

# G.A.T.E.

EE1205 : Signals and Systems  
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(EE23BTECH11206)

## I. QUESTION E.E.(32)

**Question:** Let  $f(t)$  be an even function, i.e.  $f(-t) = f(t)$  for all  $t$ . Let the Fourier transform of  $f(t)$  be defined as  $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$ . Suppose  $\frac{dF(\omega)}{d\omega} = -\omega F(\omega)$  for all  $\omega$ , and  $F(0) = 1$ . Then

(A)  $f(0) < 1$

(B)  $f(0) > 1$

(C)  $f(0) = 1$

(D)  $f(0) = 0$

$$e^{-at^2} \longleftrightarrow \sqrt{\frac{\pi}{a}} e^{-\frac{\omega^2}{4a}}; a > 0 \quad (8)$$

$$\text{At } a = \frac{1}{2}, e^{-\frac{t^2}{2}} \longleftrightarrow \sqrt{2\pi} e^{-\frac{\omega^2}{2}} \quad (9)$$

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \longleftrightarrow e^{-\frac{\omega^2}{2}} = F(\omega) \quad (10)$$

$$\text{Thus, } f(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \quad (11)$$

At  $t = 0$

$$f(0) = \frac{1}{\sqrt{2\pi}} < 1 \quad (12)$$

Hence, option (a) is correct.

(GATE EE 2021)

**Solution:** Given,

$$\frac{dF(\omega)}{d\omega} = -\omega F(\omega) \quad (1)$$

$$\frac{dF(\omega)}{d\omega} + \omega F(\omega) = 0 \quad (2)$$

$$\ln|F(\omega)| = -\frac{\omega^2}{2} + c \quad (3)$$

$$F(\omega) = K e^{-\frac{\omega^2}{2}} \quad (4)$$

Put  $\omega = 0$ ,

$$F(0) = K \quad (5)$$

$$K = 1 \quad (6)$$

$$\therefore F(\omega) = e^{-\frac{\omega^2}{2}} \quad (7)$$

$$f(t) \longleftrightarrow F(\omega)$$