MA 4_4 Exercise

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32. $\lim_{x\to 0} \frac{\cos mx - \cos nx}{x^2}$

$$\begin{aligned}
\lim_{x\to 0} \frac{\cos mx - \cos nx}{x^2} &= \lim_{x\to 0} \frac{(1-\frac{1}{2}(mx)^2) - (1-\frac{1}{2}(nx)^2)}{x^2} \\
&= \lim_{x\to 0} \frac{\frac{1}{2}(n^2-m^2)x^2}{x^2} \\
&= \lim_{x\to 0} \frac{n^2-m^2}{2}
\end{aligned}$$

39. $\lim_{x\to 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4}$

$$\begin{aligned}
\lim_{x\to 0} \frac{\cos x - 1 + \frac{1}{2}x^2}{x^4} &= \lim_{x\to 0} \frac{1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - 1 + \frac{1}{2}x^2}{x^4} \\
&= \lim_{x\to 0} \frac{\frac{1}{24}x^4}{x^4} \\
&= \frac{1}{24}
\end{aligned}$$

38. $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$

$$\begin{aligned}
\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} &= \lim_{x\to 0} \frac{(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3) - (1 - x + \frac{1}{2}x^2 - \frac{1}{6}x^3) - 2x}{x - (x - \frac{1}{6}x^3)} \\
&= \frac{2x + \frac{1}{3}x^3 - 2x}{\frac{1}{6}x^3} \\
&= 2
\end{aligned}$$

$\lim_{x\to 0} \frac{e^{x^3}-1-x^3}{(\sin 2x)^6}$

$$\begin{aligned}
\lim_{x\to 0} \frac{e^{x^3}-1-x^3}{(\sin 2x)^6} &= \lim_{x\to 0} \frac{1+x^3+\frac{1}{2}x^6-1-x^3}{(2x)^6} \\
&= \lim_{x\to 0} \frac{\frac{x^6}{2}}{2^6x^6} \\
&= \frac{1}{128}
\end{aligned}$$

$\lim_{x\to\infty}(x+\frac{1}{2})\ln(1+\frac{1}{x})$

$$\begin{aligned}
\lim_{x\to\infty}(x+\frac{1}{2})\ln(1+\frac{1}{x}) &= \lim_{x\to\infty}(x+\frac{1}{2})(\frac{1}{x}-\frac{1}{2x^2}) \\
&= \lim_{x\to\infty}(1-\frac{1}{2x}+\frac{1}{2x}-\frac{1}{4x^2}) \\
&= \lim_{x\to\infty}(1-\frac{1}{4x^2}) \\
&= 1
\end{aligned}$$

$\lim_{x\to 0}(\frac{1}{x}-\frac{1}{e^x-1})$

$$\begin{aligned}
\lim_{x\to 0}(\frac{1}{x}-\frac{1}{e^x-1}) &= \lim_{x\to 0} \frac{e^x-1-x}{x(e^x-1)} \\
&= \lim_{x\to 0} \frac{1+x+\frac{1}{2}x^2-1-x}{x(1+x+\frac{1}{2}x^2-1)} \\
&= \lim_{x\to 0} \frac{1}{2(1+\frac{x}{2})} \\
&= \frac{1}{2}
\end{aligned}$$

$\lim_{x\to 0^+} \frac{e^x-1-x}{\sqrt{1-x}-\cos\sqrt{x}}$

$$\begin{aligned}
\lim_{x\to 0^+} \frac{e^x-1-x}{\sqrt{1-x}-\cos\sqrt{x}} &= \lim_{x\to 0^+} \frac{1+x+\frac{1}{2}x^2-1-x}{1-\frac{1}{2}x-\frac{1}{8}x^2-(1-\frac{1}{2}x)} \\
&= \lim_{x\to 0^+} \frac{\frac{x^2}{2}}{-\frac{x^2}{8}} \\
&= -4
\end{aligned}$$