Exercise 15.1

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14.

$$\begin{aligned}
\iint_R \sqrt{9-y^2}dA &= \int_0^4 \int_0^2 \sqrt{9-y^2}dydx \\
&= \int_0^4 \int_0^2 -\frac{\sqrt{9-y^2}}{2}d(9-y^2)dx \\
&= \int_0^4 -\frac{1}{2} \times \frac{2}{3}(9-y^2)^{\frac{3}{2}}\bigg|_0^2 dx \\
&= \int_0^4 (9-\frac{\sqrt{125}}{3})dx \\
&= 36 - \frac{20}{3}\sqrt{5}
\end{aligned}$$

17.

Proof.

$$\begin{aligned}
\iint_R f(x,y)dA &= \iint_R kdA \\
&= \int_a^b \int_c^d kdydx \\
&= \int_a^b k(d-c)dx \\
&= k(b-a)(d-c)
\end{aligned}$$

□

18.

Proof. $\because x \in [0, \frac{1}{4}]$, $\therefore \sin \pi x \in [0, \frac{\sqrt{2}}{2}]$

$\because y \in [\frac{1}{4}, \frac{1}{2}]$, $\therefore \cos \pi x \in [0, \frac{\sqrt{2}}{2}]$

$$\therefore 0 \leq \sin \pi x \cos \pi y \leq \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

$\therefore$ by the conclusion of Exercise 17,

$$0 \times \frac{1}{4} \times \frac{1}{4} \leq \int_0^{\frac{1}{4}} \int_{\frac{1}{4}}^{\frac{1}{2}} \sin \pi x \cos \pi y dy dx \leq \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}$$

which is equivalent to

$$0 \leq \iint_R \sin \pi x \cos \pi y dA \leq \frac{1}{32}$$

$\square$