MA 5_3 Exercise

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Exercise 5.4

11. $\int \frac{x^3-2\sqrt{x}}{x}dx$

$$\begin{aligned}\int \frac{x^3-2\sqrt{x}}{x}dx &= \int x^2 - \frac{2}{\sqrt{x}}dx \\ &= \frac{1}{3}x^3 - 4\sqrt{x} + C\end{aligned}$$

12. $\int (x^2+1+\frac{1}{x^2+1})dx$

$$\int (x^2+1+\frac{1}{x^2+1})dx = \frac{1}{3}x^3 + x + \arctan x + C$$

14. $\int (\csc^2 t - 2e^t)dt$

$$\begin{aligned}\int (\csc^2 t - 2e^t)dt &= \int (\frac{1}{\sin^2} - 2e^t)dt \\ &= -\cot t - 2e^t + C\end{aligned}$$

16. $\int \sec t(\sec t + \tan t)dt$

$$\begin{aligned}\int \sec t(\sec t + \tan t)dt &= \int (\sec^2 t + \sec t \tan t)dt \\ &= \tan t + \sec t + C\end{aligned}$$

18. $\int \frac{\sin 2x}{\sin x}dx$

$$\begin{aligned}\int \frac{\sin 2x}{\sin x}dx &= \int 2\cos x dx \\ &= 2\sin x + C\end{aligned}$$

27. $\int_0^{\pi}(5e^x + 3\sin x)dx$

$$\begin{aligned}\int_0^{\pi}(5e^x + 3\sin x)dx &= (5e^x - 3\cos x)\Big|_0^{\pi} \\ &= (5e^{\pi} + 3) - (5 - 3) \\ &= 5e^{\pi} + 1\end{aligned}$$

33. $\int_1^2(\frac{x}{2} - \frac{2}{x})dx$

$$\begin{aligned}\int_1^2(\frac{x}{2} - \frac{2}{x})dx &= (\frac{x^2}{4} - 2\ln x)\Big|_1^2 \\ &= (1 - 2\ln 2) - (\frac{1}{4} - 0) \\ &= -2\ln 2 - \frac{1}{4}\end{aligned}$$

34. $\int_0^1(5x - 5^x)dx$

$$\begin{aligned}\int_0^1(5x - 5^x)dx &= (\frac{5}{2}x^2 - \frac{5^x}{\ln 5})\Big|_0^1 \\ &= (\frac{5}{2} - \frac{5}{\ln 5}) - (0 - \frac{1}{\ln 5}) \\ &= \frac{5}{2} - \frac{4}{\ln 5}\end{aligned}$$

37. $\int_0^{\frac{\pi}{4}} \frac{1+\cos^2\theta}{\cos^2\theta}d\theta$

$$\begin{aligned}\int_0^{\frac{\pi}{4}} \frac{1 + \cos^2\theta}{\cos^2\theta}d\theta &= \int_0^{\frac{\pi}{4}}(1 + \frac{1}{\cos^2\theta})d\theta \\ &= (\tan\theta + \theta)\Big|_0^{\frac{\pi}{4}} \\ &= 1 + \frac{\pi}{4}\end{aligned}$$

38. $\int_0^{\frac{\pi}{3}} \frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta}d\theta$

$$\begin{aligned}\int_0^{\frac{\pi}{3}} \frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta}d\theta &= \int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta+\sin\theta\tan^2\theta\cos^2\theta)d\theta \\ &= \int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta+\sin\theta\sin^2\theta)d\theta \\ &= \int_0^{\frac{\pi}{3}} \sin\theta d\theta \\ &= (-\cos\theta)\Big|_0^{\frac{\pi}{3}} \\ &= (-\frac{1}{2})-(-1) \\ &= \frac{1}{2}\end{aligned}$$

44. $\int_0^2 |2x-1|dx$

$$\begin{aligned}\int_0^2 |2x-1|dx &= \int_0^{1/2}(1-2x)dx+\int_{1/2}^2(2x-1)dx \\ &= (x-x^2)\Big|_0^{\frac{1}{2}}+(x^2-x)\Big|_{\frac{1}{2}}^2 \\ &= (\frac{1}{2}-\frac{1}{4})+(4-2-\frac{1}{4}+\frac{1}{2}) \\ &= \frac{5}{2}\end{aligned}$$