Calculus Exercise 2.4 Homework

计创 王樾

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- 15 18. Prove the statement using the ϵ, δ definition of a limit and illustrate with a diagram.

  Solution:

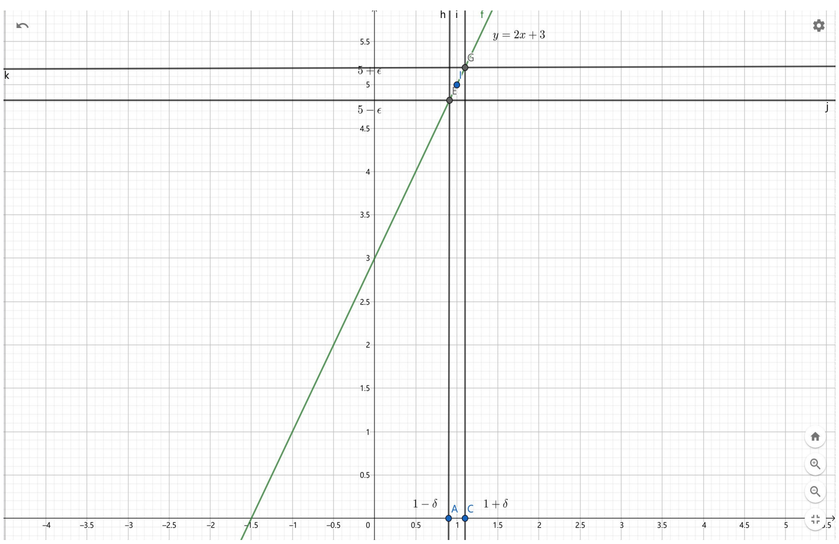
  (15) Proof:

  $\forall \epsilon \geq 0, \exists \delta = \frac{\epsilon}{2}$

  if $0 < |x - 1| < \delta$

  then $|2x + 3 - 5| = 2|x - 1| < 2\delta = \epsilon$

  illustration is below.

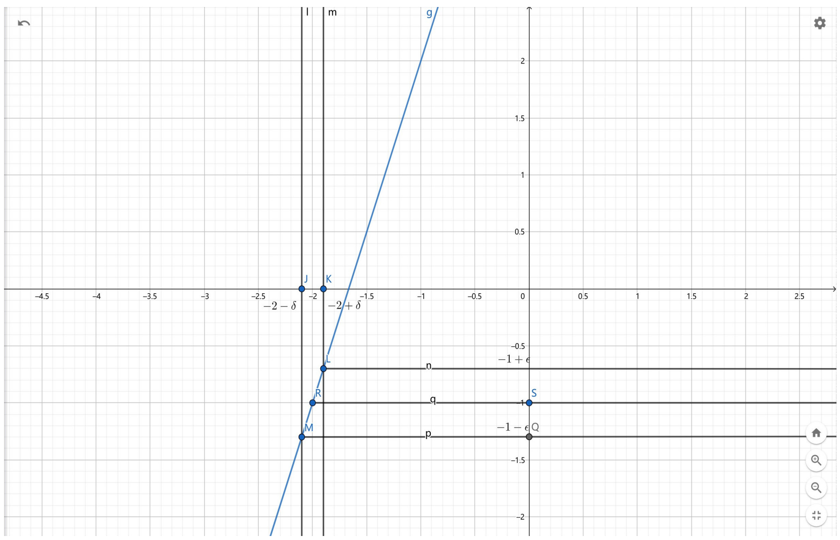
  (18) Proof:

  $\forall \epsilon \geq 0, \exists \delta = \frac{\epsilon}{3}$

if $0 < |x - (-2)| = |x + 2| < \delta$

then $|3x + 5 - (-1)| = |3x + 6| = 3|x + 2| < 3\delta = \epsilon$

illustration is below.



- 21 29 32. Prove the statement using the ϵ, δ definition of a limit.

  21. $\lim_{x \to 2} \frac{x^2+x-6}{x-2} = 5$

  29. $\lim_{x \to 2}(x^2 - 4x + 5) = 1$

  32. $lim_{x \to 2} x^3 = 8$

  Solution:

  (21) Proof:

  $\forall \epsilon \geq 0, \exists \delta = \epsilon$

  if $0 < |x - 2| < \delta$

  then $|\frac{x^2+x-6}{x-2} - 5| = |\frac{x^2-4x-4}{x-2}| = |x - 2| < \delta = \epsilon$

  (29) Proof:

  $\forall \epsilon \geq 0, \exists \delta = \sqrt{\epsilon}$

  if $0 < |x - 2| < \delta$

  then $|x^2 - 4x + 5 - 1| = |(x - 2)^2| = |x - 2|^2 < \delta^2 = \epsilon$

  (32) Proof:

$\forall \epsilon \geq 0, \exists \delta = \min(\frac{\epsilon}{19}, 1)$

if $0 < |x-2| < \delta$

then $|x^3 - 8| = |x-2||x^2+2x+4| < |x^2+2x+4|\delta$

restrict x to lie in a neighborhood of 2, such that $|x-2| < 1$

so $1 < x < 3$

then $|x^3 - 8| < |(x+1)^2 + 3|\delta < 19\delta = \epsilon$

- 36. Prove that $\lim_{x\to 2} \frac{1}{x} = \frac{1}{2}$.

  Solution:

  (36) Proof:

  $\forall \epsilon \geq 0, \exists \delta = \min(2\epsilon, 1)$

  if $0 < |x-2| < \delta$

  then $|\frac{1}{x} - \frac{1}{2}| = \frac{|x-2|}{2|x|}$

  restrict x to lie in a neighborhood of 2, such that $|x-2| < 1$

  so $1 < x < 3$

  then $\frac{|x-2|}{2|x|} < \frac{|x-2|}{2} < \frac{\delta}{2} = \epsilon$

- 39. If the function f is defined by

  $$f(x) = \begin{cases} 0 & \text{if x is rational} \\ 1 & \text{if x is irrational} \end{cases}$$

  prove that $\lim_{x\to 0} f(x)$ does not exist.

  Solution:

  (39) Proof:

  $\exists \epsilon = \frac{1}{2}, \forall \delta > 0$

  if $0 < |x| < \delta$, because of the denseness of real number, $\exists x_1 \in Q, x_2 \notin Q$

  but $|f(x_1) - f(x_2)| = 1 > \epsilon$

  so $f(x)$ doesn't have a limit when x approach 0.

- 42. Prove, using Definition 6, that $\lim_{x \to -3} \frac{1}{(x+3)^4} = \infty$

  Solution:

  (42) Proof:

  $\forall M > 0, \exists \delta = \lfloor \sqrt[4]{\frac{1}{M}} \rfloor$

  if $0 < |x - (-3)| = |x + 3| < \delta$

  then $\frac{1}{(x+3)^4} > \frac{1}{\delta^4} > \frac{1}{\frac{1}{M}} = M$