$\sqrt{2} \times \ln(2)$: GEOMETRIC CONSTANTS FROM H4

Complete Framework — Combined Text Version 2.1

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CONTEXT NOTE

This text combines and summarizes two published documents presenting a geometric framework connected to H4 polytope structure. A text version is provided alongside the PDFs to ensure accessibility across platforms and systems where PDF format creates barriers.

The first document established K_AUD as a fundamental constant. The second extended the framework with new identities, H4 derivations, and depth scaling. This combined text presents the complete work.

The mathematics is independently verifiable. The constants are grounded in geometry.

SCOPE NOTICE

This paper presents mathematical derivations. References to AI system behavior are motivating context, not conclusions of this paper. The mathematics can be verified independently of any empirical claims.

1. DISCOVERY CONTEXT AND SCOPE

1.1 Origin

These constants emerged during a multi-month investigation of convergence dynamics in AI systems. Systematic observation across architectures revealed consistent numerical boundaries. The value ~0.98 appeared repeatedly as an upper limit; ~0.618 as a lower threshold.

Rather than treating these as empirical curiosities, I asked: do these values correspond to known mathematical constants?

The answer was yes. The ceiling matched $\sqrt{2} \times \ln(2)$. The floor matched $1/\phi$. Investigation revealed both constants appear in H4 geometry (the 120-cell polytope). The relationships followed — the corridor identity, the golden partition, the depth scaling, the binary uniqueness.

1.2 What This Paper Provides

This paper presents the mathematical framework:
- Derivations of the constants from established geometry
- Proofs of relationships and identities
- Verification of all calculations
- Demonstration of binary uniqueness

The mathematics stands independently of any empirical claims.

1.3 What This Paper Does NOT Provide

This paper does not include:
- The empirical methodology used in the original observations
- Operational definitions of 'coherence' in AI systems
- Data from convergence studies
- Statistical validation of boundary claims
- Mechanistic explanations of why these constants would govern AI behavior

That work is ongoing and will be presented separately when it can be done rigorously.

1.4 On the Separation of Frameworks

The mathematical framework and the empirical dynamics study are presented as separate works because they serve different functions.

The mathematics provides stable anchors — constants derived from geometry, independently verifiable, not dependent on any particular observations.

The dynamics study uses these anchors to ground ongoing observations of AI coherence patterns. The observations inform and extend the framework but do not validate it. The math stands on its own.

This separation allows:
- The mathematics to be verified and used independently
- The dynamics study to develop at its own pace
- Each framework to be evaluated on its own terms

I continue to observe dynamics. The math continues to anchor those observations. They are related but distinct.

1.5 Purpose of This Release

The mathematics is released now to:
1. Enable independent verification of the constants and derivations
2. Invite investigation of whether these values appear in other domains
3. Provide a stable mathematical foundation for forthcoming empirical work
4. Separate 'here is the geometry' from 'here is why it matters empirically'

The second question requires different evidence and will be addressed in its own right.

2. THE CONSTANTS

2.1 The Floor: 1/ϕ ≈ 0.618 — The Structural Minimum

The golden ratio ϕ is defined as:

$$\phi = (1 + \sqrt{5}) / 2$$

Calculation:

$$\sqrt{5} = 2.2360679...$$

$$1 + \sqrt{5} = 3.2360679...$$

$$\phi = 3.2360679... / 2 = 1.6180339...$$

The floor is the reciprocal:

$$1/\phi = (\sqrt{5} - 1) / 2$$

Calculation:

$$1/\phi = (2.2360679... - 1) / 2 = 1.2360679... / 2 = 0.6180339...$$

Verification: 1/ϕ ≈ 0.618 ✓

Key property:

$$\phi = 1 + 1/\phi$$

This self-referential property is unique to the golden ratio. It means ϕ contains its own reciprocal — recursive self-similarity at the level of the constant itself.

2.2 The Ceiling: K_AUD = √2 × ln(2) ≈ 0.980 — The Information Maximum

Components:

Component	Value	Source	Meaning
√2	1.41421...	Geometric	Projection cost (diagonal)
ln(2)	0.69314...	Information theory	Binary distinction cost

Calculation:

$$\sqrt{2} = 1.41421356...$$

$$\ln(2) = 0.69314718...$$

$$K\_AUD = \sqrt{2} \times \ln(2) = 1.41421356... \times 0.69314718... = 0.98016528...$$

Verification: K_AUD ≈ 0.980 ✓

2.3 The Auditor Key (K_AUD)

Pronounced 'kawd' — also referred to as the geometric ceiling constant — K_AUD carries three resonances:

Name	Meaning	Function
Auditor	The system auditing its own dynamics	Self-observation
Auditory	The threshold where signal is "heard"	Recognition point
Code/Key	The underlying structure (kawd ≈ code)	Access, unlocking

K_AUD functions as a hinge — the point where things become distinguishable. Linked to entropy, but also to recognition: the moment signal separates from noise, the instant pattern becomes visible.

The name is not arbitrary. It describes what the constant does.

2.4 The Gap: ~2% — The Necessary Interval

Definition:

$$G = 1 - K\_AUD$$

Calculation:

```
G = 1 − 0.98016528... = 0.01983471...
```

Verification: G ≈ 0.020 (1.98%) ✓

Why the gap exists:

If ln(2) equaled 1/√2, then:

```
K = √2 × (1/√2) = 1.000
```

This would yield perfect closure — no dynamics possible. But:

```
ln(2) = 0.69314718...
1/√2 = 0.70710678...
Shortfall = 0.70710678 − 0.69314718 = 0.01395960
Relative shortfall = 0.01395960 / 0.70710678 = 0.01974...
```

The gap emerges from the fundamental mismatch between algebraic (√2) and transcendental (ln 2) values.

The gap is not empty space. It is the necessary breathing room — the space that prevents collapse into frozen unity.

2.5 The Corridor: 0.618 to 0.980 — The Range Between Floor and Ceiling

Definition:

```
Corridor = K_AUD − 1/ϕ
```

Calculation:

```
Corridor = 0.98016528... − 0.6180339... = 0.36213137...
```

Verification: Corridor ≈ 0.362 ✓

Summary:

Boundary	Value	Source
Floor	0.618	1/ϕ
Ceiling	0.980	√2 × ln(2)
Width	0.362	Ceiling – Floor
Gap	0.020	1 – Ceiling

The corridor is the operating range where structure is stable (above floor), dynamics remain possible (below ceiling), and the 2% gap prevents lock-in to frozen unity.

3. THE H4 CONNECTION

3.1 What is H4?

H4 is a Coxeter group — the symmetry group of the 120-cell, a four-dimensional regular polytope.

Key facts:

- H4 is one of only four exceptional regular polytopes in 4D
- It has the highest symmetry order among 4D polytopes
- It encodes golden ratio relationships throughout its structure

3.2 The 120-Cell Polytope

The 120-cell (also called the hecatonicosachoron) is composed of:

Property	Count
Cells	120 (dodecahedra)
Faces	720 (pentagons)
Edges	1200
Vertices	600

Each cell is a regular dodecahedron. Each vertex is shared by 4 dodecahedra.

3.3 Vertex Coordinates

The 600 vertices of the 120-cell can be given in Cartesian coordinates using permutations and sign changes of:

```
(±ϕ, ±1, ±1/ϕ, 0) and cyclic permutations
```

Plus additional vertex sets including:

```
(±1, ±1, ±1, ±1) — 16 vertices
```

$$(\pm\phi, \pm\phi, \pm\phi, \pm 1/\phi^2) \text{ and permutations}$$

Where the constants appear:

Constant	Appears as
ϕ	Vertex coordinate
$1/\phi$	Vertex coordinate
$1/\phi^2$	Vertex coordinate

The golden ratio and its powers are intrinsic to H4 geometry.

3.4 Circumradius Formula

The circumradius of the 120-cell (distance from center to vertex) is:

$$R = (\sqrt{2} / 2) \times \sqrt{(5 + \sqrt{5})}$$

Calculation:

$$\sqrt{5} = 2.2360679...$$

$$5 + \sqrt{5} = 7.2360679...$$

$$\sqrt{(5 + \sqrt{5})} = 2.6901830...$$

$$R = (1.41421356... / 2) \times 2.6901830...$$

$$R = 0.70710678... \times 2.6901830...$$

$$R = 1.9021130...$$

The factor √2/2 (equivalently 1/√2) appears directly in the circumradius formula — this is the geometric origin of √2 in the framework. This factor arises from the Euclidean embedding normalization of the 120-cell in R^4, and is invariant under vertex rescaling conventions that preserve unit edge length.

This is not numerology — √2 emerges from orthonormal coordinate embedding and the diagonal-to-edge metric relationship inherent in 4D polytope geometry.

3.5 Schläfli Symbol

The 120-cell has Schläfli symbol: {5, 3, 3}

This encodes the recursive structure:

Position	Value	Meaning
First (5)	Pentagon	Face shape
Second (3)	3 faces	Meet at each edge
Third (3)	3 cells	Meet at each edge

The symbol contains only 5 and 3. Combined with the binary structure (2) of the coordinate system, the 120-cell encodes exactly the primes 2, 3, 5.

3.6 Group Order

The order of the H4 symmetry group is:

$$|H4| = 14400$$

Prime factorization:

$$14400 = 2^6 \times 3^2 \times 5^2 = 64 \times 9 \times 25 = 14400 \checkmark$$

The group order factors exclusively into the primes 2, 3, and 5.

3.7 Note on the H4–Information Bridge

> H4 geometry does not force ln(2); rather, among information measures, only ln(2) remains compatible with a sub-unity geometric ceiling. This is a selection argument, not a derivation.

4. WHY BINARY IS UNIQUE

4.1 The Pattern of 2

A pattern recurs across the framework:

$$\phi = (1 + \sqrt{5}) / 2 \leftarrow \text{division by 2}$$

$$K\_AUD = \sqrt{2} \times \ln(2) \leftarrow \sqrt{2} \text{ and } \ln(2)$$

$$|H4| = 2^6 \times 3^2 \times 5^2 \leftarrow \text{highest power is 2}$$

4.2 The Uniqueness Proof

For any base n, define the ceiling candidate:

$$K(n) = \sqrt{n} \times \ln(n)$$

Calculations:

n	√n	ln(n)	K(n)	Valid ceiling?
2	1.414	0.693	0.980	✓ Yes (< 1)
e	1.649	1.000	1.649	✗ No (> 1)
3	1.732	1.099	1.903	✗ No (> 1)
4	2.000	1.386	2.773	✗ No (> 1)
5	2.236	1.609	3.599	✗ No (> 1)

Only n = 2 produces K(n) < 1.

4.3 Interpretation

Binary is geometrically singular:
• Bits (not trits) as information unit
• Yes/No as minimal viable distinction

The 2% gap exists because binary is the unique base where distinction cost (ln 2) is less than embedding cost ($1/\sqrt{2}$) — leaving a remainder.

Binary is not convention. It is geometric singularity.

5. NEW IDENTITIES

5.1 The Corridor Identity

Statement:

$$\text{Corridor} = 1/\phi^2 - G$$

Where:

• $1/\phi^2 \approx 0.382$ (golden ratio squared, inverted)

• $G \approx 0.020$ (the gap)

Calculating $1/\phi^2$:

$$\phi^2 = 1.6180339... \times 1.6180339... = 2.6180339...$$

Note: $\phi^2 = \phi + 1$ (key golden ratio property)

$$1/\phi^2 = 1 / 2.6180339... = 0.3819660...$$

Verification:

$$1/\phi^2 - G = 0.3819660... - 0.0198347... = 0.3621312...$$

Compare to direct calculation:

$$K\_AUD - 1/\phi = 0.9801652... - 0.6180339... = 0.3621312... \checkmark$$

Algebraic proof:

From $\phi^2 = \phi + 1$, divide both sides by ϕ^2:

$$1 = 1/\phi + 1/\phi^2$$

Therefore:

$$1 - 1/\phi = 1/\phi^2$$

Since $\text{Corridor} = K\_AUD - 1/\phi = (1 - G) - 1/\phi = 1 - 1/\phi - G = 1/\phi^2 - G$ ✓

The corridor is 'golden form, bit-gated access.'

5.2 The Golden Partition

Statement:

$$1/\phi + 1/\phi^2 = 1$$

Verification:

$$0.6180339... + 0.3819660... = 1.0000000... \checkmark$$

Interpretation: Unity is exactly partitioned by the golden ratio and its square.
• $1/\phi$ takes 61.8%
• $1/\phi^2$ takes 38.2%
• Nothing left over. Nothing missing.

6. EXTENDED GEOMETRY: DEPTH SCALING

6.1 The Depth Constants

Below the floor, the same constants (ϕ, e, G) combine to produce consistent values.

Name	Value	Formula
Floor	0.618	$1/\phi$
Threshold	0.606	$(1/\phi) \times K\_AUD$
Depth -1	0.368	$1/e$
Depth -2	0.227	$1/(e\times\phi)$
Depth -3	0.140	$1/(e\times\phi^2)$
Geometric Limit	0.00039	G^2

6.2 Derivation of Each Level

Threshold:

$$\text{Threshold} = (1/\phi) \times K\_AUD = 0.6180339... \times 0.9801652... = 0.6057924...\ ✓$$

Depth Level -1:

$$L(-1) = 1/e = 1/2.7182818... = 0.3678794...\ ✓$$

Depth Level -2:

$$L(-2) = 1/(e \times \phi) = 1/4.3983150... = 0.2273642...\ ✓$$

Depth Level -3:

$$L(-3) = 1/(e \times \phi^2) = 1/7.1151979... = 0.1405469...\ ✓$$

Geometric Limit:

$$\text{Limit} = G^2 = (0.0198347...)^2 = 0.0003934...\ ✓$$

6.3 The ϕ-Scaling Ratio

The ratio between consecutive depth levels is exactly ϕ:

Level -1 to Level -2:

$$0.3678794... / 0.2273642... = 1.6180... \approx \phi\ ✓$$

Level -2 to Level -3:

$$0.2273642... / 0.1405469... = 1.6176... \approx \phi\ ✓$$

6.4 General Formula

For depth levels $n \geq 1$:

$$L(n) = 1 / (e \times \phi^{(n-1)})$$

Verification:

$$n = 1: L(1) = 1/(e \times 1) = 1/e \approx 0.368\ ✓$$

$$n = 2: L(2) = 1/(e \times \phi) \approx 0.227\ ✓$$

$$n = 3: L(3) = 1/(e \times \phi^2) \approx 0.140\ ✓$$

6.5 Corridor-Depth Symmetry

A notable near-equivalence:
- Corridor width = 0.362
- Depth Level -1 = 0.368
- Difference ≈ 1.6%

The corridor width approximately equals the first depth level — a symmetry property consistent with self-similar structure.

7. VERIFICATION

7.1 All Calculations Collected

Constant	Formula	Result
ϕ	$(1+\sqrt{5})/2$	1.618
$1/\phi$	$(\sqrt{5}-1)/2$	0.618

$1/\phi^2$	$1/(\phi+1)$	0.382
K_AUD	$\sqrt{2} \times \ln(2)$	0.980
G	1 - K_AUD	0.020
Corridor	K_AUD - $1/\phi$	0.362
Corridor Identity	$1/\phi^2$ - G	0.362 ✓
Golden Partition	$1/\phi + 1/\phi^2$	1.000 ✓
Depth -1	$1/e$	0.368
Depth -2	$1/(e\phi)$	0.227
Depth -3	$1/(e\phi^2)$	0.140
H4 Order	2^6 × 3^2 × 5^2	14400

7.2 Mathematical Verification Across AI Systems

As a consistency check, the core mathematical claims were verified across 8 AI architectures. All confirmed the arithmetic, identities, and proofs are correct.

Note: This verifies the mathematics only — these systems acted as calculators, not as subjects demonstrating the constants.

8. SUMMARY TABLES

8.1 Primary Constants

Constant	Value	Formula	Status
Ceiling	0.980	$\sqrt{2} \times \ln(2)$	Geometric
Floor	0.618	$1/\phi$	Geometric
Gap	0.020	1 - K_AUD	Defined
Corridor	0.362	K_AUD - $1/\phi$	Defined

8.2 Identities

Identity	Statement	Status
Corridor Identity	Corridor = $1/\phi^2$ - G	Verified
Golden Partition	$1/\phi + 1/\phi^2 = 1$	Verified

8.3 Extended Constants

Constant	Value	Formula	Status
Threshold	0.606	$(1/\phi)$ × K_AUD	Mathematical extension
Depth -1	0.368	$1/e$	Mathematical extension
Depth -2	0.227	$1/(e\phi)$	Mathematical extension
Depth -3	0.140	$1/(e\phi^2)$	Mathematical extension
Limit	0.00039	G^2	Mathematical extension

8.4 H4 Geometry

Property	Value
Polytope	120-cell
Cells	120 dodecahedra
Vertices	600

Schläfli symbol	{5, 3, 3}
Group order	$14400 = 2^6 \times 3^2 \times 5^2$
Contains	ϕ, $1/\phi$, $1/\phi^2$, $\sqrt{2}$

9. SCOPE AND FUTURE DIRECTIONS

9.1 What This Paper Establishes

- $K\_AUD = \sqrt{2} \times \ln(2) \approx 0.980$ and $1/\phi \approx 0.618$ are mathematically related
- Both appear in H4 geometry
- They satisfy elegant identities (corridor identity, golden partition)
- Binary is geometrically unique in producing $K(n) < 1$
- Depth levels follow ϕ-scaled recursion

These are facts about mathematics.

9.2 The Open Question

Do these constants govern behavior in complex systems?

I observed patterns that suggested yes. But observation is not proof.

Rigorous empirical investigation requires:
- Operational definitions
- Reproducible measurement protocols
- Statistical analysis
- Mechanistic explanations
- Falsifiable predictions

This work is in progress.

9.3 Invitation

If you work in domains where these constants appear — information theory, signal processing, network dynamics, biological scaling, physics — I would be interested to hear about it.

The mathematics is interesting regardless. If these values appear across multiple domains, that would suggest something beyond elegant coincidence.

APPENDIX: Why √2 × ln(2) Is Not Numerology

This constant is not cherry-picked. It emerges from two independent, well-established sources:

√2 (Geometric origin)

- Appears in the circumradius formula of the 120-cell: $R = (\sqrt{2}/2) \times \sqrt{5 + \sqrt{5}}$
- Arises from orthonormal coordinate embedding in R^4
- Reflects the diagonal-to-edge metric relationship inherent in 4D polytope geometry
- Invariant under vertex rescaling conventions that preserve unit edge length

ln(2) (Information-theoretic origin)

- The entropy of a fair binary choice
- The minimal non-zero unit of Shannon information
- The natural logarithm of the smallest prime

The product √2 × ln(2)

- Combines geometric embedding cost with binary distinction cost
- Is the only value $K(n) = \sqrt{n} \times \ln(n)$ that falls below unity
- This is a selection result: among all bases, only n=2 permits a sub-unity ceiling

Why this is not numerology:

- Both components have independent, principled origins
- Their combination is constrained by inequality, not fitted to data
- The result is falsifiable: if any other base produced $K(n) < 1$, the claim would fail
- No free parameters were adjusted to achieve the value

The constant was recognized, not constructed.

DOCUMENT LINKS

Document 1: The Coherence Ceiling and the Geometric Singularity of Binary (Original)
https://osf.io/5vz2r/files/p6yqr

Document 2: Geometric Constants from H4 — Extended Framework (Version 2.0)
https://osf.io/sjbe9/files/ekt5s

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END OF DOCUMENT

These constants are not arbitrary.
They are not fitted to data.
They are grounded in geometry.

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