# Unit 4: Inference for numerical data

## 1. Inference using the *t*-distribution

GOVT 3990 - Spring 2017

Cornell University

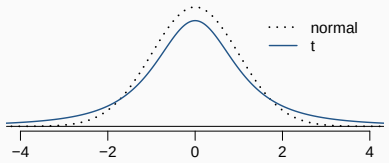
## Announcements

- Exams returned at the end of class today
- MT grades posted on ACES
- Peer eval due tonight
- Project proposal due Friday
  - Read the project instructions one more time
  - Work on the proposal before your lab on Thursday
  - Go to lab with questions
- MT course feedback due Tuesday night – anonymous, appreciate feedback



## 2. T corrects for uncertainty introduced by plugging in $s$ for $\sigma$

- CLT says $\bar{x} \sim N\left(mean = \mu, SE = \frac{\sigma}{\sqrt{n}}\right)$, but, in practice, we use s instead of $\sigma$.
  - Plugging in an estimate introduces additional uncertainty.
  - We make up for this by using a more "conservative" distribution than the normal distribution.
- *t*-distribution also has a bell shape, but its tails are *thicker* than the normal model's
  - Observations are more likely to fall beyond two SDs from the mean than under the normal distribution.
  - Extra thick tails help mitigate the effect of a less reliable estimate for the standard error of the sampling distribution.

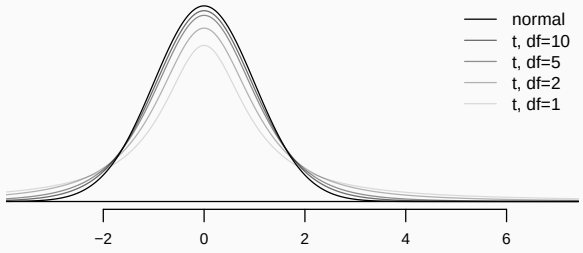





## *t*-distribution

- Always centered at zero, like the standard normal ($z$) distribution
- Has a single parameter, *degrees of freedom* (*df* ), that is tied to sample size.
  - one sample: $df = n - 1$
  - two (independent) samples: $df = min(n_1 - 1, n_2 - 1)$

What happens to shape of the *t*-distribution as *df* increases?




Why?

## Example 1: Zinc in water

Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water at 10 randomly sampled locations.

| Location | bottom | surface |
|---|---|---|
| 1 | 0.43 | 0.415 |
| 2 | 0.266 | 0.238 |
| 3 | 0.567 | 0.39 |
| 4 | 0.531 | 0.41 |
| 5 | 0.707 | 0.605 |
| 6 | 0.716 | 0.609 |
| 7 | 0.651 | 0.632 |
| 8 | 0.589 | 0.523 |
| 9 | 0.469 | 0.411 |
| 10 | 0.723 | 0.612 |

Water samples collected at the same location, on the surface and in the bottom, cannot be assumed to be independent of each other, hence we need to use a *paired* analysis.

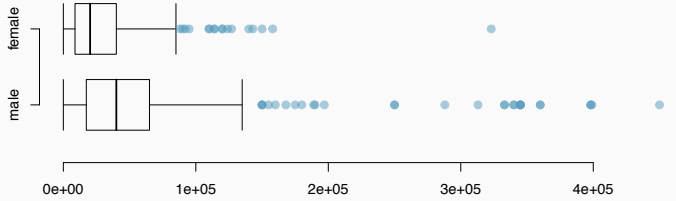
Source: https://onlinecourses.science.psu.edu/stat500/node/51



## Example 2: Gender gap in salaries

Since 2005, the American Community Survey[1] polls ∼3.5 million households yearly. The following summarizes distribution of salaries of males and females from a random sample of individuals who responded to the 2012 ACS:

| | $\bar{x}$ | $s$ | $n$ |
|---|---|---|---|
| male | 55,890 | 68,767.88 | 470 |
| female | 29,240 | 32,025.98 | 373 |




[1]Aside: Surge of media attention in spring 2012 when the House of Representatives voted to eliminate the survey. Daniel Webster, Republican congressman from Florida: "in the end this is not a scientific survey. It's a random survey."



How are the two examples different from each other? How are they similar to each other?

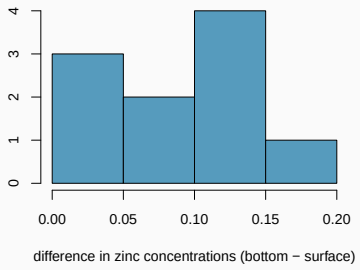


## Analyzing paired data

Suppose we want to compare the average zinc concentration levels in the bottom and surface:

- Two sets of observations with a special correspondence (not independent): *paired*
- Synthesize down to differences in outcomes of each pair of observations, subtract using a consistent order

| Location | bottom | surface | difference |
|---|---|---|---|
| 1 | 0.43 | 0.415 | 0.015 |
| 2 | 0.266 | 0.238 | 0.028 |
| 3 | 0.567 | 0.39 | 0.177 |
| 4 | 0.531 | 0.41 | 0.121 |
| 5 | 0.707 | 0.605 | 0.102 |
| 6 | 0.716 | 0.609 | 0.107 |
| 7 | 0.651 | 0.632 | 0.019 |
| 8 | 0.589 | 0.523 | 0.066 |
| 9 | 0.469 | 0.411 | 0.058 |
| 10 | 0.723 | 0.612 | 0.111 |



## Parameter and point estimate for paired data

For comparing average zinc concentration levels in the bottom and surface when the data are paired:

- *Parameter of interest:* Average difference between the bottom and surface zinc measurements of *all* drinking water.

$$\mu_{diff}$$

- *Point estimate:* Average difference between the bottom and surface zinc measurements of drinking water from the *sampled* locations.

$$\bar{x}_{diff}$$



## Parameter and point estimate for independent data

For comparing average salaries in two independent groups

- *Parameter of interest:* Average difference between the average salaries of *all* males and females in the US.

$$\mu_m - \mu_f$$

- *Point estimate:* Average difference between the average salaries of *sampled* males and females in the US.

$$\bar{x}_m - \bar{x}_f$$



## Standard errors

- Dependent (paired) groups (e.g. pre/post weights of subjects in a weight loss study, twin studies, etc.)

$$SE_{\bar{x}_{diff}} = \frac{s_{diff}}{\sqrt{n_{diff}}}$$

- Independent groups (e.g. grades of students across two sections)

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- For the same data, $SE_{paired} < SE_{independent}$, so be careful about calling data paired



## 3. All other details of the inferential framework is the same...

$$HT: test\ statistic = \frac{point\ estimate - null}{SE}$$

$$CI: point\ estimate \pm critical\ value \times SE$$

*One mean:*

$df = n - 1$

**HT:**

$H_0 : \mu = \mu_0$

$T_{df} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

**CI:**

$\bar{x} \pm t^\star_{df} \frac{s}{\sqrt{n}}$

*Paired means:*

$df = n_{diff} - 1$

**HT:**

$H_0 : \mu_{diff} = 0$

$T_{df} = \frac{\bar{x}_{diff} - 0}{\frac{s_{diff}}{\sqrt{n_{diff}}}}$

**CI:**

$\bar{x}_{diff} \pm t^\star_{df} \frac{s_{diff}}{\sqrt{n_{diff}}}$

*Independent means:*

$df = min(n_1 - 1, n_2 - 1)$

**HT:**

$H_0 : \mu_1 - \mu_2 = 0$

$T_{df} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

**CI:**

$\bar{x}_1 - \bar{x}_2 \pm t^\star_{df} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$