Unit 6: Introduction to linear regression

2. Outliers and inference for regression

GOVT 3990 - Spring 2020

Cornell University

Outline

1. Housekeeping
2. Main ideas
    1. Predicted values also have uncertainty around them
    2. R^2 assesses model fit – higher the better
    3. Inference for regression uses the t-distribution
    4. Conditions for regression
    5. Type of outlier determines how it should be handled
3. Summary

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- With any prediction we can (and should) also report a measure of uncertainty of the prediction.

Prediction intervals for specific predicted values

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- Prediction level: If we repeat the study of obtaining a regression data set many times, each time forming a XX% prediction interval at $x^\star$, and wait to see what the future

Calculating the prediction interval

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In R:

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# predict
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We are 95% confident that the annual murders per million for a county with 20% poverty rate is between 9.52 and 33.15.

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Response: annual_murders_per_mil
          Df  Sum Sq Mean Sq F value    Pr(>F)
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Residuals 18  546.86   30.38
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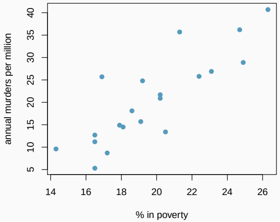
Your turn

R^2 for the regression model for predicting annual murders per million based on percentage living in poverty is roughly 71%. Which of the following is the correct interpretation of this value?



(a) 71% of the variability in percentage living in poverty is explained by the model.

(b) 84% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

(c) 71% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

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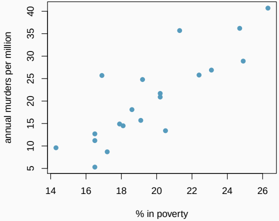

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Inference for regression uses the t-distribution

- Use a T distribution for inference on the slope, with degrees of freedom $n-2$
  - Degrees of freedom for the slope(s) in regression is $df = n - k - 1$ where k is the number of slopes being estimated in the model.

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```
                 2.5 %     97.5 %
(Intercept) -46.265631 -13.536694
perc_pov      1.740003   3.378776
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- Nearly normally distributed residuals $\rightarrow$ histogram or normal probability plot of residuals
- Constant variability of residuals (*homoscedasticity*) $\rightarrow$ no fan shape in the residuals plot
- Independence of residuals (and hence observations) $\rightarrow$ depends on data collection method, often violated for time-series data

Your turn

What condition is this linear model obviously and definitely violating?

(a) Linear relationship
(b) Non-normal residuals
(c) Constant variability
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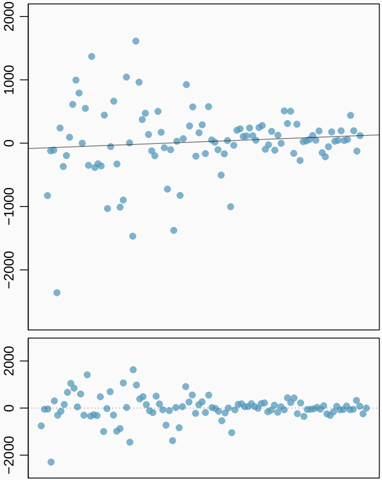

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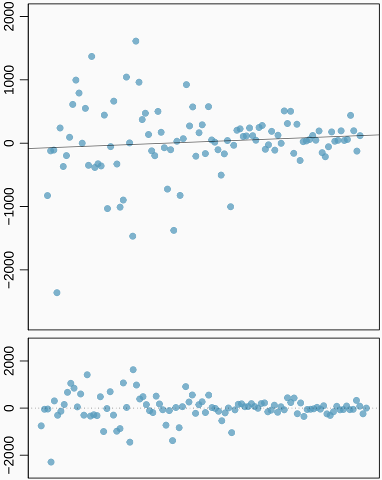

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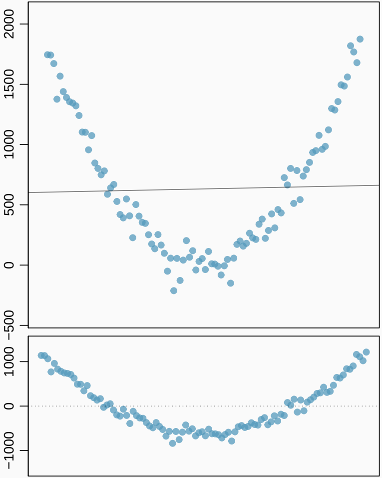

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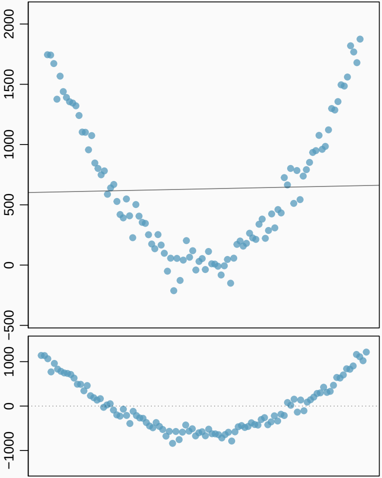

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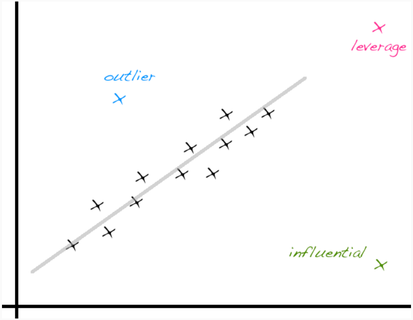

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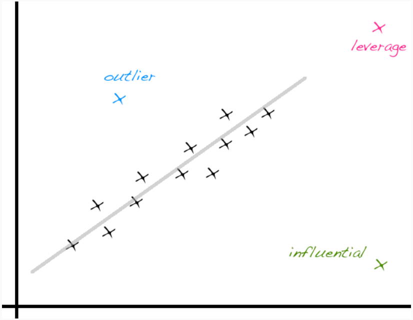

  - *Outlier* is an unusual point without these special characteristics (this one likely affects the intercept only)
  - If clusters (groups of points) are apparent in the data, it might be worthwhile to model the groups separately.

Application exercise: 6.2 Linear regression

See course website for details

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