Assignment-2

EE1205 : Signals and Systems

Indian Institute of Technology Hyderabad

Chirag Garg

(EE23BTECH11206)

I. Question 11.9.1 (5)

Write the first five terms of the sequence whose n^{th} term is : $x(n) = (-1)^{n-1} 5^{n+1}$.

II. Solution

To find the Z-transform of a sequence, we use the formula:

$$X(z) = \sum_{n=0}^{\infty} x[n] \cdot z^{-n} \tag{1}$$

Given the sequence $a_n = (-1)^{n-1} \cdot 5^{n+1}$, we can substitute this into the formula:

$$A(z) = \sum_{n=0}^{\infty} (-1)^{n-1} \cdot 5^{n+1} \cdot z^{-n} \tag{2}$$

$$= \sum_{n=0}^{\infty} (-1)^{n-1} \cdot \frac{5^{n+1}}{z^n} \tag{3}$$

Now, let's split the summation into two parts, one for even values of n and one for odd values of n:

$$A(z) = \sum_{n=0}^{\infty} (-1)^{n-1} \cdot \frac{5^{n+1}}{z^n} \tag{4}$$

$$= \sum_{k=0}^{\infty} (-1)^{2k-1} \cdot \frac{5^{2k+1}}{z^{2k}} + \sum_{k=0}^{\infty} (-1)^{2k} \cdot \frac{5^{2k+2}}{z^{2k+1}} \tag{5}$$

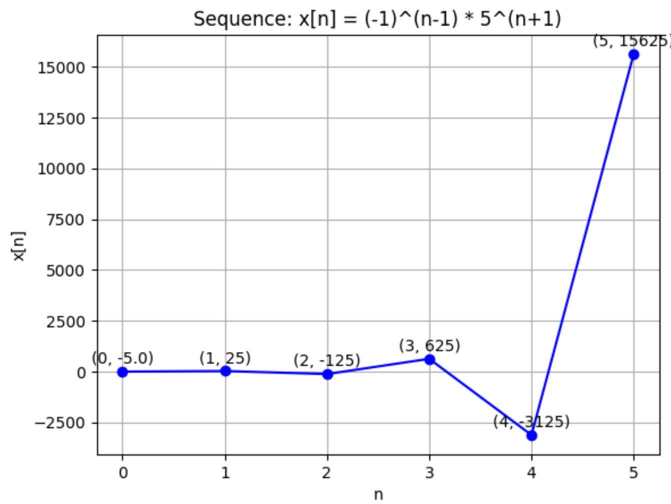
$$= -\sum_{k=0}^{\infty} \frac{5^{2k+1}}{z^{2k}} + \sum_{k=0}^{\infty} \frac{5^{2k+2}}{z^{2k+1}} \tag{6}$$

$$= -5 \sum_{k=0}^{\infty} \left(\frac{5}{z}\right)^{2k} + 5 \sum_{k=0}^{\infty} \left(\frac{5}{z}\right)^{2k+1} \tag{7}$$

$$= \frac{-5}{1 - 25 \times z^{-2}} + \frac{25 \times z^{-1}}{1 - 25 \times z^{-2}} \tag{8}$$

$$= \frac{-5 \times z}{z + 5} \tag{9}$$

$$\tag{10}$$



This result is valid for R.O.C. $\left|\frac{5}{z}\right| < 1$

$$|z| > 5 \tag{11}$$

The Z-transform of the given sequence a_n is $A(z)$ as derived above.

The n^{th} term of sequence is: $x(n) = (-1)^{n-1} 5^{n+1}$
On substituting n = 0, 1, 2, 3 and 4, we get the first five terms

Hence, the required terms are -5, 25, –125, 625, –3125 .