# Homework

March 31, 2021

$\because \begin{cases} x = 3\cos t \\ y = 2\sin t \end{cases}$
$\therefore r(t) = (3\cos t, 2\sin t, 0)$

$$\therefore r'(t) = (-3\sin t, 2\cos t, 0), r''(t) = (-3\cos t, -2\sin t, 0)$$

$$\therefore r'(t) \times r''(t) = (0, 0, 6\sin^2 t + 6\cos^2 t) = (0, 0, 6)$$

$$\therefore k = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} = \frac{6}{(\sqrt{9\sin^2 t + 4\cos^2 t})^3}$$

$\therefore$ When $t = 0$, $k = \frac{6}{\sqrt{4}^3} = \frac{3}{4}$
$\therefore$ When $t = \frac{\pi}{2}$, $k = \frac{6}{\sqrt{9}^3} = \frac{2}{9}$
$\because$ Generally, for a circle $x^2 + y^2 = R^2$, we have:

$$r(t) = (R\cos t, R\sin t, 0)$$

$\therefore r'(t) = (-R\sin t, R\cos t, 0), r''(t) = (-R\cos t, -R\sin t, 0)$

$$\therefore k = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} = \frac{R^2}{R^3} = \frac{1}{R}$$

$\therefore$ When $t = 0$, $r(t) = (3, 0, 0)$, the corresponding circle of curvature has radius $\frac{1}{k} = \frac{4}{3}$, and the center is at $(\frac{5}{3}, 0)$, which satisfies the equation:

$$(x - \frac{5}{3})^2 + y^2 = \frac{16}{9}$$

$\therefore$ When $t = \frac{\pi}{2}$, $r(t) = (0, 2, 0)$, the corresponding circle of curvature has radius $\frac{1}{k} = \frac{9}{2}$, and the center is at $(0, -\frac{5}{2})$, which satisfies the equation:

$$x^2 + (y + \frac{5}{2})^2 = \frac{81}{4}$$