Name : Guizhi Xu

CWID: 20008710

P1: draw top 7 cards from 52-card deck.

a) 7 cards include 3 aces.

b) 7 cards include 2 kings

c) 7 cards include 3 aces, or 2 kings, or both.

a) $P(3A)$: 7 cards include 3 aces

$$P(3A) = \frac{C_4^3 \cdot C_{48}^4}{C_{52}^7} = \frac{C_4^1 \cdot C_{48}^4}{C_{52}^7}$$

$$\because C_n^m = \frac{n!}{m!(n-m)!}$$

$$\therefore P(3A) = \frac{4 \cdot 194580}{133784560} = 0.00581771$$

b) $P(2K)$: 7 cards include 2 kings

$$P(2K) = \frac{C_4^2 \cdot C_{48}^5}{C_{52}^7}$$

$$= \frac{6 \cdot 1712304}{133784560} = 0.07679379$$

c) $P(3A2KB)$ = 7 cards include 3 aces, or 2 kings, or both

$$P(3A2KB) = P(3A \cup 2K) = P(3A) + P(2K) - P(3A \cap 2K)$$

$$= 0.00581771 + 0.07679379 - \left(\frac{C_4^3 \cdot C_4^2 \cdot C_{44}^2}{C_{52}^7}\right)$$

$$= 0.0826115 - \frac{24 \cdot 946}{133784560}$$

$$= 0.0826115 - 0.00016971$$

$$= 0.08244179$$

P2: 2n+1 coins, Bob tosses n+1 coins

Alice tosses n coins (ramaining)

probablity Bob got more heads than Alice is $\frac{1}{2}$

B: Bob got more heads than Alice.

A: Alice got more heads than Bob.

W: Bob got more heads than Alice, after n rounds

L: Alice got more heads the Bob, after n rounds

T: Bob and Alice got same heads.

$\because P(B|W) = 1$ $\quad \therefore P(B) = P(W) \cdot P(B|W) + P(L) \cdot P(B|L) + P(T) \cdot P(B|T)$

$P(B|L) = 0$ $\quad = P(W) + \frac{1}{2} \cdot P(T)$

$P(B|T) = \frac{1}{2}$ $\quad = \frac{1}{2} \cdot [P(W) + P(L) + P(T)]$

$= \frac{1}{2}$

P3: 3 coins, both heads, both tails, one heads one tails

pick one, it's heads, probabillty the opposite face is tails?

TH: both heads was chosen

H: tosses heads

THH: both heads was chosen and tosses heads

RS: the opposite face is tails.

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$\because P(THH) = \frac{1}{3}$

$P(H) = \frac{1}{2}$

$\therefore P = 1 - P(TH \mid H)$

$= 1 - \left(\frac{P(THH)}{P(H)}\right)$

$= 1 - \frac{\frac{1}{3}}{\frac{1}{2}}$

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$

P_4: k jars, m white and n black balls.

chose a ball from jar 1 to jar 2 ......

Show the probability of first ball is white is same as last ball

~~S: last ball is white~~

S_k: K ball is white

S_j: j ball is white

$P(S_j)\ \cancel{P(S_k)} = \frac{m}{m+n}.$

$\therefore P(S_k) = P(A_{k-1})P(A_k \mid A_{k-1}) + P(A_{k-1}^c)P(A_k \mid A_{k-1}^c)$

$= \frac{m}{m+n} \cdot \frac{m+1}{m+n+1} + \frac{n}{m+n} \cdot \frac{m}{m+n+1}$

$= \left(\frac{m+1}{m+n+1} + \frac{n}{m+n+1}\right) \cdot \frac{m}{m+n}$

$= 1 \cdot \frac{m}{m+n}$

$= \frac{m}{m+n}$

$\therefore P(S_1) = P(S_j) = P(S_k) = \frac{m}{m+n}$

P5 : n plants   plant i fails probability p_i

(a) probability city black-out (at least 1 plant works)

(b) probability city black-out (at least 2 plants works)

(a) : A: city black-out (1 plant works)

$$P(A) = \prod_{i=1}^{n} p_i$$

(b) : B: city black-out (2 plants works)

$$P(B) = P(A) + \sum_{i=1}^{n} \left((1-p_i) \prod_{j \neq i} p_j \right)$$

$$= \prod_{i=1}^{n} p_i + \sum_{i=1}^{n} \left((1-p_i) \prod_{j \neq i} p_j \right)$$