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EE1205 : Signals and Systems
Indian Institute of Technology Hyderabad

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I. Question E.E.(32)

Question: Let $f(t)$ be an even function, i.e.$f(-t) = f(t)$ for all t.Let the Fourier transform of $f(t)$ be defined as $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$. Suppose $\dfrac{dF(\omega)}{d\omega} = -\omega F(\omega)$ for all ω , and $F(0) = 1$. Then

(A) $f(0) < 1$

(B) $f(0) > 1$

(C) $f(0) = 1$

(D) $f(0) = 0$

(GATE EE 2021)

Solution: Given,

$$\frac{dF(\omega)}{d\omega} = -\omega F(\omega) \tag{1}$$

$$\frac{dF(\omega)}{d\omega} + \omega F(\omega) = 0 \tag{2}$$

$$ln|F(\omega)| = -\frac{\omega^2}{2} + c \tag{3}$$

$$F(\omega) = Ke^{-\frac{\omega^2}{2}} \tag{4}$$

Put $\omega = 0$,

$$F(0) = K \tag{5}$$

$$1 = K \tag{6}$$

$$\therefore F(\omega) = e^{-\frac{\omega^2}{2}} \tag{7}$$

$$f(t) \longleftrightarrow F(\omega)$$

$$e^{-at^2} \longleftrightarrow \sqrt{\frac{\pi}{a}}e^{-\frac{\omega^2}{4a}} \;;\; a > 0 \tag{8}$$

$$At\ a = \frac{1}{2},\ e^{-\frac{t^2}{2}} \longleftrightarrow \sqrt{2\pi}e^{-\frac{\omega^2}{2}} \tag{9}$$

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}} \longleftrightarrow e^{-\frac{\omega^2}{2}} = F(\omega) \tag{10}$$

$$\text{Thus ,}\ f(t) = \frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}} \tag{11}$$

At $t = 0$

$$f(0) = \frac{1}{\sqrt{2\pi}} < 1 \tag{12}$$

Hence , option (a) is correct.