Exercise 9.3

Wang Yue from CS Elite Class

June 17, 2021

12. $\frac{dy}{dx} = \frac{\ln x}{xy}, y(1) = 2$

$$\frac{\ln x}{x}dx = \ln x d \ln x = y dy$$

$$\int \ln x d \ln x = \int y dy$$

$$\frac{(\ln x)^2}{2} = \frac{y^2}{2} + C$$

$\because \frac{(\ln 1)^2}{2} = 2 + C$
$\therefore C = -2$

$$\therefore y^2 = (\ln x)^2 + 4$$

14. $y' = \frac{xy \sin x}{y+1}, y(0) = 1$

$$\frac{dy}{dx} = \frac{xy \sin x}{y+1}$$

If $y \neq 0$, then

$$\frac{y+1}{y}dy = x \sin x dx$$

$$\int \frac{y+1}{y}dy = \int x \sin x dx$$

$$y + \ln |y| = -\int x d \cos x = -x \cos x + \sin x + C$$

$\because 1 + \ln 1 = -0 + 0 + C$
$\therefore C = 1$

$$\therefore y + \ln |y| = -x \cos x + \sin x + 1$$

$\because y = 0$ satisfies $\frac{dy}{dx} = \frac{xy \sin x}{y+1} = 0$
$\therefore \begin{cases} y + \ln |y| = -x \cos x + \sin x + 1, & y \neq 0 \\ y = 0, & y = 0 \end{cases}$

20. $f'(x) = f(x)(1 - f(x)), f(0) = \frac{1}{2}$

$$\frac{dy}{dx} = y(1-y)$$

If $y \neq 0$ and $y \neq 1$, then

$$\int \frac{1}{y(1-y)} dy = \int dx$$

$$\int (\frac{1}{y} + \frac{1}{1-y}) dy = \ln|y| + \ln|1-y| = x + C$$

$$e^{\ln|y|} e^{\ln|1-y|} = |y||1-y| = e^C e^x$$

$\because \frac{1}{4} = e^C$
$\therefore |y||1-y| = \frac{1}{4}e^x$
If $y = 0$ or $y = 1$, then $\frac{dy}{dx} = 0$ is satisified
$\therefore \begin{cases} |y||1-y| = \frac{1}{4}e^x, & y \neq 0 \text{ and } y \neq 1 \\ y = 0, & y = 0 \\ y = 1, & y = 1 \end{cases}$

22. $xy' = y + xe^{\frac{y}{x}}$

Let $v = \frac{y}{x}$, then $y = vx$, then

$$y' = \frac{dy}{dx} = \frac{dv}{dx}x + v$$

$$\therefore x\frac{dv}{dx} = e^v$$

$$\int \frac{1}{e^v} dv = \int \frac{1}{x} dx$$

$$-e^{-v} = \ln|x| + C$$

where C is an arbitrary constant

33. $y(x) = 2 + \int_2^x [t - ty(t)]dt$

Taking derivative on the both sides, we get

$$\frac{dy}{dx} = x - xy$$

and the initial condition is $y(2) = 2 + \int_2^2 [t - ty(t)]dt = 2$
If $y \neq 1$, then

$$\int \frac{1}{1-y} dy = \int x dx$$

$$-\ln|1-y| = \frac{x^2}{2} + C$$

$\because -\ln|1-2| = \frac{2^2}{2} + C$
$\therefore C = -2$

$$\therefore |\frac{1}{1-y}| = e^{-2}e^{\frac{x^2}{2}}$$

If $y = 1$, then $\frac{dy}{dx} = 0$
$\therefore \begin{cases} |\frac{1}{1-y}| = e^{-2}e^{\frac{x^2}{2}}, & y \neq 1 \\ y = 1, & y = 1 \end{cases}$

34.

Taking derivative on the both sides, we get

$$\frac{dy}{dx} = \frac{1}{xy}$$

and the initial condition $y(1) = 2$

$$\therefore \int y dy = \int \frac{1}{x} dx$$

Since $x > 0$, then

$$\frac{y^2}{2} = \ln x + C_1$$

$$\therefore y^2 = 2\ln x + C$$

where C_1 and C is arbitrary constants

35.

Taking derivative on the both sides, we get

$$\frac{dy}{dx} = 2x\sqrt{y}$$

and the initial condition $y(0) = 4$
If $y > 0$, then

$$\int \frac{1}{2\sqrt{y}} dy = \int x dx$$

$$\sqrt{y} = \frac{x^2}{2} + C_1$$

$$y = \frac{x^4}{4} + C$$

$\because 4 = 0 + C$
$\therefore C = 4$

$$\therefore y = \frac{x^4}{4} + 4$$

If $y = 0$, then $\frac{dy}{dx} = 0$

$\therefore \begin{array}{ll} y = \frac{x^4}{4} + 4, & y \neq 0 \\ y = 0, & y = 0 \end{array}$