Chapter 3 First Exercise

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Exercise 3.1

35. $y = x^4 + 2e^x, \quad (0, 2)$

Solution:

$\because y' = 4x^3 + 2e^x$
$\therefore y'|_{x=0} = 4 \times 0 + 2 \times 1 = 2$
$\therefore$ the slope of the tangent line to the curve at $(0, 2)$ is 2
$\therefore$ the equation of the tangent line is

$$y - 2 = 2(x - 0) \iff y = 2x + 2$$

$\therefore$ the slope of the normal line to the curve at $(0, 0)$ is $-\frac{1}{2}$
$\therefore$ the equation of the normal line is

$$y - 2 = -\frac{1}{2}(x - 0) \iff x + 2y - 4 = 0$$

53. Show that the curve $y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2.

Proof. $\because y' = 2e^x + 3 + 15x^2$
$\because e^x > 0, x^2 \geq 0$
$\therefore y' = 2e^x + 15x^2 + 3 > 3 > 2$
$\therefore y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2. $\square$

54. Find an equation of the tangent line ot the curve $y = x\sqrt{x}$ that is parallel to the line $y = 1 + 3x$.

Solution:

Obviously, $(1 + 3x)' = 3$
And we will find a tangent line to $y = x\sqrt{x}$ whose slope is 3.
Let $f(x) = x\sqrt{x}$, and

$$f'(x) = 1 \times \sqrt{x} + x \times \frac{1}{2\sqrt{x}} = \frac{3}{2}\sqrt{x}$$

Solving the equation $\frac{3}{2}\sqrt{x} = 3$, we can get $x = 4$
$\because$ When $x = 4$, $x\sqrt{x} = 4 \times 2 = 8$
$\therefore$ an tangent line to the curve at $x = 4$ is

$$y - 8 = 3(x - 4) \iff y = 3x - 4$$

75. Let

$$f(x) = \begin{cases} x^2 & \textbf{if } x \leq 2 \\ mx + b & \textbf{if } x > 2 \end{cases}$$

Find the values of m and b that make f differentiable everywhere.

Solution:
$\because$ f is differentiable everywhere
$\therefore$ f is continuous everywhere
$\therefore$

$$f(2) = \lim_{x \to 2^+} f(x), f'(2) = \lim_{x \to 2^+} f'(x)$$

$\therefore \{ \ \lim_{x \to 2^+} f(x) = 2m + b = f(2) = 4 \lim_{x \to 2^+} f'(x) = m = f'(2) = 4$
Solving this equation set, we can get $\begin{cases} m = 4 \\ b = -4 \end{cases}$

Exercise 3.2

52. (c) $y = \frac{x^2}{f(x)}$

$$y' = \frac{2xf(x) - x^2 f'(x)}{f^2(x)}$$

52. (d) $y = \frac{1 + xf(x)}{\sqrt{x}}$

$$y' = \frac{(f(x) + xf'(x))\sqrt{x} - (1 + xf(x))\frac{1}{2\sqrt{x}}}{x}$$

46. If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}(\frac{h(x)}{x})|_{x=2}$$

Let $h(x) = -3x + 10$, which satisfies $h(2) = 4$ and $h'(2) = -3$
$\frac{h(x)}{x} = -3 + \frac{10}{x}$
$(\frac{h(x)}{x})' = (\frac{10}{x})' = \frac{-10}{x^2}$
$\therefore \frac{d}{dx}(\frac{h(x)}{x})|_{x=2} = \frac{-10}{4} = -\frac{5}{2}$

24. $f(x) = \frac{1-xe^x}{x+e^x}$

$$f'(x) = \frac{-[(x+1)e^x](x+e^x) - (1-xe^x)(1+e^x)}{(x+e^x)^2}$$