Exercise 9.5

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10. $y' + y = \sin e^x$

Integrating factor: $e^{\int 1dx} = e^x$

$$e^x y' + e^x y = \frac{d}{dx}(ye^x) = e^x \sin e^x$$

$$\int \frac{d}{dx}(ye^x)dx = \int \sin e^x dx$$

$$ye^x = -\cos e^x + C$$

$$y = -\cos x + Ce^{-x}$$

where C is an arbitrary constant

11. $\sin x \frac{dy}{dx} + (\cos x)y = \sin(x^2)$

$$\int \frac{d}{dx}(y \sin x)dx = \int \sin(x^2)dx$$

$$y = \frac{\int \sin(x^2)dx + C}{\sin x}$$

where $\int \sin(x^2)dx$ is an integral that cannot be solved directly, C is an arbitrary constant.

14. $t \ln t \frac{dr}{dt} + r = te^t$

$$\frac{dr}{dt} + \frac{1}{t \ln t} r = \frac{e^t}{\ln t}$$

Integrating factor: $e^{\int \frac{1}{t \ln t} dt} = e^{\int \frac{1}{\ln t} d \ln t} = e^{\ln(\ln t)} = \ln t$

$$\ln t \frac{dr}{dt} + \frac{1}{t} r = \frac{d}{dx}(r \ln t) = e^t$$

$$r\ln t = \int e^t dt = e^t + C$$

$$\therefore r\ln t = e^t + C$$

where C is an arbitrary constant

15.

$$x^2y' + 2xy = \frac{d}{dx}(x^2y) = \ln x$$

$$x^2y = \int \ln x dx = x\ln x - x + C$$

Since $2 = 0 - 1 + C$, $C = 3$

$$\therefore x^2y = x\ln x - x + 3$$

16.

$$\frac{d}{dt}(t^3y) = \cos t$$

$$t^3y = \int \cos t dt = \sin t + C$$

Since $\pi^3 \times 0 = \sin\pi + C$, $C = 0$

$$\therefore t^3y = \sin t$$

23.

Let $u = y^{1-n}$, then

$$\frac{du}{dx} = \frac{du}{dy}\frac{dy}{dx} = (1-n)y^{-n}\frac{dy}{dx}$$

Multiplied by $(1-n)y^{-n}$ on the both sides, we get

$$(1-n)y^{-n}\frac{dy}{dx} + (1-n)y^{1-n}P(x) = (1-n)Q(x)$$

which is equivalent to

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x)$$

25.

Let $u = y^{1-3} = y^{-2}$

Multiplied by $(1-3)y^{-3}$ on the both sides, we get

$$(-2)y^{-3}y' + (-2)\frac{2}{x}y^{-2} = \frac{-2}{x^2}$$

which is equivalent to

$$\frac{du}{dx} - \frac{4}{x}u = \frac{-2}{x^2}$$

Integrating factor: $e^{\int -\frac{4}{x}dx} = e^{-4\ln x} = x^{-4}$

$$x^{-4}\frac{du}{dx} - 4x^{-5}u = \frac{d}{dx}(x^{-4}u) = -2x^{-6}$$

$$x^{-4}u = \int -2x^{-6}dx + C = \frac{2}{5}\int -5x^{-6}dx + C = \frac{2}{5}x^{-5} + C$$

$$\therefore u = \frac{2}{5}x^{-1} + Cx^4$$

where C is an arbitrary constant

26.

Let $u = y'$, then

$$x\frac{du}{dx} + 2u = 12x^2 \iff \frac{du}{dx} + \frac{2}{x}u = 12x$$

Integrating factor: $e^{\int \frac{2}{x}dx} = x^2$

$$x^2\frac{du}{dx} + 2xu = \frac{d}{dx}(x^2u) = 12x^3$$

$$x^2u = \int 12x^3dx = 3x^4 + C$$

$$u = 3x^2 + \frac{C}{x^2}$$

where C is an arbitrary constant