

Unit 4: Inference for numerical data

1. Inference using the t -distribution

GOVT 3990 - Spring 2020

Cornell University

1. Housekeeping

2. Main ideas

1. T corrects for uncertainty introduced by plugging in s for σ
2. When comparing means of two groups, details depend on paired or independent
3. All other details of the inferential framework is the same...

- ▶ General check in

Announcements

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 - Lab 4 Apr 10
 - Lab 5 Apr 15
 - Problem Set 3 Apr 22

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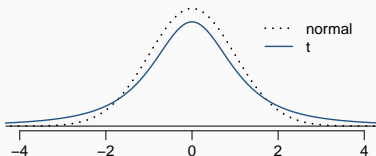
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- ▶ CLT says $\bar{x} \sim N\left(\text{mean} = \mu, SE = \frac{\sigma}{\sqrt{n}}\right)$, but, in practice, we use s instead of σ .
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 - Plugging in an estimate introduces additional uncertainty.
 - We make up for this by using a more “conservative” distribution than the normal distribution.
- ▶ t -distribution also has a bell shape, but its tails are *thicker* than the normal model's
 - Observations are more likely to fall beyond two SDs from the mean than under the normal distribution.
 - Extra thick tails help mitigate the effect of a less reliable estimate for the standard error of the sampling distribution.



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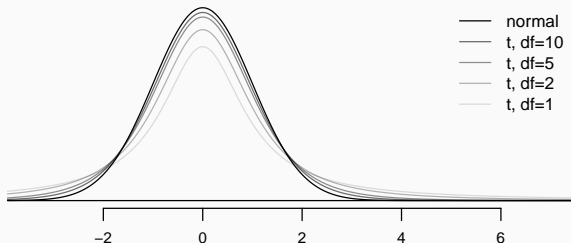
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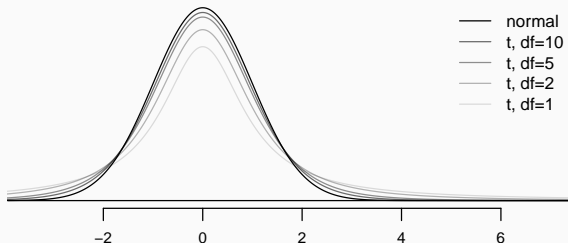
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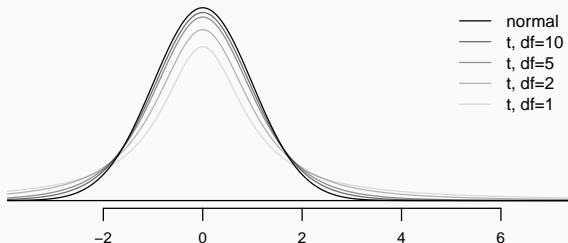


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Why?

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Example 1: Zinc in water

Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water and surface water at 10 randomly sampled locations.

| Location | bottom | surface |
|----------|--------|---------|
| 1 | 0.43 | 0.415 |
| 2 | 0.266 | 0.238 |
| 3 | 0.567 | 0.39 |
| 4 | 0.531 | 0.41 |
| 5 | 0.707 | 0.605 |
| 6 | 0.716 | 0.609 |
| 7 | 0.651 | 0.632 |
| 8 | 0.589 | 0.523 |
| 9 | 0.469 | 0.411 |
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Water samples collected at the same location, on the surface and in the bottom, cannot be assumed to be independent of each other, hence we need to use a *paired* analysis.

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The New York Times |

The Beginning of the End of the Census?

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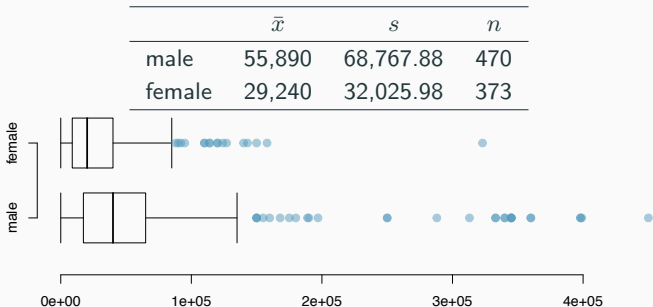
“This is a program that intrudes on people’s lives, just like the Environmental Protection Agency or the bank regulators,” said **Daniel Webster**, a first-term Republican congressman from Florida who sponsored the relevant legislation.

“We’re spending \$70 per person to fill this out. That’s just not cost effective,” he continued, “especially since in the end this is not a scientific survey. It’s a random survey.”

In fact, the randomness of the survey is precisely what makes the survey scientific, statistical experts say.

Example 2: Gender gap in salaries

Since 2005, the American Community Survey polls ~ 3.5 million households yearly. The following summarizes distribution of salaries of males and females from a random sample of individuals who responded to the 2012 ACS:



How are the two examples different from each other? How are they similar to each other?

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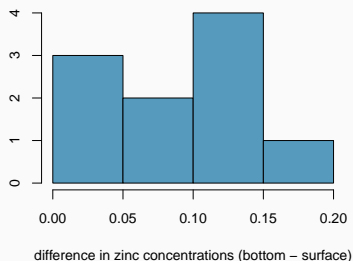
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Analyzing paired data

Suppose we want to compare the average zinc concentration levels in the bottom and surface:

- ▶ Two sets of observations with a special correspondence (not independent): *paired*
- ▶ Synthesize down to differences in outcomes of each pair of observations, subtract using a consistent order

| Location | bottom | surface | difference |
|----------|--------|---------|------------|
| 1 | 0.43 | 0.415 | 0.015 |
| 2 | 0.266 | 0.238 | 0.028 |
| 3 | 0.567 | 0.39 | 0.177 |
| 4 | 0.531 | 0.41 | 0.121 |
| 5 | 0.707 | 0.605 | 0.102 |
| 6 | 0.716 | 0.609 | 0.107 |
| 7 | 0.651 | 0.632 | 0.019 |
| 8 | 0.589 | 0.523 | 0.066 |
| 9 | 0.469 | 0.411 | 0.058 |
| 10 | 0.723 | 0.612 | 0.111 |



Parameter and point estimate for paired data

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- ▶ *Parameter of interest:* Average difference between the bottom and surface zinc measurements of *all* drinking water.

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- ▶ *Point estimate:* Average difference between the bottom and surface zinc measurements of drinking water from the *sampled* locations.

$$\bar{x}_{diff}$$

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- ▶ *Point estimate:* Average difference between the average salaries of *sampled* males and females in the US.

$$\bar{x}_m - \bar{x}_f$$

- ▶ Dependent (paired) groups (e.g. pre/post weights of subjects in a weight loss study, twin studies, etc.)

$$SE_{\bar{x}_{diff}} = \frac{s_{diff}}{\sqrt{n_{diff}}}$$

- ▶ Independent groups (e.g. grades of students across two sections)

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

- ▶ For the same data, $SE_{paired} < SE_{independent}$, so be careful about calling data paired

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$$df = n - 1$$

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$$H_0 : \mu = \mu_0$$

$$T_{df} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

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$$\bar{x} \pm t_{df}^* \frac{s}{\sqrt{n}}$$

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Paired means:

$$df = n_{diff} - 1$$

HT:

$$H_0 : \mu_{diff} = 0$$

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Independent means:

$$df = \min(n_1 - 1, n_2 - 1)$$

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$$H_0 : \mu_1 - \mu_2 = 0$$

$$T_{df} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

CI:

$$\bar{x}_1 - \bar{x}_2 \pm t_{df}^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$