Unit 3: Foundations for inference

3. Hypothesis tests

LBJ - SDA - Spring 2024

University of Texas

Outline

1. Housekeeping

2. Main ideas

    1. Use hypothesis tests to make decisions about population parameters

    2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

    3. Results that are statistically significant are not necessarily practically significant

    4. Hypothesis tests are prone to decision errors

3. Summary

Outline

1. Housekeeping

2. Main ideas

    1. Use hypothesis tests to make decisions about population parameters
    2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
    3. Results that are statistically significant are not necessarily practically significant
    4. Hypothesis tests are prone to decision errors

3. Summary

Outline

1. Housekeeping

2. Main ideas

    1. Use hypothesis tests to make decisions about population parameters

    2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

    3. Results that are statistically significant are not necessarily practically significant

    4. Hypothesis tests are prone to decision errors

3. Summary

1. Use hypothesis tests to make decisions about population parameters

Hypothesis testing framework:

1. Set the hypotheses.
2. Check assumptions and conditions.
3. Calculate a *test statistic* and a p-value.
4. Make a decision, and interpret it in context of the research question.

Hypothesis testing for a population mean

1. Set the hypotheses
   - $H_0 : \mu = null\ value$
   - $H_A : \mu <$ or $>$ or $\neq null\ value$

Hypothesis testing for a population mean

1. Set the hypotheses
   - $H_0 : \mu = null\ value$
   - $H_A : \mu <$ or $>$ or $\neq null\ value$
2. Check assumptions and conditions
   - Independence: random sample/assignment, 10% condition when sampling without replacement
   - Sample size / skew: $n \geq 30$ (or larger if sample is skewed), no extreme skew

Hypothesis testing for a population mean

1. Set the hypotheses
   - $H_0 : \mu = null\ value$
   - $H_A : \mu <$ or $>$ or $\neq null\ value$
2. Check assumptions and conditions
   - Independence: random sample/assignment, 10% condition when sampling without replacement
   - Sample size / skew: $n \geq 30$ (or larger if sample is skewed), no extreme skew
3. Calculate a *test statistic* and a p-value (draw a picture!)

$$Z = \frac{\bar{x} - \mu}{SE}, \ \ where\ SE = \frac{s}{\sqrt{n}}$$

Hypothesis testing for a population mean

1. Set the hypotheses
   - $H_0 : \mu = null\ value$
   - $H_A : \mu <$ or $>$ or $\neq null\ value$
2. Check assumptions and conditions
   - Independence: random sample/assignment, 10% condition when sampling without replacement
   - Sample size / skew: $n \geq 30$ (or larger if sample is skewed), no extreme skew
3. Calculate a *test statistic* and a p-value (draw a picture!)

$$Z = \frac{\bar{x} - \mu}{SE},\ \ where\ SE = \frac{s}{\sqrt{n}}$$

4. Make a decision, and interpret it in context of the research question
   - If p-value $< \alpha$, reject H_0, data provide evidence for H_A
   - If p-value $> \alpha$, do not reject H_0, data do not provide

Application exercise: 3.2 Hypothesis testing for a single mean

See course website for details.

Your turn

Which of the following is the correct interpretation of the p-value from App Ex 3.2?

(a) The probability that average GPA of UT students has changed since 2001.

(b) The probability that average GPA of UT students has not changed since 2001.

(c) The probability that average GPA of UT students has not changed since 2001, if in fact a random sample of 63 UT students this year have an average GPA of 3.58 or higher.

(d) The probability that a random sample of 63 UT students have an average GPA of 3.58 or higher, if in fact the average GPA has not changed since 2001.

(e) The probability that a random sample of 63 UT students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.

Your turn

Which of the following is the correct interpretation of the p-value from App Ex 3.2?

(a) The probability that average GPA of UT students has changed since 2001.

(b) The probability that average GPA of UT students has not changed since 2001.

(c) The probability that average GPA of UT students has not changed since 2001, if in fact a random sample of 63 UT students this year have an average GPA of 3.58 or higher.

(d) The probability that a random sample of 63 UT students have an average GPA of 3.58 or higher, if in fact the average GPA has not changed since 2001.

(e) *The probability that a random sample of 63 UT students have an average GPA of 3.58 or higher or 3.16 or lower, if in fact the average GPA has not changed since 2001.*

1. P-value is the probability that the null hypothesis is true
   A p-value is the probability of getting a sample that results in a test statistic as or more extreme than what you actually observed (and in favor of the null hypothesis) if in fact the null hypothesis is correct. It is a conditional probability, conditioned on the null hypothesis being correct.

Common misconceptions about hypothesis testing

1. P-value is the probability that the null hypothesis is true
   A p-value is the probability of getting a sample that results in a test statistic as or more extreme than what you actually observed (and in favor of the null hypothesis) if in fact the null hypothesis is correct. It is a conditional probability, conditioned on the null hypothesis being correct.
2. A high p-value confirms the null hypothesis.
   A high p-value means the data do not provide convincing evidence for the alternative hypothesis and hence that the null hypothesis can't be rejected.

Common misconceptions about hypothesis testing

1. P-value is the probability that the null hypothesis is true
   A p-value is the probability of getting a sample that results in a test statistic as or more extreme than what you actually observed (and in favor of the null hypothesis) if in fact the null hypothesis is correct. It is a conditional probability, conditioned on the null hypothesis being correct.
2. A high p-value confirms the null hypothesis.
   A high p-value means the data do not provide convincing evidence for the alternative hypothesis and hence that the null hypothesis can't be rejected.

3. A low p-value confirms the alternative hypothesis.

Common misconceptions about hypothesis testing

1.
2. A high p-value confirms the null hypothesis.
   A high p-value means the data do not provide convincing evidence for the alternative hypothesis and hence that the null hypothesis can't be rejected.
3. A low p-value confirms the alternative hypothesis.
   A low p-value means the data provide convincing evidence for the alternative hypothesis, but not necessarily that it is confirmed.

Outline

1. Housekeeping

2. Main ideas

    1. Use hypothesis tests to make decisions about population parameters

    2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

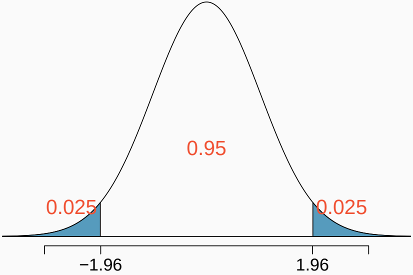
    3. Results that are statistically significant are not necessarily practically significant

    4. Hypothesis tests are prone to decision errors

3. Summary

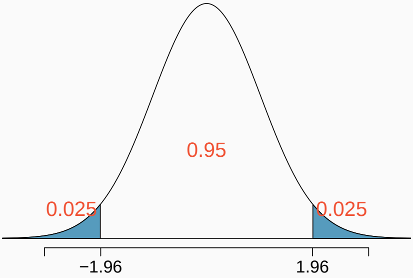
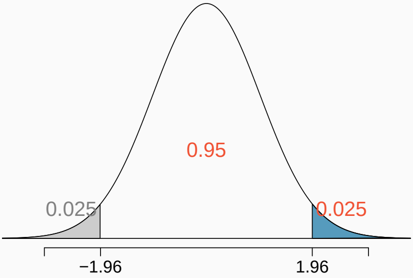
2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

Two sided



95% confidence level
is equivalent to
two sided HT with $\alpha = 0.05$

2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

Two sided	One sided
	
95% confidence level is equivalent to two sided HT with $\alpha = 0.05$	95% confidence level is equivalent to one sided HT with $\alpha = 0.025$

Your turn

What is the confidence level for a confidence interval that is equivalent to a two-sided hypothesis test at the 1% significance level? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.80

(b) 0.90

(c) 0.95

(d) 0.98

(e) 0.99

Your turn

What is the confidence level for a confidence interval that is equivalent to a two-sided hypothesis test at the 1% significance level? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.80
(b) 0.90
(c) 0.95
(d) 0.98
(e) ***0.99***

Your turn

What is the confidence level for a confidence interval that is equivalent to a one-sided hypothesis test at the 1% significance level? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.80

(b) 0.90

(c) 0.95

(d) 0.98

(e) 0.99

Your turn

What is the confidence level for a confidence interval that is equivalent to a one-sided hypothesis test at the 1% significance level? *Hint: Draw a picture and mark the confidence level in the center.*

(a) 0.80

(b) 0.90

(c) 0.95

(d) *0.98*

(e) 0.99

Your turn

A 95% confidence interval for the average normal body temperature of humans is found to be (98.1 F, 98.4 F). Which of the following is true?

(a) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.05$ in favor of $H_A : \mu \neq 98.2$.

(b) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.025$ in favor of $H_A : \mu > 98.2$.

(c) The hypothesis $H_0 : \mu = 98$ would be rejected using a 90% confidence interval.

(d) The hypothesis $H_0 : \mu = 98.2$ would be rejected using a 99% confidence interval.

Your turn

A 95% confidence interval for the average normal body temperature of humans is found to be (98.1 F, 98.4 F). Which of the following is true?

(a) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.05$ in favor of $H_A : \mu \neq 98.2$.

(b) The hypothesis $H_0 : \mu = 98.2$ would be rejected at $\alpha = 0.025$ in favor of $H_A : \mu > 98.2$.

(c) ***The hypothesis $H_0 : \mu = 98$ would be rejected using a 90% confidence interval.***

(d) The hypothesis $H_0 : \mu = 98.2$ would be rejected using a 99% confidence interval.

Outline

1. Housekeeping

2. Main ideas

    1. Use hypothesis tests to make decisions about population parameters

    2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

    3. Results that are statistically significant are not necessarily practically significant

    4. Hypothesis tests are prone to decision errors

3. Summary

3. Results that are statistically significant are not necessarily practically significant

Your turn

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$
(b) $n = 10,000$

3. Results that are statistically significant are not necessarily practically significant

Your turn

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$
(b) $n = 10,000$

3. Results that are statistically significant are not necessarily practically significant

Your turn

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$
(b) $n = 10,000$

Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu > 4.5$.

3. Results that are statistically significant are not necessarily practically significant

Your turn

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$
(b) $n = 10,000$

Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu > 4.5$.

$$Z_{n=100} = \frac{5 - 4.5}{\frac{2}{\sqrt{100}}}$$

3. Results that are statistically significant are not necessarily practically significant

Your turn

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$
(b) $n = 10,000$

Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu > 4.5$.

$$Z_{n=100} = \frac{5 - 4.5}{\frac{2}{\sqrt{100}}} = \frac{5 - 4.5}{\frac{2}{10}} = \frac{0.5}{0.2} = 2.5, \quad \textit{p-value} = 0.0062$$

3. Results that are statistically significant are not necessarily practically significant

Your turn

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$

(b) $n = 10,000$

Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu > 4.5$.

$$Z_{n=100} = \frac{5 - 4.5}{\frac{2}{\sqrt{100}}} = \frac{5 - 4.5}{\frac{2}{10}} = \frac{0.5}{0.2} = 2.5, \quad \textit{p-value} = 0.0062$$

$$Z_{n=10000} = \frac{5 - 4.5}{\frac{2}{\sqrt{10000}}}$$

3. Results that are statistically significant are not necessarily practically significant

Your turn

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$

(b) $n = 10,000$

Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu > 4.5$.

$$
\begin{aligned}
Z_{n=100} &= \frac{5 - 4.5}{\frac{2}{\sqrt{100}}} = \frac{5 - 4.5}{\frac{2}{10}} = \frac{0.5}{0.2} = 2.5, \quad \textit{p-value} = 0.0062 \\
Z_{n=10000} &= \frac{5 - 4.5}{\frac{2}{\sqrt{10000}}} = \frac{5 - 4.5}{\frac{2}{100}} = \frac{0.5}{0.02} = 25, \quad \textit{p-value} \approx 0
\end{aligned}
$$

3. Results that are statistically significant are not necessarily practically significant

Your turn

All else held equal, will p-value be lower if $n = 100$ or $n = 10,000$?

(a) $n = 100$
(b) $n = 10,000$

Suppose $\bar{x} = 5$, $s = 2$, $H_0 : \mu = 4.5$, *and* $H_A : \mu > 4.5$.

$$\begin{aligned} Z_{n=100} &= \frac{5 - 4.5}{\frac{2}{\sqrt{100}}} = \frac{5 - 4.5}{\frac{2}{10}} = \frac{0.5}{0.2} = 2.5, \quad \textit{p-value} = 0.0062 \\ Z_{n=10000} &= \frac{5 - 4.5}{\frac{2}{\sqrt{10000}}} = \frac{5 - 4.5}{\frac{2}{100}} = \frac{0.5}{0.02} = 25, \quad \textit{p-value} \approx 0 \end{aligned}$$

Outline

1. Housekeeping

2. Main ideas

    1. Use hypothesis tests to make decisions about population parameters

    2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree

    3. Results that are statistically significant are not necessarily practically significant

    4. Hypothesis tests are prone to decision errors

3. Summary

4. Hypothesis tests are prone to decision errors

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true		
	H_A true		

4. Hypothesis tests are prone to decision errors

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true		
	H_A true		

4. Hypothesis tests are prone to decision errors

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	
	H_A true		

4. Hypothesis tests are prone to decision errors

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	$\checkmark$	*Type 1 Error, α*
	H_A true		

- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true: α
  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error*, α
	H_A true	*Type 2 Error*, β	

- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true: α
  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α
- A *Type 2 Error* is failing to reject the null hypothesis when H_A is true: β

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error,* α
	H_A true	*Type 2 Error,* β	*Power,* $1 - \beta$

- A *Type 1 Error* is rejecting the null hypothesis when H_0 is true: α
  - For those cases where H_0 is actually true, we do not want to incorrectly reject it more than 5% of those times
  - Increasing α increases the Type 1 error rate, hence we prefer to small values of α
- A *Type 2 Error* is failing to reject the null hypothesis when H_A is true: β
- *Power* is the probability of correctly rejecting H_0, and hence the complement of the probability of a Type 2

Outline

1. Housekeeping
2. Main ideas
    1. Use hypothesis tests to make decisions about population parameters
    2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
    3. Results that are statistically significant are not necessarily practically significant
    4. Hypothesis tests are prone to decision errors
3. Summary

Summary of main ideas

1. Use hypothesis tests to make decisions about population parameters
2. Hypothesis tests and confidence intervals at equivalent significance/confidence levels should agree
3. Results that are statistically significant are not necessarily practically significant
4. Hypothesis tests are prone to decision errors