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EE1205 : Signals and Systems
Indian Institute of Technology Hyderabad

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(EE23BTECH11206)

I. Question E.C.(45)

Question: Let a frequency modulated (FM) signal : $x(t) = A\cos(\omega_c t + k_f \int_{-\infty}^{t} m(\lambda)d\lambda)$, where $m(t)$is a message signal of bandwidth W. It is passed through a non-linear system with output $y(t) = 2x(t) + 5(x(t))^2$. Let B_T denote the FM bandwidth. The minimum value of ω_c required to recover $x(t)$ from $y(t)$ is:

(A) $B_T + W$

(B) $\frac{3}{2}B_T$

(C) $2B_T + W$

(D) $\frac{5}{2}B_T$

(GATE EC 2023)

Solution:

Parameter	Value	Description
$\phi(t)$	$k_f \int_{-\infty}^{t} m(\lambda)d\lambda$	Phase
$x(t)$	$A\cos(\omega_c t + \phi(t))$	FM Signal
$y(t)$	$2x(t) + 5(x(t))^2$	Output from system

TABLE 1:Given Parameters

$$BW[x(t)] \longrightarrow B_T = 2(\omega + \triangle f) \tag{1}$$

Now , for $x^2(t)$

$$\triangle f' = 2\triangle f \tag{2}$$
$$\omega_C' = 2\omega_C \tag{3}$$

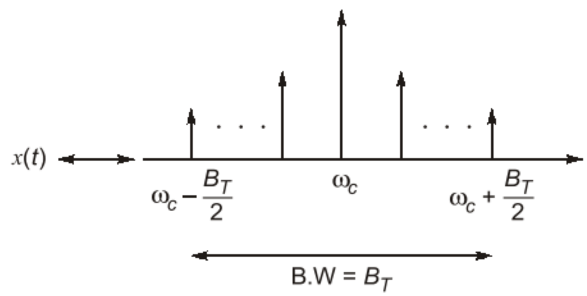

Fig. 1

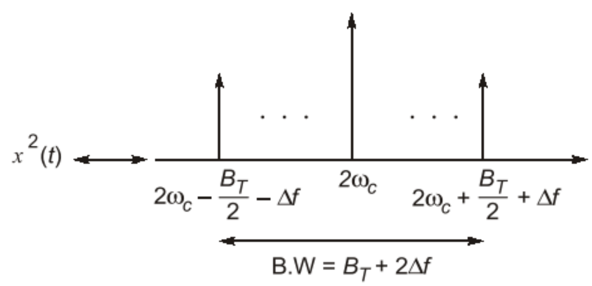

Fig. 2

$$\begin{aligned} BW[x^2(t)] &= 2(\triangle f' + \omega) && (4)\\ &= 2(2\triangle f + \omega) && (5)\\ &= B_T + 2\triangle f && (6)\end{aligned}$$

$$\begin{aligned} y(t) &= 2x(t) + 5(x(t))^2 && (7)\\ &= 2A\cos(\omega_c t + \phi(t)) + 5A^2\cos^2(\omega_c t + \phi(t)) && (8)\\ &= 2A\cos(\omega_c t + \phi(t)) + \frac{5}{2}A^2[1 + \cos(2\omega_c t + 2\phi(t))] && (9)\end{aligned}$$

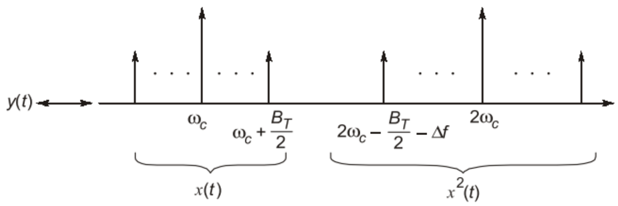

Fig. 3

To recover $x(t)$,

$$2\omega_C - \frac{B_T}{2} - \triangle f > \omega_C + \frac{B_T}{2} \quad (10)$$

$$\omega_C > \triangle f + B_T \quad (11)$$

$$\omega_C > \triangle f + 2(\omega + \triangle f) \quad (12)$$

$$\omega_C > 3\triangle f + 2\omega \quad (13)$$

$$\omega_C > \frac{3}{2}[2(\triangle f + \omega)] - \omega \quad (14)$$

$$\omega_C > \frac{3}{2}B_T - \omega \quad (15)$$

$$\therefore (\omega_C)_{min} = \frac{3}{2}B_T \quad (16)$$

Hence the correct option is (b)