Analytic Solution

The given logistic map is

$$x_{n+1} = r\frac{x_n}{1+x_n^2}.$$

Assuming there is a stable fixed point such that $x_{n+1} = x_n = x_s$ and inserting this into the logistic map yields

$$x_s = \pm\sqrt{r-1}.$$

In class it was shown for an arbitrary logistic map $f(x)$ the condition $|f'(x_s)| < 1$ defines a region of stability. Appling this condition gives $|\frac{r}{1+x_s^2} - \frac{2rx_s^2}{(1+x_s^2)^2}| < 1$. Substituting x_s into this relation and disributing the absolute value leaves the inequality

$$-1 < 1 - \frac{2(r^2-r)}{r^2} < 1 \rightarrow 0 > -\frac{1}{r} > -1 \rightarrow r > 1.$$

Comments on B and C

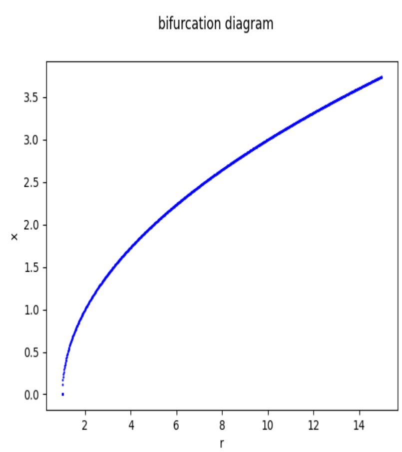

Figure 1: Bifurcation Diagram $x_{n+1} = r\frac{x_n}{1+x_n^2}$

As can be seen in Figure 1, the logistic map is stable in the range $1 \geq r \leq 15$. This agrees with the analytic solution attained above. There is no apparent period doubling.

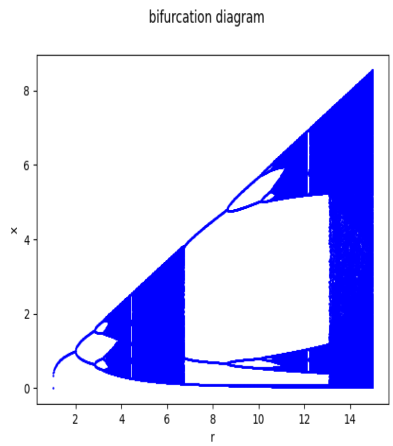

Figure 2: Bifurcation Diagram $x_{n+1} = r\frac{x_n}{1+x_n^4}$

As seen in figure 2 there period doubling which occurs at $r = 2$ and further period multiplicity which reduces at $r = 7$ before ramping up again.