Unit 2: Probability and distributions

1. Probability and conditional probability

GOVT 3990 - Spring 2020

Cornell University

Outline

1. Main ideas

    1. Disjoint and independent do not mean the same thing
    2. Application of the addition rule depends on disjointness of events
    3. Probability trees are useful for conditional probability calculations
    4. Bayesian inference: start with a prior, collect data, calculate posterior, make a decision or iterate
    5. Posterior probability and p-value do not mean the same thing

2. Summary

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  - But they might be a Republican and a Moderate at the same time – *non-disjoint events*
  - For disjoint A and B: $P(A \ and \ B) = 0$
- If A and B are *independent events*, having information on A does not tell us anything about B (and vice versa)
  - If A and B are independent:
    - $P(A \mid B) = P(A)$
    - $P(A \ and \ B) = P(A) \times P(B)$

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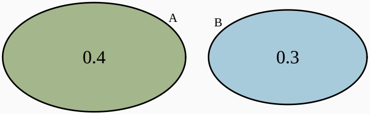

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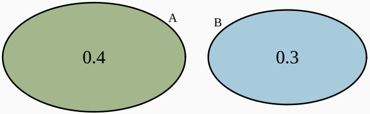

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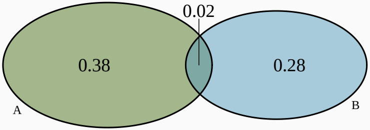

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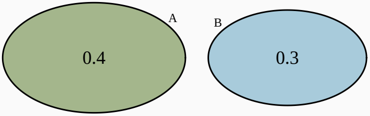

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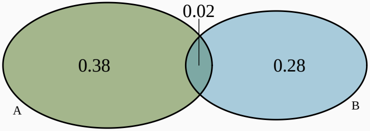

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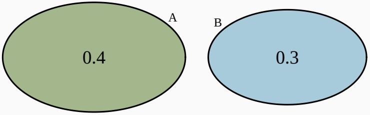
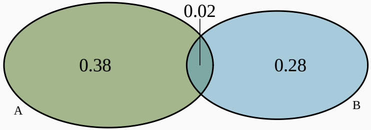


non-disjoint events:

P(A or B)

= P(A) + P(B) - P(A and B)

= 0.4 + 0.3 - 0.02 = 0.68



Application exercise: 2.1 Probability and conditional probability

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2. Summary

1. Probability trees are useful for conditional probability calculations

- Probability trees are useful for organizing information in conditional probability calculations
- They're especially useful in cases where you know P(A | B), along with some other information, and you're asked for P(B | A)

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- You can iterate this process.

Dice game

We'll play a game to demonstrate this approach:

- Two dice: 6-sided and 12-sided
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- The "good die" is the 12-sided die.
- Ultimate goal: come to a class consensus about whether the die on the cellphone or the die on the computer is the "good die"
- We will start with priors, collect data, and calculate posteriors, and make a decision or iterate until we're ready to make a decision

Prior probabilities

- At each roll I tell you whether you won or not (win $= \geq 4$)
  - P(win | 6-sided die) $=$ 0.5 $\rightarrow$ bad die
  - P(win | 12-sided die) $=$ 0.75 $\rightarrow$ good die

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  H_2: Good die is on the computer
- Since initially you have no idea which is true, you can assign equal *prior probabilities* to the hypotheses

  P(H_1 is true) = 0.5

  P(H_2 is true) = 0.5

Rules of the game

- You won't know which die I'm holding in which hand, left (Computer) or right (Cellphone). left = YOUR left
- You pick die (L or R), I roll it, and I tell you if you win or not, where winning is getting a number ≥ 4. If you win, you get a piece of candy. If you lose, I get to keep the candy.
- We'll play this multiple times with different contestants.
- I will not swap the sides the dice are on at any point.
- You get to pick how long you want play, but there are costs associated with playing longer.

	Truth	
Decision	L good, R bad	L bad, R good
Pick L	*You get candy!*	*You lose all the candy :(*
Pick R	*You lose all the candy :(*	*You get candy!*

Sampling isn't free!

At each trial you risk losing pieces of candy if you lose (the die comes up < 4). Too many trials means you won't have much candy left. And if we spend too much class time and we may not get through all the material.

Data and decision making

	Choice (L or R)	Result (win or loss)
Roll 1		
Roll 2		
Roll 3		
Roll 4		
Roll 5		
Roll 6		
Roll 7		
...		

What is your decision? How did you make this decision?

Most companies drug test their employees before they start employment, and sometimes regularly during their employment as well. Suppose that a drug test for an illegal drugs is 97% accurate in the case of a user of that drug, and 92% accurate in the case of a non-user for that drug. Suppose also that 5% of the entire population uses that drug.

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- You are the hiring manager at a company that drug tests their employees. You have recently decided to hire a new employee. What is the prior probability that this employee is a user of this drug? (You may assume that this prospective employee is a randomly drawn person from the population.)
  - *P(drug user)* $= 0.05$

- The prospective employee gets drug tested, and the test comes out to be positive. What is the probability that they are actually a user for the drug? What is this probability called? Sketch a probability tree for this question.

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