Discrete

EE1205 : Signals and Systems
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I. Question 11.9.5 (18)

If a and b are the roots of $x^2 - 3x + p = 0$ and c , d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. Prove that $(q + p) : (q - p)$ = 17:15 .

II. Solution

Given a and b are the roots of $x^2–3x + p = 0$ So, we have :

$$a + b = 3 \tag{1}$$
$$ab = p \tag{2}$$

Also, c and d are the roots of $x^2–12x + q = 0$, so,

$$c + d = 12 \tag{3}$$
$$cd = q \tag{4}$$

And given a, b, c, d are in G.P. Let's take $a = x, b = xr, c = xr^2, d = xr^3$,where x and r are first term and common ratio of the G.P. respectively.
From (1) and (3) , we get ,

$$x + xr = 3 \tag{5}$$
$$x(1 + r) = 3 \tag{6}$$

And ,

$$xr^2 + xr^3 = 12 \tag{7}$$
$$xr^2(1 + r) = 12 \tag{8}$$

On dividing eq. (5) and eq. (6), we get

$$\frac{xr^2(1 + r)}{x(1 + r)} = \frac{12}{3} \tag{9}$$
$$r^2 = 4 \tag{10}$$
$$r = \pm 2 \tag{11}$$

When r = 2, x = 3/(1 + 2) = 3/3 = 1
When r = -2, x = 3/(1 – 2) = 3/-1 = -3

Case 1 : When $r = 2$ and $x = 1$

$$p = ab \tag{12}$$
$$p = 2 \tag{13}$$
$$q = cd \tag{14}$$
$$q = 32 \tag{15}$$
$$\frac{q + p}{q - p} = \frac{32 + 2}{32 - 2} \tag{16}$$
$$= \frac{17}{15} \tag{17}$$

Case 2 : When $r = -2$ and $x = -3$

$$p = ab \tag{18}$$
$$p = -18 \tag{19}$$
$$q = cd \tag{20}$$
$$q = 288 \tag{21}$$
$$\frac{q + p}{q - p} = \frac{288 - 18}{288 + 18} \tag{22}$$
$$= \frac{135}{153} \tag{23}$$

Hence , case 1 satisfies the condition .