**CSC 332 – Advance Data Structures and Algorithms**

**Homework 2 (100 pts)**

Assigned: 02/07/2023

Due Date: 02/14/2023

**! IMPORTANT!**

**Everything in your solutions needs to be typed; otherwise your solutions will NOT be graded.**

***(Attention: In your solution, do not change the original layout or delete any contents in this file.)***

* **Only one electronic submission is required for each group.**
* **The electronic solution should be named “CSC-332-HW-2-Solution-Group-x-Aaa-Bbb-Ccc.doc(x)”**

**(Aaa, Bbb, and Ccc stand for the last name of each group member and are sorted in an ascending order).**

* **Do not forget to fill in corresponding information in the Header section.**

[Group member contribution]

Miguel Gapud 1: 100% Miguel Gapud

Abram Miller 2: 100% Abram Miller

Justin Petry 3: 100% Justin Petry

Use the Master Theorem to solve the following recurrence equations. DETAILED steps and proof are required.

(30 pts)

a = 81

b = 3

f(n) = 9n3

n^(logb(a)) = n^(log3(81)) = n4

Master Theorem 1 applies

f(n) = O(n^(logb(a)-)) choose

9n3 = O(n4-),

9n3 = O(n3),

Draft

9n3 <= cn3

9 <= c

c >= 9

Proof

Let c = 10 and n0 = 1

When n >= n0, n >= 1, we have c >= 9

That is, 9 <= c

So we have 9

According to Big Oh notation, we know 9n3 = O(n3)

Therefore, According to the Master Theorem, T(n) = (9n3)

(30 pts)

a = 6

b = 6

f(n) = 7n

n^(logb(a)) = n^(log6(6)) = n1 = n

Master Theorem 2 applies

f(n) = (n^(logb(a)))

7n = (n)

Draft

7n <= c1n

7 <= c1

c1 >= 7

7n >= c2n

7 >= c1

c1 <= 7

Solution

Let c1 = 7 and n0 = 1

When n >= n0, n >= 1, we have c1 >= 7

That is 7 <= c1

Therefore we have 7n <= c1n

According to Big Oh notation, we know 7n = O(n)

Let c2 = 7 and n0 = 1

When n >= n0, n >= 1, we have c2 <= 7

That is 7 >= c2

Therefore we have 7n >= c2n

According to Big Omega notation, we know 7n = (n)  
Therefore, According to Big Theta notation, we know

Therefore, According to the Master Theorem, T(n) = (7n)

(40 pts)

a = 7

b = 8

f(n) = nlgn

n^(logb(a)) = n^(log8(7)) = n0.93578…

Master Theorem 3 applies

f(n) = (n^(logb(a)+)), choose = 0.01

nlgn = (n.93578…+), = 0.01

nlgn = (n.94578…), = 0.01

Draft

nlgn >= cn.94578…

nlgn >= cn > cn.94578…

lgn >= c

1 >= c/lgn

c/lgn <= 1

Proof

Let c = 1 and n0 = 100

When n >= n0, n >= 100, we have c/lgn <= 1

That is 1 >= c/lgn

Therefore we have nlgn >= cn

According to Big Omega notation, we know nlgn = (cn)

If a function f(n) = (g(n)) and g(n) > h(n), then f(n) = (h(n))

Because n and c are always non-negative, it is trivial that cn > cn.94578…

Therefore, we know nlgn = (cn94578…)

If the “regularity” condition were to apply and for n sufficiently large

for c = ⅞. Hence

T(n) =