Question 1 A

In[96]:=
```
Gfunction[function_, {x_, a_, order_}] :=
 Sum[((D[function, {x, n}]) /. x → a) * ((x - a) ^ n) / (n!), {n, 0, order}]
```

Question 1 B

In[98]:=
```
Gfunction[Sin[x], {x, 0, 10}]
```

Out[98]= $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362\,880}$

In[152]:=
```
o = Series[Sin[x], {x, 0, 10}]
```

Out[152]= $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362\,880} + O[x]^{11}$

Question 2 B (II)

In[100]:=
```
Gfunction[Cos[x], {x, 0, 10}]
```

Out[100]= $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40\,320} - \frac{x^{10}}{3\,628\,800}$

In[101]:=
```
Series[Cos[x], {x, 0, 10}]
```

Out[101]= $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40\,320} - \frac{x^{10}}{3\,628\,800} + O[x]^{11}$

Question 2 B (III)

In[102]:=
```
Gfunction[Exp[x], {x, 0, 10}]
```

Out[102]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40\,320} + \frac{x^9}{362\,880} + \frac{x^{10}}{3\,628\,800}$

In[103]:=
```
Series[Exp[x], {x, 0, 10}]
```

Out[103]= $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40\,320} + \frac{x^9}{362\,880} + \frac{x^{10}}{3\,628\,800} + O[x]^{11}$

Question 2 B (IV)

In[104]:=
```
Gfunction[(1 - x) ^ -1, {x, 0, 10}]
```

Out[104]= $1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

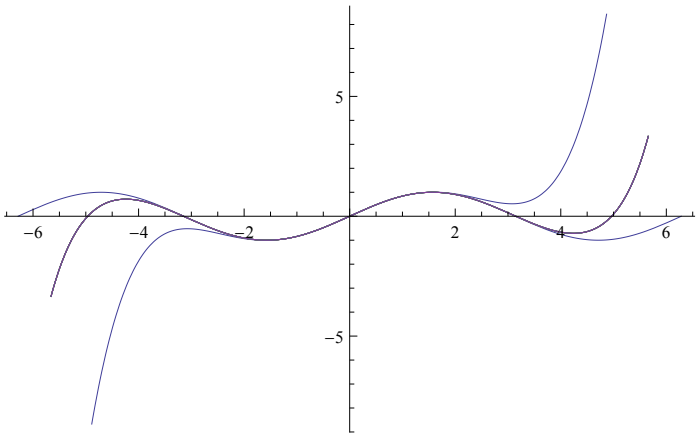
In[105]:=
```
Series[(1 - x) ^ -1, {x, 0, 10}]
```

Out[105]= $1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + O[x]^{11}$

Question 1 C

In[172]:=
```
Show[Plot[Gfunction[Sin[x], {x, 0, 5}] // Evaluate, {x, -2 Pi, 2 Pi}],
 Plot[Sin[x], {x, -2 Pi, 2 Pi}], Plot[
  Evaluate[Table[Normal[Series[Sin[x], {x, 0, 10}]], {n, 10}]], {x, -2 Pi, 2 Pi}]]
```

Out[172]=



Question 1 D

```
D[(1 + x) ^ 2, {x, 1}]
```

$2\,(1+x)$

```
D[(1 + x) ^ 2, {x, 2}]
```

2

```
D[(1 + x) ^ 2, {x, 3}]
```

0

In[108]:=
```
Gfunction[(1 + x)^2, {x, 0, 2}]
```

Out[108]= $1 + 2\,x + x^2$

Question 1 E (I)

In[109]:=
```
Gfunction[(1 + x)^(1/2), {x, 0, 10}]
```

Out[109]= $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5\,x^4}{128} + \frac{7\,x^5}{256} - \frac{21\,x^6}{1024} + \frac{33\,x^7}{2048} - \frac{429\,x^8}{32\,768} + \frac{715\,x^9}{65\,536} - \frac{2431\,x^{10}}{262\,144}$

In[110]:=
```
Series[(1 + x)^(1/2), {x, 0, 10}]
```

Out[110]= $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5\,x^4}{128} + \frac{7\,x^5}{256} - \frac{21\,x^6}{1024} + \frac{33\,x^7}{2048} - \frac{429\,x^8}{32\,768} + \frac{715\,x^9}{65\,536} - \frac{2431\,x^{10}}{262\,144} + O[x]^{11}$

Question 1 E (II)

In[111]:=
```
Gfunction[(1 - x)^(1/2), {x, 0, 10}]
```

Out[111]= $1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} - \frac{5\,x^4}{128} - \frac{7\,x^5}{256} - \frac{21\,x^6}{1024} - \frac{33\,x^7}{2048} - \frac{429\,x^8}{32\,768} - \frac{715\,x^9}{65\,536} - \frac{2431\,x^{10}}{262\,144}$

In[114]:=
```
Series[(1 - x)^(1/2), {x, 0, 10}]
```

Out[114]= $1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} - \frac{5\,x^4}{128} - \frac{7\,x^5}{256} - \frac{21\,x^6}{1024} - \frac{33\,x^7}{2048} - \frac{429\,x^8}{32\,768} - \frac{715\,x^9}{65\,536} - \frac{2431\,x^{10}}{262\,144} + O[x]^{11}$

Question 1 E (III)

In[112]:=
```
Gfunction[(1 - x)^(1/3), {x, 0, 10}]
```

Out[112]=
$$1 - \frac{x}{3} - \frac{x^2}{9} - \frac{5\,x^3}{81} - \frac{10\,x^4}{243} - \frac{22\,x^5}{729} - \frac{154\,x^6}{6561} - \frac{374\,x^7}{19\,683} - \frac{935\,x^8}{59\,049} - \frac{21\,505\,x^9}{1\,594\,323} - \frac{55\,913\,x^{10}}{4\,782\,969}$$

In[115]:=
```
Series[(1 - x)^(1/3), {x, 0, 10}]
```

Out[115]=
$$1 - \frac{x}{3} - \frac{x^2}{9} - \frac{5\,x^3}{81} - \frac{10\,x^4}{243} - \frac{22\,x^5}{729} - \frac{154\,x^6}{6561} - \frac{374\,x^7}{19\,683} - \frac{935\,x^8}{59\,049} - \frac{21\,505\,x^9}{1\,594\,323} - \frac{55\,913\,x^{10}}{4\,782\,969} + O[x]^{11}$$

Question 1 E (IV)

In[113]:=
```
Gfunction[CubeRoot[1 + x^4], {x, 0, 10}]
```

Out[113]=
$$1 + \frac{x^4}{3} - \frac{x^8}{9}$$

In[119]:=
```
Series[CubeRoot[1 + x^4], {x, 0, 10}]
```

Out[119]=
$$1 + \frac{x^4}{3} - \frac{x^8}{9} + O[x]^{11}$$