Unit 4: Inference for numerical data

1. Inference using the t-distribution

GOVT 3990 - Spring 2018

Cornell University

1. Main ideas

    1. T corrects for uncertainty introduced by plugging in s for σ
    2. When comparing means of two groups, details depend on paired or independent
    3. All other details of the inferential framework is the same…

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2. T corrects for uncertainty introduced by plugging in s for σ

- CLT says $\bar{x} \sim N\left(mean = \mu, SE = \frac{\sigma}{\sqrt{n}}\right)$, but, in practice, we use s instead of σ.
  - Plugging in an estimate introduces additional uncertainty.
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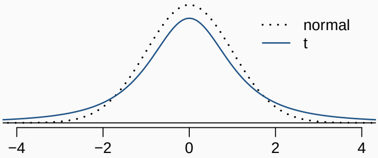
  - We make up for this by using a more “conservative” distribution than the normal distribution.
- t-distribution also has a bell shape, but its tails are *thicker* than the normal model’s
  - Observations are more likely to fall beyond two SDs from the mean than under the normal distribution.
  - Extra thick tails help mitigate the effect of a less reliable estimate for the standard error of the sampling distribution.



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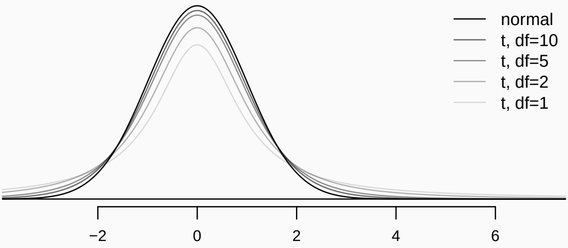

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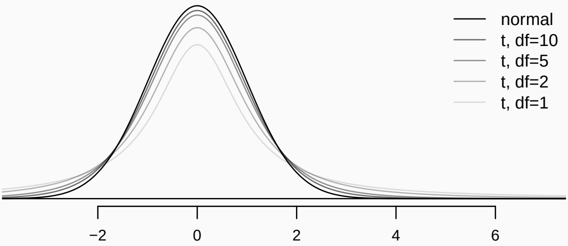

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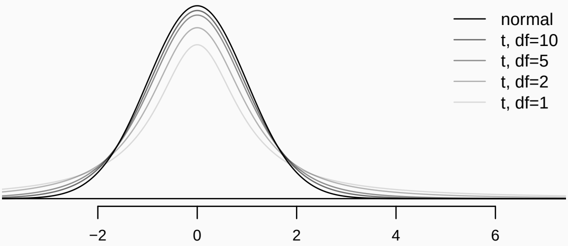

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Why?

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Example 1: Zinc in water

Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water at 10 randomly sampled locations.

Location	bottom	surface
1	0.43	0.415
2	0.266	0.238
3	0.567	0.39
4	0.531	0.41
5	0.707	0.605
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Water samples collected at the same location, on the surface and in the bottom, cannot be assumed to be independent of each other, hence we need to use a *paired* analysis.

Source: *https://onlinecourses.science.psu.edu/stat500/node/51*

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The New York Times

The Beginning of the End of the Census?

“This is a program that intrudes on people’s lives, just like the Environmental Protection Agency or the bank regulators,” said Daniel Webster, a first-term Republican congressman from Florida who sponsored the relevant legislation.

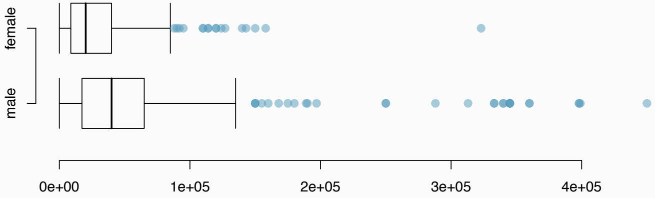
“We’re spending $70 per person to fill this out. That’s just not cost effective,” he continued, “especially since in the end this is not a scientific survey. It’s a random survey.”

In fact, the randomness of the survey is precisely what makes the survey scientific, statistical experts say.

Example 2: Gender gap in salaries

Since 2005, the American Community Survey polls ~3.5 million households yearly. The following summarizes distribution of salaries of males and females from a random sample of individuals who responded to the 2012 ACS:

	$\bar{x}$	s	n
male	55,890	68,767.88	470
female	29,240	32,025.98	373



How are the two examples different from each other? How are they similar to each other?

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Location	bottom	surface	difference
1	0.43	0.415	0.015
2	0.266	0.238	0.028
3	0.567	0.39	0.177
4	0.531	0.41	0.121
5	0.707	0.605	0.102
6	0.716	0.609	0.107
7	0.651	0.632	0.019
8	0.589	0.523	0.066
9	0.469	0.411	0.058
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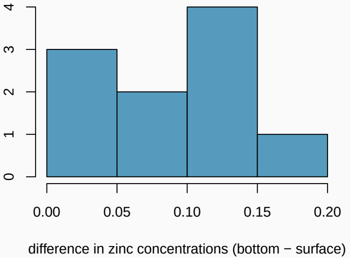

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- *Point estimate:* Average difference between the bottom and surface zinc measurements of drinking water from the *sampled* locations.

$$\bar{x}_{diff}$$

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- *Point estimate:* Average difference between the average salaries of *sampled* males and females in the US.

$$\bar{x}_m - \bar{x}_f$$

- Dependent (paired) groups (e.g. pre/post weights of subjects in a weight loss study, twin studies, etc.)

$$SE_{\bar{x}_{diff}} = \frac{s_{diff}}{\sqrt{n_{diff}}}$$

- Independent groups (e.g. grades of students across two sections)

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- For the same data, $SE_{paired} < SE_{independent}$, so be careful about calling data paired

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$df = n_{diff} - 1$

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Independent means:

$df = min(n_1 - 1, n_2 - 1)$

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$T_{df} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

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