Exercise 14.1

Wang Yue from CS Elite Class

April 3, 2021

11.

(a) $f(1,1,1) = \sqrt{1} + \sqrt{1} + \sqrt{1} + \ln(4 - 1 - 1 - 1) = 3$

(b) $\because f(x,y,z) = \sqrt{x} + \sqrt{y} + \sqrt{z} - \ln(4 - x - y - z)$ is defined as

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ z \geq 0 \\ 4 - x - y - z > 0 \end{cases}$$

$\therefore$ The domain of f is

$$\{(x,y,z)|4 - x - y - z > 0 \text{ and } x \geq 0 \text{ and } y \geq 0 \text{ and } z \geq 0\}$$
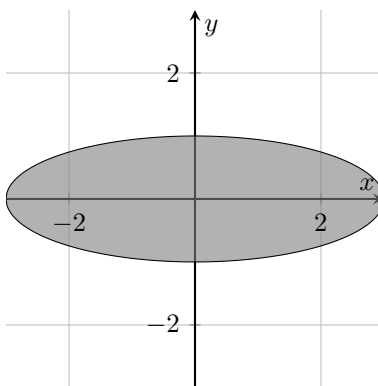
15.

$\because f(x,y) = \ln(9 - x^2 - 9y^2)$ is defined as

$$9 - x^2 - 9y^2 > 0 \qquad \text{or} \qquad \frac{x^2}{9} + y^2 < 1$$

$\therefore$ The domain of f is

$$\{(x,y)|\frac{x^2}{9} + y^2 < 1\}$$

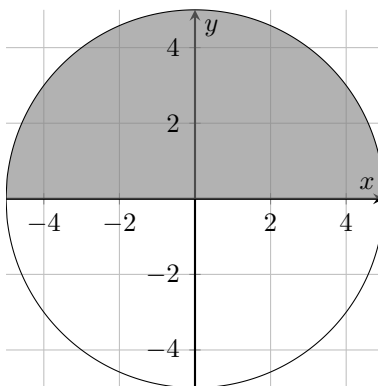and the graph of $D(f)$ is illustrated below.

18.

$\because f(x, y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}$ is defined as

$$\begin{cases} y \geq 0 \\ 25 - x^2 - y^2 \geq 0 \end{cases}$$

$\therefore$ The domain of f is

$$\{(x, y) | x^2 + y^2 \leq 25 \text{ and } y \geq 0\}$$

and the graph of $D(f)$ is illustrated below.
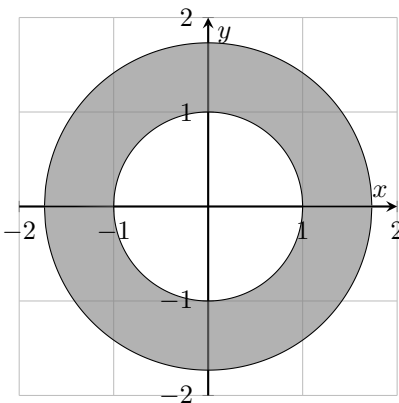

20.

$\because f(x, y) = \arcsin(x^2 + y^2 - 2)$ is defined as

$$-1 \leq x^2 + y^2 - 2 \leq 1$$

$\therefore$ The domain of f is

$$\{(x, y) | 1 \leq x^2 + y^2 \leq 3\}$$

and the graph of $D(f)$ is illustrated below.

2

21.

∵ $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$ is defined when

$$1 - x^2 - y^2 - z^2 \geq 0$$

∴ The domain of f is

$$\{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$$

and the graph of $D(f)$ is illustrated below.