Analog 12.7.4

EE1205 : Signals and Systems
Indian Institute of Technology Hyderabad

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Question: A 60 μ F capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

Solution:

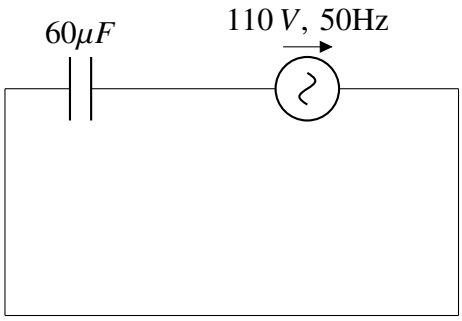

Fig. 1

Symbol	Value	Description
C	$60\,\mu F$	Capacitance
V_0	$110\sqrt{2}V$	Peak Voltage
I_0	$V_0 \times \omega C$	Peak Current
f	$60Hz$	Frequency
ω	$2\pi f$	Angular Frequency
$H(s)$	$\frac{V(s)}{I(s)}$	Transfer Function

Table 1 : Given Parameters

The voltage as a function of time can be given as :

$$V(t) = V_0 \cos(\omega t) \tag{1}$$

So the current as a function of time will be given as :

$$I(t) = V_0 \times \omega C \cos(\omega t + \frac{\pi}{2}) \tag{2}$$

$$= I_0 \sin(\omega t) \tag{3}$$

The I_{rms} value is defined as :-

$$I_{\text{rms}}^2 = \frac{1}{T}\int_0^T [I(t)]^2\,dt \tag{4}$$

$$= f\int_0^{\frac{1}{f}} I_0^2 \cdot \sin^2(2\pi f t)\,dt \tag{5}$$

$$= \frac{1}{2}I_0^2\left(1 - \frac{1}{f}\left[\frac{\sin(4\pi f t)}{4\pi f}\right]_0^{\frac{1}{f}}\right) \tag{6}$$

$$= \frac{1}{2}I_0^2\left(1 - \frac{1}{f}\cdot\frac{\sin(4\pi) - \sin(0)}{4\pi f}\right) \tag{7}$$

$$= \frac{I_0^2}{2} \tag{8}$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} \tag{9}$$

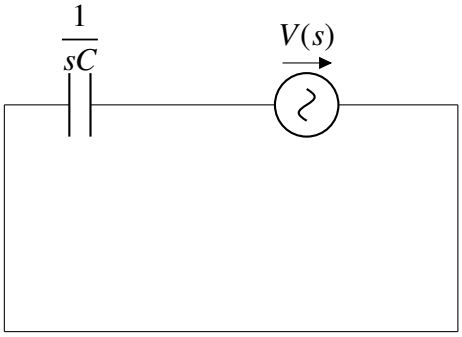

Fig. 2 : Equivalent s domain circuit

We know that,

$$H(s) = \frac{1}{sC} \tag{10}$$

$$= \frac{1}{j\omega C} \tag{11}$$

$$|H(j\omega)| = \left|\frac{1}{j\omega C}\right| \tag{12}$$

$$= \frac{1}{(2\pi) \times (60) \times (60) \times (10^{-6})} \tag{13}$$

$$\approx 44.21 \tag{14}$$

$$I(s) = \frac{V(s)}{H(s)} \tag{15}$$

$$= \frac{110}{44.21} \tag{16}$$

$$\approx 2.49A \tag{17}$$