Pages from Infinitesimal Assignment

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1 Calculate the following limits.

1.1 $\lim_{n\to\infty}(\cos\frac{x}{n})^{n^2}$

$\because$ cosine function is continuous function,
$\therefore \lim_{n\to\infty}\cos(\frac{x}{n}) = \cos\lim_{n\to\infty}\frac{x}{n} = \cos 0 = 1$
$\therefore \lim_{n\to\infty}(\cos\frac{x}{n})^{n^2} = \lim_{n\to\infty}1^{n^2} = 1$

1.2 $\lim_{x\to 2}\frac{2^x-x^2}{x-2}$

Let $f(x) = 2^x$, and obviously $f'(x) = 2^x \ln 2$

$$\begin{aligned}\lim_{x\to 2}\frac{2^x-x^2}{x-2} &= \lim_{x\to 2}\frac{2^x-4-(x^2-4)}{x-2}\\ &= \lim_{x\to 2}\frac{2^x-4}{x-2} + \lim_{x\to 2}(x+2)\\ &= f'(2)+4\\ &= 4(1+\ln 2)\end{aligned}$$