MA3_3 Exercise

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Exercise 3.10

6. Find the linear approximation of $f(x) = \sqrt[3]{1+x}$ at $a = 0$ and use it to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$. Illustrate by graphing f and the tangent line.

$\because g'(x) = \frac{1}{3}(1+x)^{-\frac{2}{3}}, g(0) = \sqrt[3]{1} = 1, g'(0) = \frac{1}{3}$

$\therefore$ the linear approximation of $g(x)$ at $a = 0$ is

$$y - 1 = \frac{1}{3}(x - 0) \iff y = \frac{1}{3}x + 1$$

$\therefore \sqrt[3]{1+x} \approx \frac{1}{3}x + 1 \qquad$ (when x is near 0)

$\because 1 \approx 0.95$

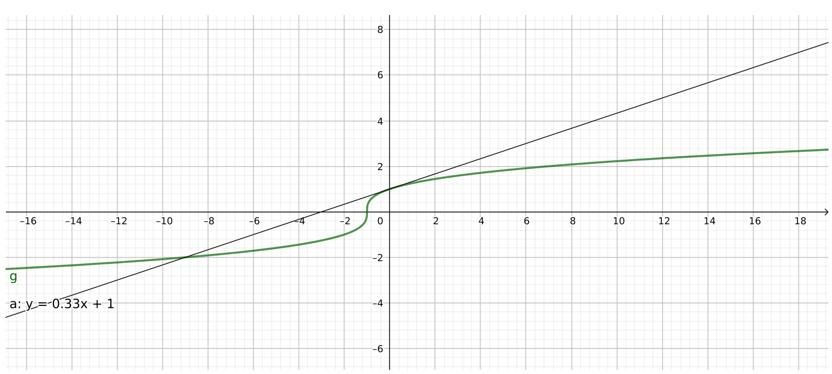
$\therefore$ let $x = -0.05$, and

$$\sqrt[3]{1-0.05} = \sqrt[3]{0.95} \approx \frac{61}{60}$$

$\because 1 \approx 1.1$

$\therefore$ let $x = 0.1$, and

$$\sqrt[3]{1+0.1} = \sqrt[3]{1.1} \approx \frac{31}{30}$$



11. **(b)** $y = \ln\sqrt{1+x^2}$

$$\mathrm{d}y = \frac{1}{\sqrt{1+x^2}}\frac{1}{2\sqrt{1+x^2}}2x\mathrm{d}x = \frac{x}{1+x^2}\mathrm{d}x$$

12. **(a)** $y = \frac{s}{1+2s}$

$$\mathrm{d}y = \frac{(1+2s)-2s}{(1+2s)^2}\mathrm{d}s = \frac{1}{(1+2s)^2}\mathrm{d}s$$

13. **(a)** $y = \tan\sqrt{t}$

$$\mathrm{d}y = \sec^2(\sqrt{t})\frac{1}{2\sqrt{t}}\mathrm{d}t = \frac{1}{2\sqrt{t}\cos^2\sqrt{t}}\mathrm{d}t$$

14. **(a)** $y = e^{\tan \pi t}$

$$\mathrm{d}y = e^{\tan \pi t}\sec^2(\pi t)\pi \mathrm{d}t = \frac{\pi e^{t\tan\pi t}}{\cos^2 \pi t}\mathrm{d}t$$

24. $e^{-0.015}$

$\because -0.015 \approx 0$
$\therefore$ we can use the linear approximation of e^x at $a = 0$
$\because (e^x)'|_{x=0} = 1, (e^x)|_{x=0} = 1$
$\therefore$ the linear approximation is

$$y - 1 = 1(x-0) \iff y = x+1$$

$\therefore e^x \approx x+1$ (when x is near 0)

$$\therefore e^{-0.015} \approx 1 - 0.015 = 0.985$$

26. $\frac{1}{4.002}$

Let $f(x) = \frac{1}{x}$, and obviously $f'(x) = -\frac{1}{x^2}$
$\because 4.002 \approx 4$
$\therefore$ we can use linear approximation of $f(x)$ at $a = 4$
$\because f'(4) = -\frac{1}{16}, f(4) = \frac{1}{4}$
$\therefore$ the linear approximation is

$$y - \frac{1}{4} = -\frac{1}{16}(x-4) \iff y = -\frac{1}{16}x + \frac{1}{2}$$

$\therefore \frac{1}{x} \approx -\frac{1}{16}x + \frac{1}{2}$ (when x is near 4)
$\therefore \frac{1}{4.002} \approx -\frac{4.002}{16} + \frac{1}{2} = \frac{6001}{8000}$