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EE1205 : Signals and Systems Indian Institute of Technology Hyderabad

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I. Question E.C.(45)

Question: Let a frequency modulated (FM) signal: $x(t) = A\cos(\omega_c t + k_f \int_{-\infty}^t m(\lambda) d\lambda)$, where m(t) is a message signal of bandwidth W. It is passed through a non-linear system with output $y(t) = 2x(t) + 5(x(t))^2$. Let B_T denote the FM bandwidth. The minimum value of ω_c required to recover x(t) from y(t) is:

(A)
$$B_T + W$$

(B)
$$\frac{3}{2}B_T$$

(C)
$$2B_T + W$$

(D)
$$\frac{5}{2}B_T$$

Solution:

$$x(t) = A\cos(\omega_c t + k_f \int_{-\infty}^t m(\lambda)d\lambda)$$
 (1)

$$y(t) = 2x(t) + 5(x(t))^2$$
 (2)

$$= 2A\cos(\omega_c t + k_f \int_{-\infty}^t m(\lambda)d\lambda) + 5A^2\cos^2(\omega_c t + k_f \int_{-\infty}^t m(\lambda)d\lambda)$$
 (3)

$$= 2A\cos(\omega_c t + k_f \int_{-\infty}^t m(\lambda)d\lambda) + \frac{5}{2}A^2\cos(2\omega_c t + k_f \int_{-\infty}^t m(\lambda)d\lambda) + \frac{5}{2}A^2$$
 (4)

To recover x(t) from y(t)

$$2\omega_c - B_T > \omega_c + \frac{B_T}{2} \tag{5}$$

$$\omega_c > \frac{3B_T}{2} \tag{6}$$

$$\therefore (\omega_c)_{min} = \frac{3B_T}{2} \tag{7}$$