Exercise 15.1

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14.

$$\begin{aligned}\iint_R \sqrt{9-y^2}dA &= \int_0^4\int_0^2 \sqrt{9-y^2}dydx \\ &= \int_0^2\int_0^4 \sqrt{9-y^2}dxdy \\ &= 4\int_0^2 \sqrt{9-y^2}dy\end{aligned}$$

Let $y = 3\sin t$, where t, then

$$\begin{aligned}4\int_0^2 \sqrt{9-y^2}dy &= 4\int_0^{\arcsin\frac{2}{3}} 3|\cos t|3\cos t dt \\ &= 36\int_0^{\arcsin\frac{2}{3}} \cos^2 t dt \\ &= 18\int_0^{\arcsin\frac{2}{3}} (1+\cos 2t)dt \\ &= 18(t+\frac{1}{2}\sin 2t)\Big|_0^{\arcsin\frac{2}{3}} \\ &= 18(\arcsin\frac{2}{3}+\frac{2}{3}\cos\arcsin\frac{2}{3}) \\ &= 18(\frac{41.810\pi}{180}+0.4969) \\ &= 22.079\end{aligned}$$

17.

Proof.

$$\begin{aligned}\iint_R f(x,y)dA &= \iint_R k dA \\ &= \int_a^b \int_c^d k dy dx \\ &= \int_a^b k(d-c)dx \\ &= k(b-a)(d-c)\end{aligned}$$

□

18.

Proof. $\because x \in [0, \frac{1}{4}]$, $\therefore \sin \pi x \in [0, \frac{\sqrt{2}}{2}]$

$\because y \in [\frac{1}{4}, \frac{1}{2}]$, $\therefore \cos \pi x \in [0, \frac{\sqrt{2}}{2}]$

$$\therefore 0 \leq \sin \pi x \cos \pi y \leq \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$$

$\therefore$ by the conclusion of Exercise 17,

$$0 \times \frac{1}{4} \times \frac{1}{4} \leq \int_0^{\frac{1}{4}} \int_{\frac{1}{4}}^{\frac{1}{2}} \sin \pi x \cos \pi y dy dx \leq \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}$$

which is equivalent to

$$0 \leq \iint_R \sin \pi x \cos \pi y dA \leq \frac{1}{32}$$

□