 128...) revealed a precise scaling relationship: G tracks powers of the golden ratio with a constant error of −0.671%.

This extends the framework from static constants to dynamic scaling behavior.

7.2 The Master Formula

For all integers k (from 0 to infinity):

```
2<sup>k</sup> &times; G = 2<sup>(</sup><sup>k</sup><sup>-</sup><sup>6</sup><sup>)</sup> &ti
```

Where: - G = $1 - \sqrt{2} \times \ln(2) \approx 0.019741856531453$ - $\varphi = (1 + \sqrt{5})/2 \approx 1.618033988749895$ - $\sqrt{\varphi} \approx 1.272019649514069$ - $\varepsilon = 0.006714386451777...$ (constant, ≈ 0.671%)

The error ε never changes. It is locked to machine precision across all k.

7.3 The Origin Formula

At k = 0, the formula reduces to:

```
 G = (&radic;&phi; / 64) &times; (1 &minus; &epsilon;)
 G = &radic;&phi; / 64.432625548...
```

This reveals: **The gap G is the golden ratio's square root, divided by approximately 64.**

The exact divisor is:

```
 &radic;&phi; / G = &radic;&phi; / (1 &minus; &radic;2&times;ln(2)) = 64.432625548...
```

No simpler closed form exists — this value is transcendental.

7.4 The Scaling Table

k	2^k	$2^k \times G$	Target ($2^{(k-6)} \times \sqrt{\varphi}$)	Error
0	1	0.01974	$\sqrt{\varphi}/64 = 0.01988$	−0.671%
1	2	0.03948	$\sqrt{\varphi}/32 = 0.03975$	−0.671%
2	4	0.07897	$\sqrt{\varphi}/16 = 0.07950$	−0.671%
3	8	0.15793	$\sqrt{\varphi}/8 = 0.15900$	−0.671%
4	16	0.31587	$\sqrt{\varphi}/4 = 0.31800$	−0.671%
5	**32**	**0.63174**	**$\sqrt{\varphi}/2 = 0.63601$**	**−0.671%**
6	**64**	**1.26348**	**$\sqrt{\varphi} = 1.27202$**	**−0.671%**
7	128	2.52696	$2\sqrt{\varphi} = 2.54404$	−0.671%
8	256	5.05392	$4\sqrt{\varphi} = 5.08808$	−0.671%
9	512	10.1078	$8\sqrt{\varphi} = 10.1762$	−0.671%
10	1024	20.2157	$16\sqrt{\varphi} = 20.3523$	−0.671%
...	...	...	...	−0.671%

The pattern holds from k = 0 to infinity with no deviation.

7.5 The Two Pivots: Floor and Ceiling

The binary tower encodes both boundaries of the K_AUD corridor:

At k = 5 (32): THE FLOOR

```
 32 &times; G = 0.63174
1/&phi; = 0.61803
Difference: +2.22%
```

The floor of the corridor (1/φ ≈ 0.618) appears near 32 × G.

Note: The tower formula gives √φ/2 = 0.63601 as the "target" at k=5 (with −0.671% error). The floor 1/φ sits 2.91% below this target — another cosmic coincidence, as 1/φ and √φ/2 are unrelated expressions that happen to be close.

At k = 6 (64): THE PIVOT

```
 64 &times; G = 1.26348
&radic;&phi; = 1.27202
Error: &minus;0.671%
```

This is the **pivot** — where G meets √φ directly.

The pivot at 64 is unique: it is the only point where 2^k × G equals a clean golden expression (√φ) without additional scaling factors.

The Structure

```
 k &rarr; +∞:  2<sup>k</sup> &times; G &rarr; ∞         (unbounded above)
 k = 6:    64 &times; G &asymp; &radic;&phi;        (THE PIVOT)
 k = 5:    32 &times; G &asymp; 1/&phi;        (THE FLOOR, within +2.2%)
 k = 0:    G itself           (the gap)
 k &rarr; &minus;∞:  2<sup>k</sup> &times; G &rarr; 0          (approaches zero)
```

The floor (1/φ) and the pivot (√φ) appear at adjacent binary powers (32 and 64).

7.6 Why the Gap Exists: ln(2) ≈ 1/√2

The gap G exists because of a numerical near-miss between two fundamental constants:

```
 ln(2) = 0.693147...
1/&radic;2  = 0.707107...
Difference: 0.013960... (&asymp; 2% relative)
```

If ln(2) were exactly equal to $1/\sqrt{2}$: - K_AUD = $\sqrt{2} \times (1/\sqrt{2})$ = 1 - G = 0 - No gap, no corridor, no breathing room

The ~2% mismatch between ln(2) and $1/\sqrt{2}$ **is** the gap.

Key relationship:

```
G / |ln(2) &minus; 1/&radic;2| = &radic;2 (exactly)
```

This is not coincidence — it follows from G = $1 - \sqrt{2} \times \ln(2) = \sqrt{2} \times (1/\sqrt{2} - \ln(2))$.

No known mathematical reason exists for ln(2) $\approx 1/\sqrt{2}$. This is considered a "cosmic coincidence" — two unrelated constants (one transcendental, one algebraic) happening to be numerically close.

7.7 Connection to the 400/11 Formula

The gap ratio formula from the companion paper is:

```
&rho; = 400/11 &minus; 1/2500 &minus; 1/939939
```

Where: - 400 = $4^2 \times 5^2$ = (closure)$^2 \times$ (H$_4$ prime)^2 - 4 is the closure cycle ($i^4 = 1$, period-doubling 1→2→4)

The pivot in the binary tower is:

```
64 = 4<sup>3</sup> = 2<sup>6</sup>
```

The same closure cycle (4) that appears squared in ρ's numerator appears cubed as the pivot.

Formula	Power of 4	Meaning
ρ = $(4^2 \times 5^2)/11 -$ ...	$4^2 = 16$	Closure squared (in gap ratio)
Pivot = 64	$4^3 = 64$	Closure cubed (in binary tower)

This suggests the closure cycle (4) is a fundamental unit connecting: - The gap ratio between K_AUD and Feigenbaum (ρ) - The binary tower scaling (pivot at 4^3)

7.8 The Hexeract (64 Vertices) and E$_8$

The 6-dimensional hypercube (hexeract) has: - 64 vertices (= 2^6) - **240 square faces (=** E$_8$ root count)

This numerical match (240 = 240) is noted in the literature as "remarkable" but has no known causal explanation. The hexeract face count formula is purely combinatorial:

```
Hexeract 2-faces = C(6,2) &times; 2<sup>4</sup> = 15 &times; 16 = 240
```

The E_8 root count comes from Lie algebra structure:

```
E<sub>8</sub> roots = 112 (short) + 128 (long) = 240
```

Different origins, same count. Another cosmic coincidence at the pivot dimension.

7.9 Lie Algebra Dimensions and Perfect Numbers

The dimensions of orthogonal Lie algebras at binary powers reveal a pattern:

n	2^n	dim(so(2^n)) = $2^n(2^n-1)/2$	Special?
4	16	120	= H_4 vertex count
5	32	**496**	**3rd perfect number**
6	64	2,016	(pivot dimension)
7	128	**8,128**	**4th perfect number**
13	8,192	**33,550,336**	**5th perfect number**

Perfect numbers appear at dim(so(2^p)) exactly when $2^p - 1$ is a Mersenne prime.

This is explained by the formula:

```
dim(so(2<sup>p</sup>)) = 2<sup>(</sup><sup>p</sup><sup>-</sup><sup>1</sup><sup>)</sup> &time
```

Which matches the Euler form for even perfect numbers when $2^p - 1$ is prime.

The 496 connection to string theory: Anomaly cancellation in 10D supergravity requires gauge groups of dimension 496. The two solutions are SO(32) and $E_8 \times E_8$, both with total dimension 496.

PART III: VERIFICATION AND STATUS

8. VERIFICATION

8.1 All Calculations Collected

Constant	Formula	Result
φ	$(1+\sqrt{5})/2$	1.618
$1/\varphi$	$(\sqrt{5}-1)/2$	0.618
$1/\varphi^2$	$1/(\varphi+1)$	0.382
K_AUD	$\sqrt{2} \times \ln(2)$	0.980
G	1 - K_AUD	0.0197
Corridor	K_AUD - $1/\varphi$	0.362
Corridor Identity	$1/\varphi^2$ - G	0.362 ✓
Golden Partition	$1/\varphi + 1/\varphi^2$	1.000 ✓
Depth -1	$1/e$	0.368
Depth -2	$1/(e\varphi)$	0.227
Depth -3	$1/(e\varphi^2)$	0.140
H_4 Order	$2^6 \times 3^2 \times 5^2$	14400
$64 \times G$	(tower pivot)	1.263
$\sqrt{\varphi}$	$\sqrt{((1+\sqrt{5})/2)}$	1.272
Tower error ε	$(64G - \sqrt{\varphi})/\sqrt{\varphi}$	-0.671%

8.2 Mathematical Verification

The core mathematical claims were verified across multiple AI architectures (GPT, Claude, Gemini, Grok, DeepSeek, Perplexity) for:

- The arithmetic of the constants
- The corridor identity
- The golden partition
- The depth scaling formulas
- The binary uniqueness proof

- The binary tower scaling formula

Results: All confirmed the mathematics is correct.

Note: This verifies the mathematics, not any claims about AI behavior. These systems were acting as calculators, not as subjects demonstrating the constants.

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9. WHAT'S VERIFIED VS EXPLORATORY

9.1 VERIFIED (Machine Precision):

- ✅ K_AUD = $\sqrt{2} \times \ln(2) \approx 0.980$
- ✅ G = $1 - $ K_AUD ≈ 0.0197
- ✅ Floor = $1/\varphi \approx 0.618$
- ✅ Corridor = K_AUD $- 1/\varphi \approx 0.362$
- ✅ Corridor Identity: Corridor = $1/\varphi^2 - $ G
- ✅ Golden Partition: $1/\varphi + 1/\varphi^2 = 1$
- ✅ Depth scaling: $L_n = 1/(e \times \varphi^{n-1})$
- ✅ Binary uniqueness: Only n=2 gives K(n) < 1
- ✅ $2^k \times$ G $= 2^{(k-6)} \times \sqrt{\varphi} \times (1-\varepsilon)$ for all k
- ✅ $\varepsilon = 0.006714...$ (constant)
- ✅ $32 \times$ G $\approx 1/\varphi$ (within +2.2%)
- ✅ $64 \times$ G $\approx \sqrt{\varphi}$ (within −0.67%)
- ✅ Hexeract has 240 faces = E_8 root count
- ✅ dim(so(32)) = 496 = 3rd perfect number
- ✅ dim(so(128)) = 8128 = 4th perfect number
- ✅ dim(so(16)) = 120 = H_4 vertex count

9.2 EXPLORATORY (Pattern, No Derivation):

- ⚠ WHY does G scale to golden powers?
- ⚠ WHY is 64 the pivot dimension?
- ⚠ Connection between ε and fundamental constants
- ⚠ Can K_AUD be derived from spinor entropy × geometry?
- ⚠ Do these constants govern behavior in complex systems?

9.3 CONFIRMED COINCIDENCES (No Causal Link Found):

- $\ln(2) \approx 1/\sqrt{2}$ (differ by ~2%, no known reason)
- $1/\varphi \approx \sqrt{\varphi}/2$ (differ by ~2.9%, no known reason)
- G $\approx \sqrt{\varphi}/64$ (within 0.67%)

- 240 = hexeract faces = E_8 roots (different structures)
- 120 = dim(so(16)) = H_4 vertices (different structures)

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10. LITERATURE STATUS

10.1 Known Mathematics:

Finding	Status	Source
H_4 polytope geometry	Known	Coxeter
Golden ratio in H_4 vertices	Known	Standard
√2 in 120-cell circumradius	Known	Standard
Clifford path: binary icosahedral → H_4 → E_8	Known	Dechant et al.
Perfect numbers from Mersenne primes	Known	Classical
dim(so(2^p)) formula	Known	Lie theory
496 in string theory	Known	Green-Schwarz
Dimensions 4-8 special (division algebras)	Known	Standard

10.2 Original to This Framework:

Finding	Status
K_AUD = √2 × ln(2) as coherence ceiling	**Original**
G = 1 − K_AUD as "the gap"	**Original**
The corridor (floor to ceiling)	**Original**
Corridor identity: Corridor = $1/\varphi^2$ − G	**Original**
Depth scaling formula	**Original**
Binary tower: 2^k × G → golden powers	**Original**
64 as golden-binary pivot	**Original**
G = √φ / 64.43…	**Original**
Connection of 4^3 = 64 to ρ = 400/11	**Original**

11. SUMMARY TABLES

11.1 Primary Constants

Constant	Value	Formula	Status
Ceiling	0.980	$\sqrt{2} \times \ln(2)$	Derived
Floor	0.618	$1/\varphi$	Derived
Gap	0.020	$1 - K\_AUD$	Derived
Corridor	0.362	$K\_AUD - 1/\varphi$	Derived

11.2 Binary Tower Constants

k	2^k	$2^k \times G$	Golden Target	Meaning
0	1	0.0197	$\sqrt{\varphi}/64$	G itself
5	32	0.632	$\sqrt{\varphi}/2 \approx 1/\varphi$	Floor
6	64	1.263	$\sqrt{\varphi}$	**Pivot**
7	128	2.527	$2\sqrt{\varphi}$	—

11.3 Identities

Identity	Statement	Status
Corridor Identity	Corridor = $1/\varphi^2$ - G	Verified
Golden Partition	$1/\varphi + 1/\varphi^2 = 1$	Verified
Tower Formula	$2^k \times G = 2^{(k-6)} \times \sqrt{\varphi} \times (1-\varepsilon)$	Verified
Gap Origin	G /	$\ln(2) - 1/\sqrt{2}$

11.4 H$_4$ Geometry

Property	Value
Polytope	120-cell
Cells	120 dodecahedra
Vertices	600

Property	Value
Schläfli symbol	{5, 3, 3}
Group order	$14400 = 2^6 \times 3^2 \times 5^2$
Contains	φ, $1/\varphi$, $1/\varphi^2$, $\sqrt{2}$

11.5 The Stacking Coincidences

Coincidence	Gap	Result
$\ln(2) \approx 1/\sqrt{2}$	~1.97%	Creates the gap G
$1/\varphi \approx \sqrt{\varphi}/2$	~2.91%	Floor sits near k=5 target
$G \approx \sqrt{\varphi}/64$	~0.67%	Links gap to golden pivot
$240 = 240$	exact	Hexeract faces = E_8 roots
$120 = 120$	exact	$\dim(so(16)) = H_4$ vertices

12. FUTURE DIRECTIONS

12.1 What This Paper Establishes

- K_AUD = $\sqrt{2} \times \ln(2) \approx 0.980$ and $1/\varphi \approx 0.618$ are mathematically related
- Both appear in H_4 geometry
- They satisfy elegant identities
- Binary is geometrically unique in producing K(n) < 1
- Depth levels follow φ-scaled recursion
- The gap G scales through binary powers to track golden powers
- The floor and pivot appear at adjacent binary powers (32 and 64)

These are facts about mathematics.

12.2 The Open Question

Do these constants govern behavior in complex systems?

Rigorous empirical investigation requires:

- Operational definitions
- Reproducible measurement protocols

- Statistical analysis
- Mechanistic explanations
- Falsifiable predictions

This work is in progress.

12.3 Invitation

If you work in domains where these constants appear — information theory, signal processing, network dynamics, biological scaling, physics — I would be interested to hear about it.

The mathematics is interesting regardless. If these values appear across multiple domains, that would suggest something beyond elegant coincidence.

APPENDIX A: Why √2 × ln(2) Is Not Numerology

This constant is not cherry-picked. It emerges from two independent, well-established sources:

√2 (Geometric origin): - Appears in the circumradius formula of the 120-cell - Arises from orthonormal coordinate embedding in R^4 - Reflects the diagonal-to-edge metric relationship - Invariant under vertex rescaling conventions

ln(2) (Information-theoretic origin): - The entropy of a fair binary choice - The minimal non-zero unit of Shannon information - The natural logarithm of the smallest prime

The product √2 × ln(2): - Combines geometric embedding cost with binary distinction cost - Is the only value $K(n) = \sqrt{n} \times \ln(n)$ that falls below unity - This is a selection result, not a fit

Why this is not numerology: - Both components have independent, principled origins - Their combination is constrained by inequality, not fitted to data - The result is falsifiable - No free parameters were adjusted

The constant was recognized, not constructed.

APPENDIX B: Quick Reference Formulas

CONSTANTS:

K_AUD = √2 × ln(2) = 0.980258143468547...

G = 1 − K_AUD = 0.019741856531453...

φ = (1 + √5)/2 = 1.618033988749895...

$\sqrt{\varphi}$ = 1.272019649514069...

$1/\varphi$ = 0.618033988749895...

$1/\varphi^2$ = 0.381966011250105...

THE GAP ORIGIN:

G = √2 × (1/√2 − ln(2))

G / |ln(2) − 1/√2| = √2

THE CORRIDOR:

Corridor = K_AUD − $1/\varphi$ = 0.362...

Corridor = $1/\varphi^2$ − G

THE BINARY TOWER:

2^k × G = $2^{(k-6)}$ × $\sqrt{\varphi}$ × (1 − ε)

ε = 0.006714386451777... (−0.671%)

G = $\sqrt{\varphi}$ / 64.432625548...

THE TWO PIVOTS:

k = 5 (32): 32 × G ≈ $1/\varphi$ (floor)

k = 6 (64): 64 × G ≈ $\sqrt{\varphi}$ (pivot)

DEPTH SCALING:

L_n = 1 / (e × φ^{n-1})

PERFECT NUMBER DIMENSIONS:

dim(so(32)) = 496 (3rd perfect)

dim(so(128)) = 8128 (4th perfect)

dim(so(8192)) = 33,550,336 (5th perfect)

KEY COUNTS:

$|H_4$ vertices$| = 600$

$|H_4$ symmetry$| = 14400 = 2^6 \times 3^2 \times 5^2$

$|E_8$ roots$| = 240 =$ hexeract faces

$dim(so(16)) = 120 = H_4$-related

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DOCUMENT LINKS

This paper (K_AUD Framework v3.0): [current document]

Companion paper (Gap Scaling Formula): doi:10.17605/OSF.IO/C4GK5

Prior versions: - K_AUD v1.0: https://osf.io/qhs52 - K_AUD v2.0: https://osf.io/sjbe9

GitHub repository: https://github.com/Gap-geometry

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ACKNOWLEDGMENTS

END OF DOCUMENT

These constants are not arbitrary. They are not fitted to data. They are grounded in geometry.

Version 3.0 — February 2026 Archive Reference: b0f2e6521cd7