

$$1. a. f(x) = \sqrt{x+4}$$

$$\begin{aligned}f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x+4+\Delta x} - \sqrt{x+4}}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x+\Delta x+4} - \sqrt{x+4}}{\Delta x} \cdot \frac{\sqrt{x+\Delta x+4} + \sqrt{x+4}}{\sqrt{x+\Delta x+4} + \sqrt{x+4}} \\&= \lim_{\Delta x \rightarrow 0} \frac{x+\Delta x+4 - (x+4)}{\Delta x(\sqrt{x+\Delta x+4} + \sqrt{x+4})} \\&= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x(\sqrt{x+\Delta x+4} + \sqrt{x+4})} \\&= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x+4} + \sqrt{x+4}} \\&= \frac{1}{2\sqrt{x+4}}\end{aligned}$$

$$b. f(x) = \frac{3}{x}$$

$$\begin{aligned}f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{\frac{3}{x+\Delta x} - \frac{3}{x}}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{\frac{3x - 3(x+\Delta x)}{x(x+\Delta x)}}{\Delta x} \\&= \lim_{\Delta x \rightarrow 0} \frac{-3\Delta x}{x(x+\Delta x)} \\&= \lim_{\Delta x \rightarrow 0} \frac{-3\Delta x^2}{x(x+\Delta x)\Delta x} \\&= -\frac{3}{x^2}\end{aligned}$$

$$2. \quad a. \quad f(x) = 3x^3 - \frac{4}{x^2}$$

$$\begin{aligned}f'(x) &= 3 \cdot 3x^2 - 4 \cdot (-2) \cdot x^{-3} \\&= 9x^2 + 8 \cdot \frac{1}{x^3} \\&= 9x^2 + \frac{8}{x^3}\end{aligned}$$

$$b. \quad f(x) = (4-x^2)^3$$

$$\begin{aligned}f'(x) &= 3 \cdot (4-x^2)^2 \cdot (0-2x) \\&= 3(4-x^2)^2 \cdot (-2x) \\&= -6x(4-x^2)^2\end{aligned}$$

$$c. \quad f(x) = e^{\sin(x)}$$

$$\begin{aligned}f'(x) &= e^{\sin(x)} \cdot \frac{d}{dx}[\sin(x)] \\&= \cos(x) e^{\sin(x)}\end{aligned}$$

$$d. \quad f(x) = \ln(x+2)$$

$$\begin{aligned}f'(x) &= \frac{1}{x+2} \cdot \frac{d}{dx}(x+2) \\&= \frac{1}{x+2} \cdot 1 \\&= \frac{1}{x+2}\end{aligned}$$

$$e. \quad f(x) = x^2 \cos(x) + x \tan(x)$$

$$\begin{aligned}f'(x) &= 2x \cdot \cos(x) + x^2 \cdot (-\sin(x)) + 1 \cdot \tan(x) + x \cdot \sec^2(x) \\&= \tan(x) - x^2 \sin(x) + x \sec^2(x) + 2x \cos(x)\end{aligned}$$

$$f. f(x) = \sqrt{3x^2 + 2}$$

$$\begin{aligned}f'(x) &= \frac{1}{2}(3x^2 + 2)^{-\frac{1}{2}} \cdot \frac{d}{dx}(3x^2 + 2) \\&= \frac{1}{2}(3x^2 + 2)^{-\frac{1}{2}} \cdot 6x \\&= \frac{3x}{\sqrt{3x^2 + 2}} \\&= \frac{3x\sqrt{3x^2 + 2}}{3x^2 + 2}\end{aligned}$$

$$g. f(x) = \frac{x}{4} \sin^{-1}(x)$$

$$\begin{aligned}f'(x) &= \frac{1}{4} \cdot (\arcsin(x) + x \cdot \frac{1}{\sqrt{1-x^2}}) \\&= \arcsin(x) + \frac{x}{\sqrt{1-x^2}}\end{aligned}$$

$$h. xy = (y+2) + xy \sin(x)$$

$$(2x)y' + x^2y' = y \sin(x) + \cos(x) \cdot xy$$

$$x^2y' = y \sin(x) + \cos(x) \cdot xy - (2x)y'$$

$$y' = \frac{y \sin(x) + y \cos(x) \cdot x - 2y}{x^2}$$

$$y' = \frac{y(\sin(x) + \cos(x) \cdot x - 2)}{x^2}$$

3. a. hour 39.

$$\begin{aligned}f'(39) &= \frac{f(39+10) - f(39-10)}{(2 \times 10)} \\&= \frac{f(49) - f(29)}{20} \\&= \frac{145 - 115}{20} \\&= 1.5\end{aligned}$$

physical meaning: The speed of wind is increasing at 1.5 mph/hour at the time of 39 hours.

b. hour 83

$$\begin{aligned}f'(83) &= \frac{f(83+2) - f(83-2)}{(2 \times 2)} \\&= \frac{f(85) - f(81)}{4} \\&= \frac{95 - 125}{4} \\&= -7.5\end{aligned}$$

physical meaning: The speed of wind is decreasing at 7.5 mph/hour at the time of 83 hours