

# Index Class

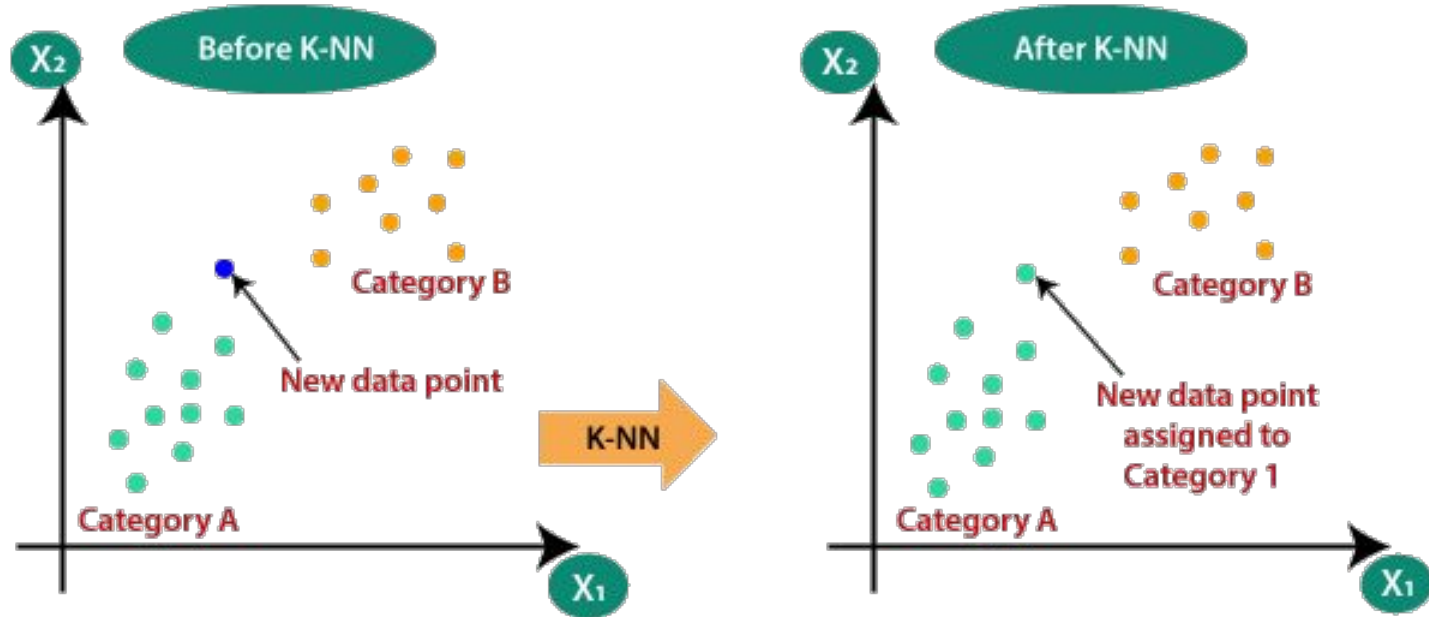
1. KNN
2. Problems and improvements
3. Overfitting and bias-variance trade off
4. From LR to KNN

# K Nearest Neighbor

Supervised learning

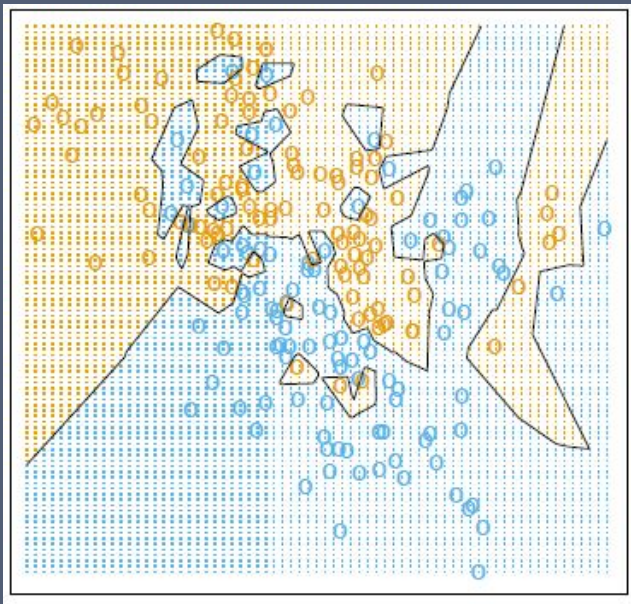
Non-parametric

# K Nearest Neighbor

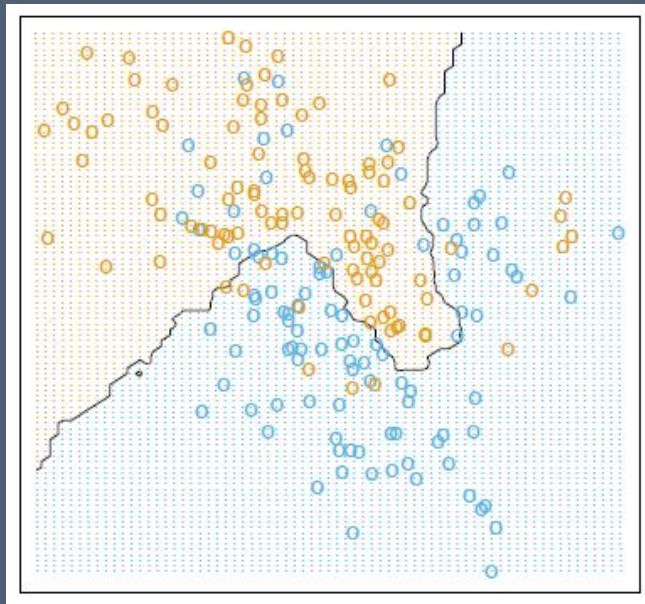


# From parameters to hyper-parameters

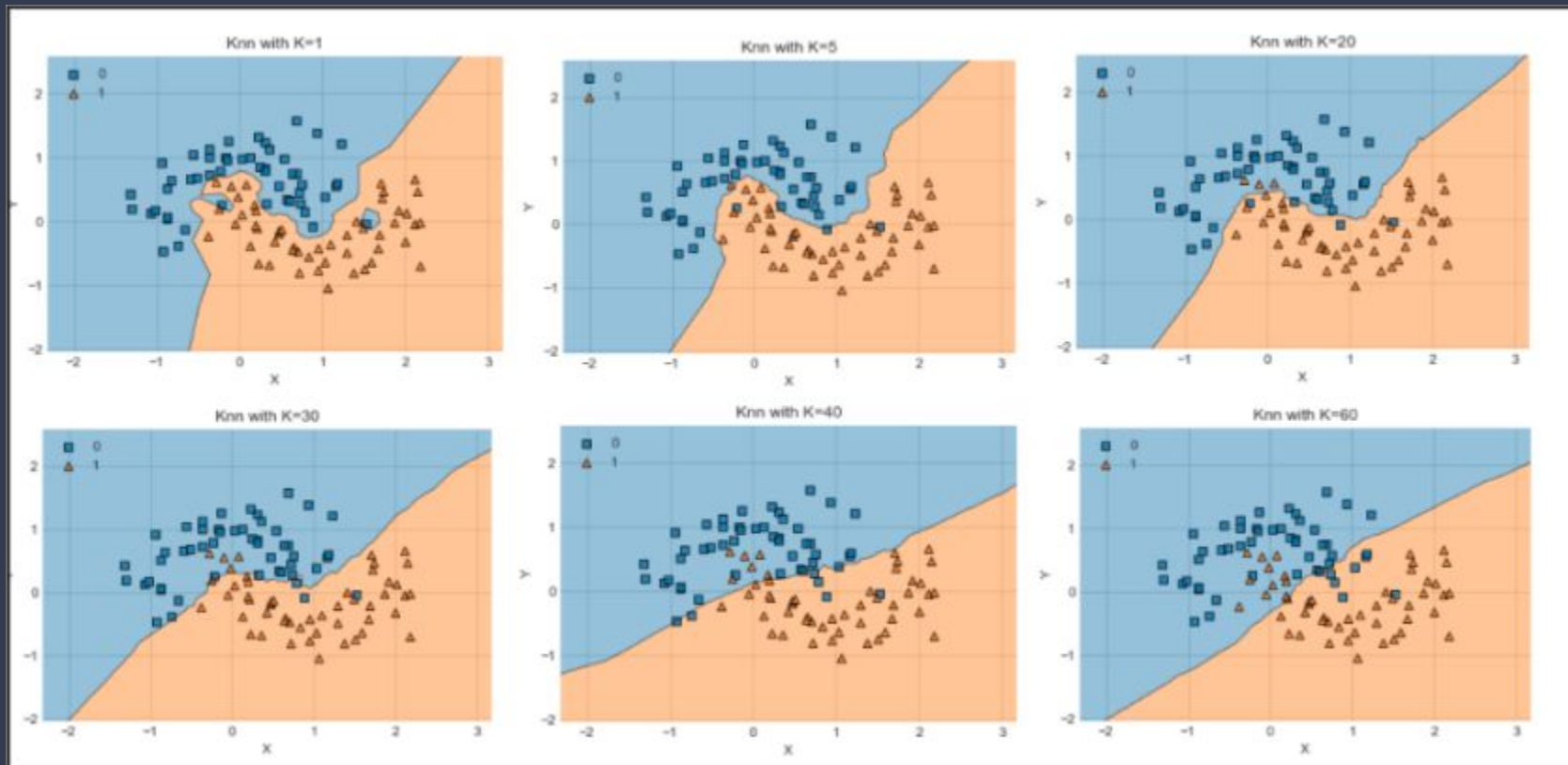
$K = 1$



$K = 20$



# K Nearest Neighbor



# K Nearest Neighbour

$N_k(x)$  is the neighborhood of  $x$  defined by the  $K$  closest points  $x_i$  in the training sample.

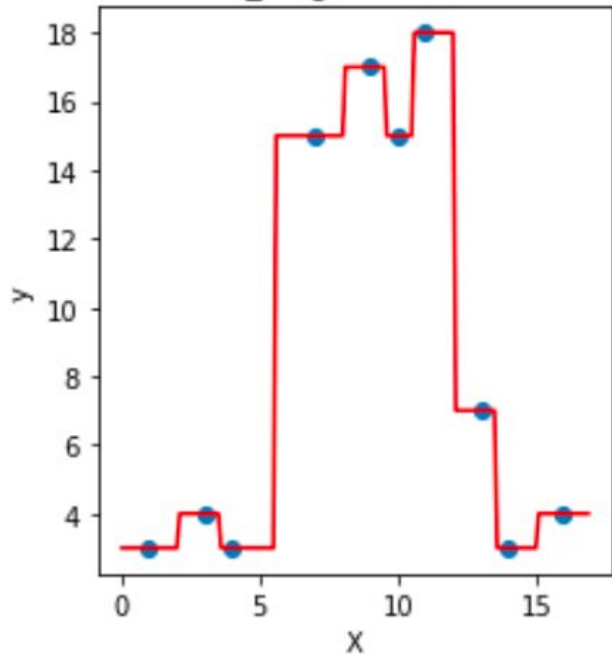
Closest implies a metric, we assume Euclidean distance, but any distance will be fine.

The response is the average response of the  $K$  closest observations (classification or regression)

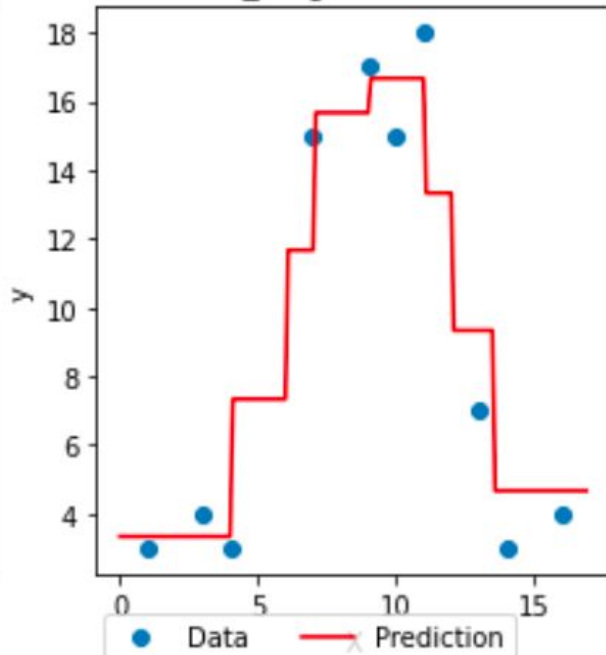
$$\hat{Y}(x) = \frac{1}{k} \sum_{x_i \in N_k(x)} y_i,$$

# KNN Regressor

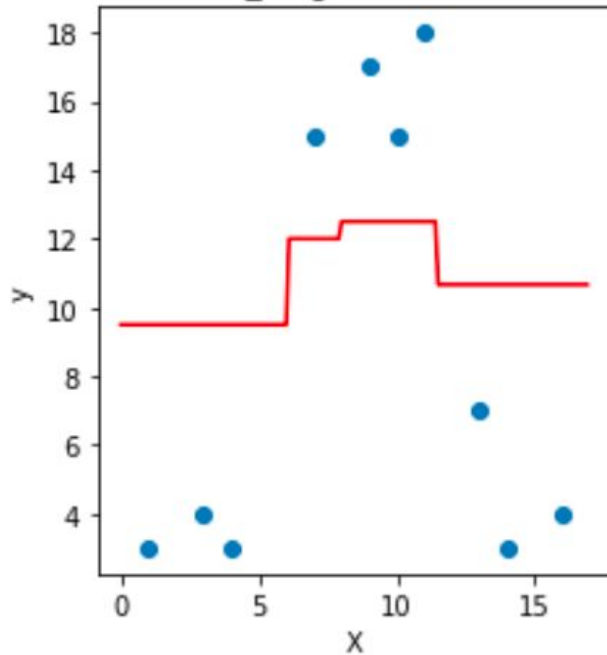
n\_neighbors = 1



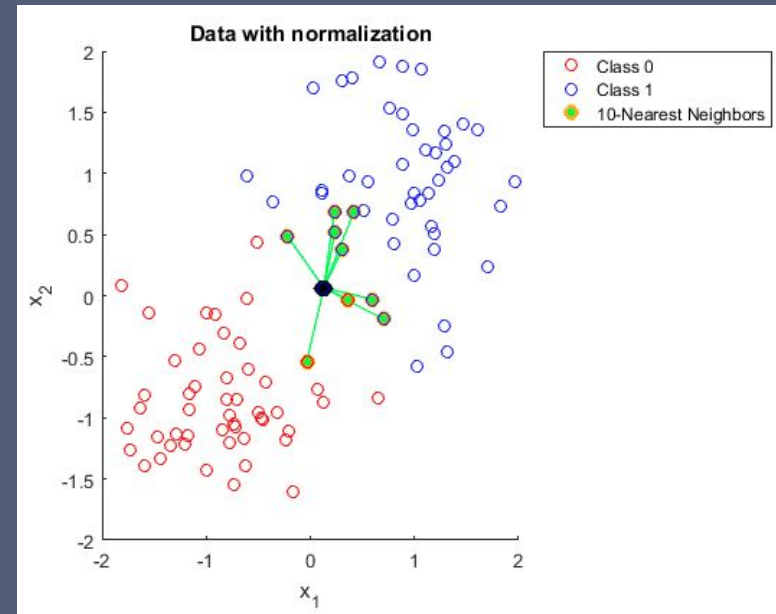
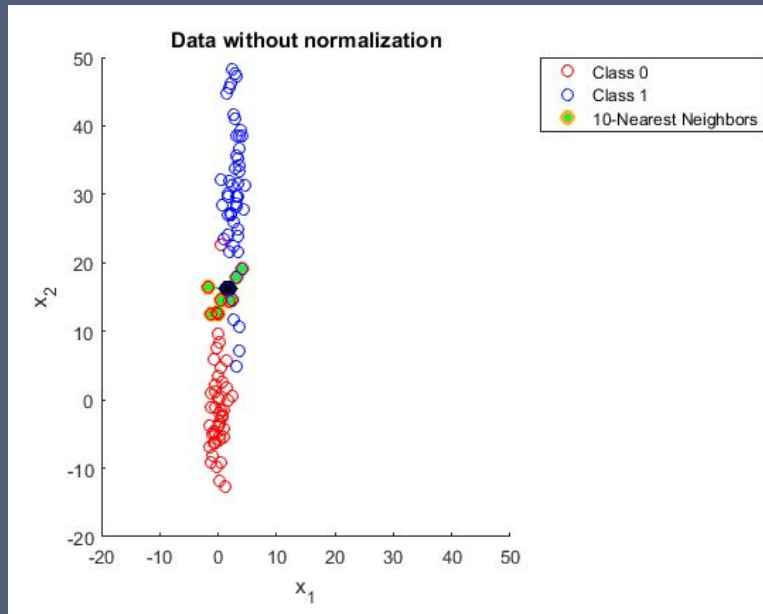
n\_neighbors = 3



n\_neighbors = 6



# Problems in KNN: Distance





# Problems in KNN: Many features

- High Dimensionality

Predictions get random as the number of features grow.

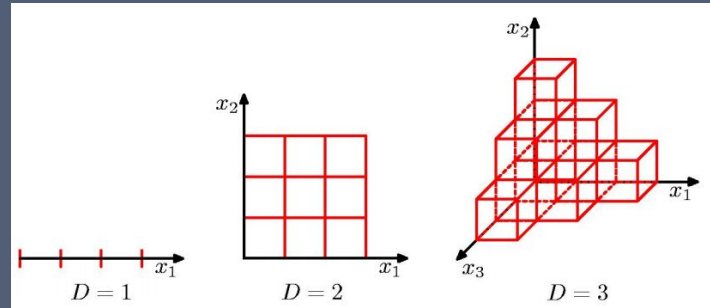
In the discrete case, number of neighbors increases:

1D: 2 - 4 - 6 - 8 - ...

2D: 4 - 12 - 20 - ...

3D: 6 - 18 - 32 - ...

It is highly recommended to use  
Dimensionality reduction (PCA)



# Improvements

- Weighted voted:  
Give a weight to each neighbor. Weight inversely depends on distance or we can use kernels
- Define your own distance:  
Each feature has a distance  
Each feature distance has a weight

$$D(x_i, x_{i'}) = \sum_{j=1}^P w_j \cdot d_j(x_{ij}, x_{i'j}); \quad \sum_{j=1}^P w_j = 1.$$

# From LogReg to KNN

LogReg:

- Low variance high bias
- Linear restriction

KNN:

- High variance and low bias
- No restriction
- Depends on training (selection of  $k$ )

