Exercise 15.3

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24.

$$\begin{aligned}
\int_0^4 \int_{-\sqrt{x}}^{\sqrt{x}} (1+x^2y^2)dydx &= \int_0^4 2\int_0^{\sqrt{x}} (1+x^2y^2)dydx \\
&= \int_0^4 2(y+x^2\frac{y^3}{3})\Big|_{y=0}^{y=\sqrt{x}} dx \\
&= \int_0^4 (2\sqrt{x}+\frac{x^{\frac{7}{2}}}{3})dx \\
&= (\frac{4}{3}x^{\frac{3}{2}}+\frac{2}{27}x^{\frac{9}{2}})\Big|_0^4 \\
&= \frac{32}{3}+\frac{1024}{27} \\
&= \frac{1312}{27}
\end{aligned}$$

40.

$$\begin{aligned}
&\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (8-x^2-2y^2-2x^2-y^2)dydx \\
&= \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (8-3x^2-3y^2)dydx \\
&= \int_0^{2\pi} \int_0^1 (8r-3r^3)drd\theta \\
&= \int_0^{2\pi} (4r^2-\frac{3r^4}{4})\Big|_0^1 d\theta \\
&= \int_0^{2\pi} (4-\frac{3}{4})d\theta \\
&= \frac{13\pi}{8}
\end{aligned}$$

49.

$$\begin{aligned}\int_0^1\int_{3y}^3 e^{x^2}dxdy &= \int_0^3\int_0^{\frac{x}{3}} e^{x^2}dydx\\ &= \int_0^3 \frac{x}{3}e^{x^2}dx\\ &= \int_0^3 \frac{1}{6}e^{x^2}dx^2\\ &= \frac{1}{6}e^{x^2}\Big|_0^3\\ &= \frac{e^9-1}{6}\end{aligned}$$

53.

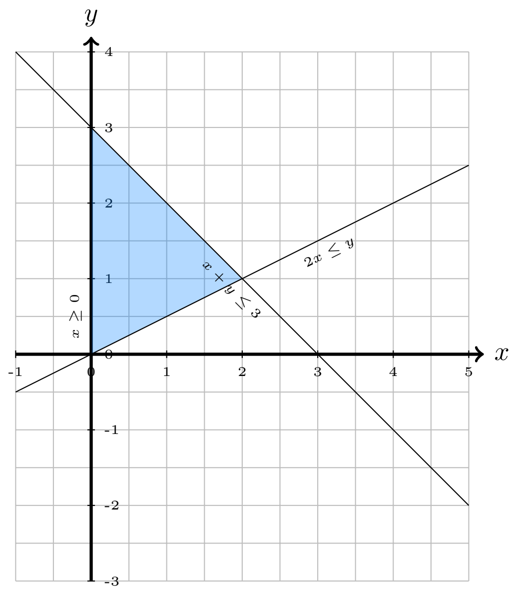
$$\begin{aligned}\int_0^1\int_{\arcsin y}^{\frac{\pi}{2}} \cos x\sqrt{1+\cos^2 x}dxdy &= \int_0^{\frac{\pi}{2}}\int_0^{\sin x} \cos x\sqrt{1+\cos^2 x}dydx\\ &= \int_0^{\frac{\pi}{2}} \sin x\cos x\sqrt{1+\cos^2 x}dx\\ &= -\frac{1}{2}\int_0^{\frac{\pi}{2}} \sqrt{1+\cos^2 x}d(1+\cos^2 x)\\ &= -\frac{1}{2}\times\frac{2}{3}(1+\cos^2 x)^{\frac{3}{2}}\Big|_0^{\frac{\pi}{2}}\\ &= -\frac{1}{3}(0-1)\\ &= \frac{1}{3}\end{aligned}$$

54.

$$\begin{aligned}\int_0^8\int_{\sqrt[3]{y}}^2 e^{x^4}dxdy &= \int_0^2\int_0^{x^3} e^{x^4}dydx\\ &= \int_0^2 x^3e^{x^4}dx\\ &= \frac{1}{4}\int_0^2 e^{x^4}dx^4\\ &= \frac{1}{4}e^{x^4}\Big|_0^2\\ &= \frac{e^{16}-1}{4}\end{aligned}$$

62.

$$D = \{(x,y) | x \geq 0 \text{ and } y \geq \frac{x}{2} \text{ and } y \leq 3 - x\}$$



$$\iint_D f(x,y)dA = \int_0^2 \int_{\frac{x}{2}}^{3-x} f(x,y)dydx$$

63.

$$\begin{aligned}
\iint_D (x+2)dA &= \int_0^3 \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} (x+2)dxdy \\
&= \int_0^3 (\frac{x^2}{2} + 2x)\Big|_{x=-\sqrt{9-y^2}}^{x=\sqrt{9-y^2}} dy \\
&= 4\int_0^3 \sqrt{9-y^2}dy \\
&= 4 \times \frac{3^2\pi}{4} = 9\pi
\end{aligned}$$

66.

$$\iint_D (2+x^2y^3-y^2\sin x)dA$$
$$= \int_{-1}^{0}\int_{-y-1}^{y+1}(2+x^2y^3-y^2\sin x)dxdy + \int_{0}^{1}\int_{y-1}^{1-y}(2+x^2y^3-y^2\sin x)dxdy$$
$$= \int_{-1}^{0}[4(y+1)+\frac{2(y+1)^3y^3}{3}]dy + \int_{0}^{1}[4(1-y)+\frac{2(1-y)^3y^3}{3}]dy$$
$$= \int_{-1}^{0}4(y+1)dy + \int_{0}^{1}4(1-y)dy + \frac{2}{3}\int_{0}^{1}(1-y)^3y^3dy + \frac{2}{3}\int_{-1}^{0}(y+1)^3y^3dy$$
$$= 4(\frac{y^2}{y}+y)\Big|_{-1}^{0} + 4(y-\frac{y^2}{2})\Big|_{0}^{1} + \frac{2}{3}(\frac{y^4}{4}-\frac{3y^5}{5}+\frac{3y^6}{6}-\frac{y^7}{7})\Big|_{0}^{1} + (\frac{y^7}{7}+\frac{3y^6}{6}+\frac{3y^5}{5}+\frac{y^4}{4})\Big|_{-1}^{0}$$
$$= 4+\frac{2}{3}(\frac{1}{4}-\frac{3}{5}+\frac{1}{2}-\frac{1}{7}+\frac{1}{7}-\frac{1}{2}+\frac{3}{5}-\frac{1}{4})$$
$$= 4$$