Exercise 6.x

Wang Yue from CS Elite Class

December 5, 2020

Exercise 6.1

19. $y = \cos \pi x, y = 4x^2 - 1$

Solving $4x^2 - 1 = \cos \pi x$, we get $x = \pm\frac{1}{2}$, and we draw the graph:

The area of the region enclosed by the curves is

$$\begin{aligned}
\int_{-\frac{1}{2}}^{\frac{1}{2}} |4x^2 - 1 - \cos \pi x| dx &= \int_{-\frac{1}{2}}^{\frac{1}{2}} (\cos \pi x - 4x^2 + 1) dx \\
&= (\frac{1}{\pi} \sin \pi x - \frac{4}{3} x^3 + x)\Big|_{-\frac{1}{2}}^{\frac{1}{2}} \\
&= \frac{1}{\pi} - \frac{1}{6} + \frac{1}{2} - (-\frac{1}{\pi} + \frac{1}{6} - \frac{1}{2}) \\
&= \frac{2}{\pi} + \frac{2}{3}
\end{aligned}$$

20. $x = y^4, y = \sqrt{2 - x}, y = 0$

We can draw the graph:

Solving $\begin{cases} x = y^4 \\ y = \sqrt{2 - x} \end{cases}$, can we get $\begin{cases} x = 1 \\ y = 1 \end{cases}$

$\because$ When $y > 0$, $x = y^4 \implies y = \sqrt[4]{x}$

$\therefore$ The area of the region enclosed by the curves is

$$\begin{aligned}
\int_0^1 \sqrt[4]{x} dx + \int_1^2 \sqrt{2 - x} dx &= (\frac{4}{5} x^{\frac{5}{4}})\Big|_0^1 - [\frac{2}{3}(2 - x)^{\frac{3}{2}}]\Big|_1^2 \\
&= \frac{4}{5} - (0 - \frac{2}{3}) \\
&= \frac{22}{15}
\end{aligned}$$