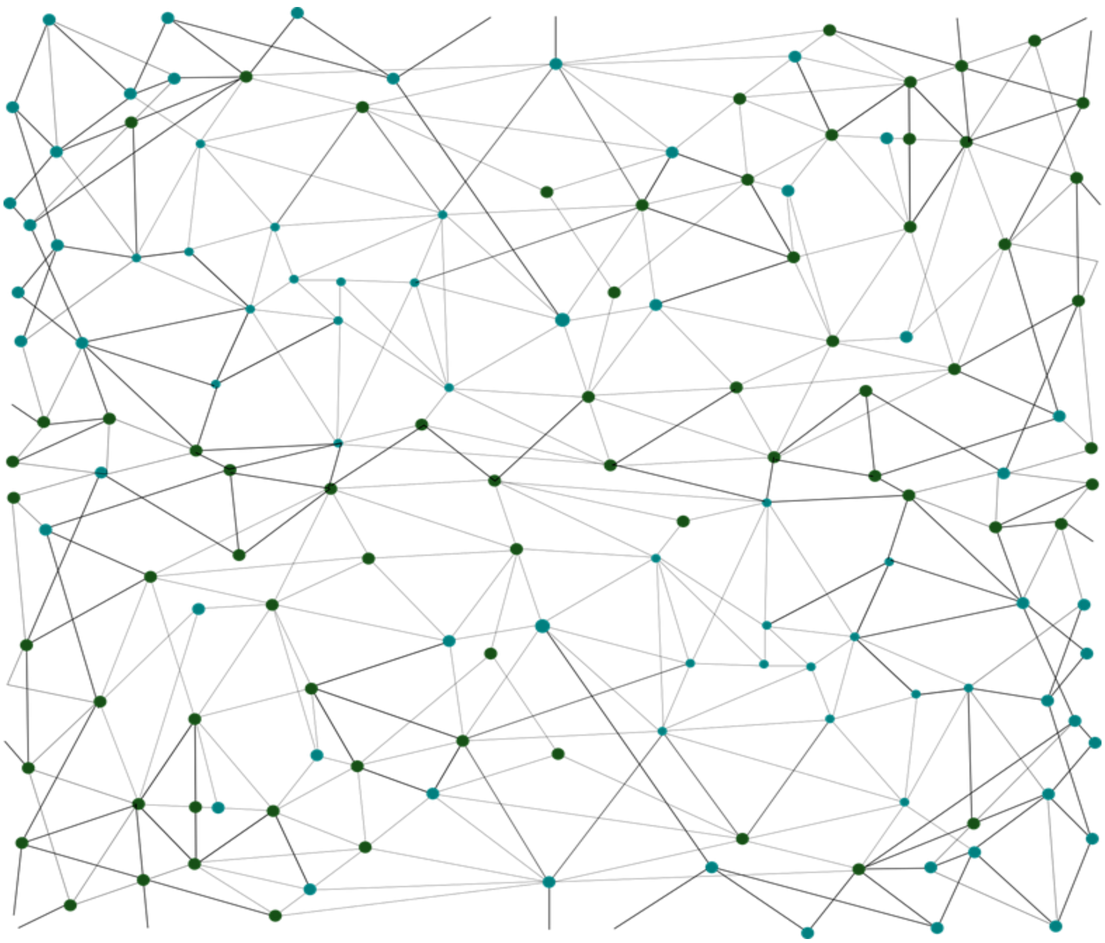
STAT 471

Predict Inflation Based on Consumer Opinions/Sentiments Using a Voter Model

Group Report



Team Hyperinflation

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Introduction

Expected inflation is one of the most important metrics a central bank must be able to measure and keep track of as expected inflation can cause inflation in the short run and expected inflation has influence on real interest rates (the Fisher equation r=i-e) which many other economic mechanics depend on. To see what future inflation may be, central banks must be able to measure inflation expectations accurately and with that they must be able to keep expectations stable to keep real interest rates and economic growth stable as growth relies on real interest rates. E.g., In the New Keynesian three equation model, real interest rate is defined as:

$$r = \theta + \sigma g + n$$

Where r is the real interest rate, is the representative households discount rate, is the constant relative risk aversion parameter, g is the exogenous technological growth rate, and n is population growth rate. It is common to take = 1 leading to

$$r = \theta + g + n = \theta + GDPgrowth \qquad (1)$$

So, we see that from the Fisher equation and (1), when inflation expectations are low, GDPgrowth will be low (leaving i the nominal interest rate fixed) and vice versa. So, r is linked to many economic mechanics such as consumption/saving, exchange rates, financial sector, etc. So, with more accurate measurements of inflation expectations will make more accurate estimates of real interest rates and possibly better estimates of future inflation.

With our model, we look at how networks affect inflation expectations and how those inflation expectations influence other individuals. With this information, central banks can get a better idea of the dynamics of inflation expectations and inflation so they can make better policy decisions. With random graphs we simulate “networks” of people sharing their inflation expectations to their connections via a Voter Model interacting particle system. The graphs simulate transfer of information of individuals inflation expectations throughout the network which causes the inflation expectations to change.

Model

Review of Model:

Initialize: Generate random graph and set initial inflation expectations

Let $A_{i,j}$ be its relation matrix for the random graph and let $N \in \mathbb{N}$, $P = \{1,2, \dots, N\}$ be the population on the random graph.

Set $\pi_{i,0} \sim N(2,\ 0.4115)\ \forall i \in P$ is the initial expectations.

For each $t > 0$, do:

1. Inflation update:

$B_j = \{i \in P\colon A_{i,j} = 1, i \neq j\}$ is the set of neighbors for person $j \in P$.

Calculate:

$$\pi_{j,t}^{\pm} = \frac{1}{|B_j|}\sum_{i \in B_j} \pi_{i,t-1}^{\pm} + \varepsilon_{j,t}\ ,\ t > 0,$$

$$\varepsilon_{j,t} = \begin{cases} \xi_{j,t}, & p = 0.7 \\ -\xi_{j,t}, & p = 0.3 \end{cases}$$

$$\xi_{j,t} \sim N(0,\ 0.4115).$$

2. After all expectations are updated, calculate:

$\bar{\pi}_t = \frac{1}{|P|}\sum_{i \in P} \pi_{j,t}^{\pm}$, and $f(\bar{\pi}_t) = f(\frac{1}{|P|}\sum_{i \in P} \pi_{j,t}^{\pm})$, $f(x)$ is a regression model

Overview of Model Construction:

Here we will go over how we decided what probabilities and distributions to use. The $N(\mu = 2, \sigma = 0.4115)$ Initial inflation expectations were derived using 5-year Breakeven inflation data from FRED by just calculating summary statistics from this data and using them for the normal distribution. Then the probabilities for were derived using twitter data and some basic natural language processing.

We chose $N(\mu = 0, \sigma = 0.4115)$ just to keep the inflation expectations around 2% throughout time. Finally, for the regression model, we downloaded 5-year breakeven inflation from and non-core CPI from FRED. With the breakeven inflation data, we derived 1-year inflation expectations and fitted a ridge regression model to those expectations and the yearly CPI data.

Simulation And Results

For normally distributed initial inflation expectations, we will approximate the distribution of an individual's inflation expectation by taking $|B_j| = \alpha = \left\lfloor \frac{M}{n} \right\rfloor$:

$$\pi_{j,1}^{\pm} = \frac{1}{\alpha}\sum_{i=1}^{\alpha} \pi_{i,0}^{\pm} + \varepsilon_{j,1} = N\left(\mu = 2, \sigma^2 = \frac{0.4115^2}{\alpha}\right) + N(\mu = 0, \sigma^2 = 0.4115^2)$$

$$= N\left(\mu = 2, \sigma^2 = \frac{0.4115^2}{\alpha} + 0.4115^2\right)$$

Here we approximate the distribution at time k:

$$\pi_{j,2}^{\pm} = \frac{1}{\alpha}\sum_{i=1}^{\alpha} \pi_{i,1}^{\pm} + \varepsilon_{j,2} = N\left(\mu = 2, \sigma^2 = \frac{0.4115^2}{\alpha^2} + \frac{0.4115^2}{\alpha}\right) + N(\mu = 0, \sigma^2 = 0.4115^2)$$

$$= N\left(\mu = 2, \sigma^2 = \frac{0.4115^2}{\alpha^2} + \frac{0.4115^2}{\alpha} + 0.4115^2\right)$$

....

$$\pi_{j,k}^{\pm} = \frac{1}{\alpha}\sum_{i=1}^{\alpha} \pi_{i,k-1}^{\pm} + \varepsilon_{j,k}$$

$$= N\left(\mu = 2, \sigma^2 = \frac{0.4115^2}{\alpha^k} + \frac{0.4115^2}{\alpha^{k-1}} + \cdots + \frac{0.4115^2}{\alpha}\right) + N(\mu = 0, \sigma^2 = 0.4115^2)$$

$$= N\left(\mu = 2, \sigma^2 = \frac{0.4115^2}{\alpha^k} + \frac{0.4115^2}{\alpha^{k-1}} + \cdots + \frac{0.4115^2}{\alpha} + 0.4115^2\right)$$

$$\rightarrow N\left(\mu = 2, \sigma^2 = \frac{0.4115^2}{1 - (\frac{1}{\alpha})}\right)$$

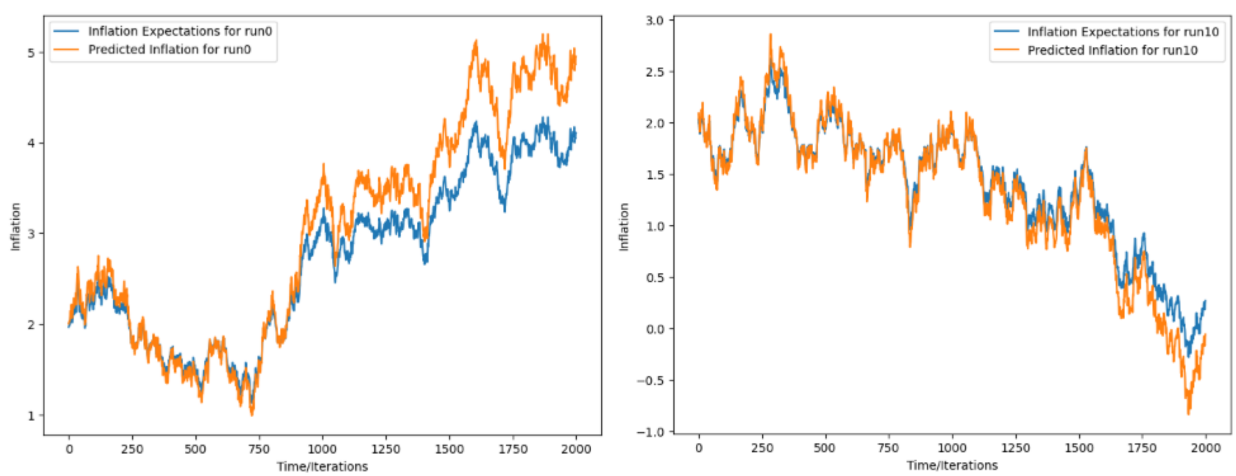
As $k \rightarrow \infty$

So $\pi^{\pm}_{j,k} \sim N(\mu = 2, \sigma^2 = \gamma)$, where k = # of iterations, and $\gamma = \frac{0.4115^2}{1-(\frac{1}{\alpha})}$This is an approximation due to the randomness of the random graph. So, we should expect inflation expectations to follow the described normal distribution as we will continually be adding normals to normals.

Simulations:

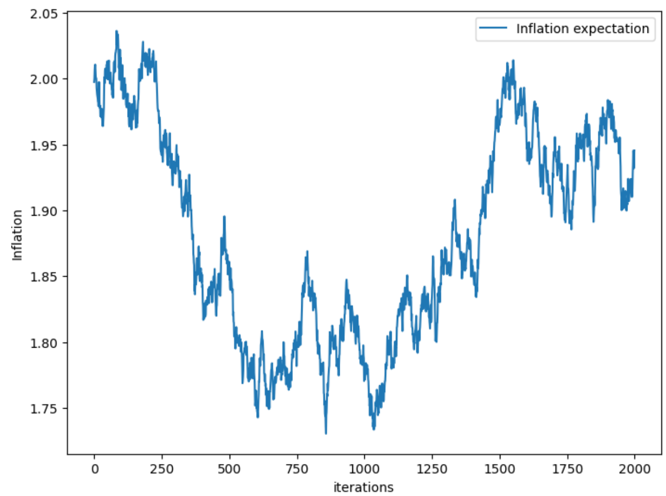
We first simulate with population = 100, edges = 400, 2000 iterations, and 50 random graphs and plot some of the 50 runs individually (one plot is a run using one random graph) and look at some summary statistics to get an idea of what's going on.

Inflation Expectations Generated From 1-st and 10-th Random Graph

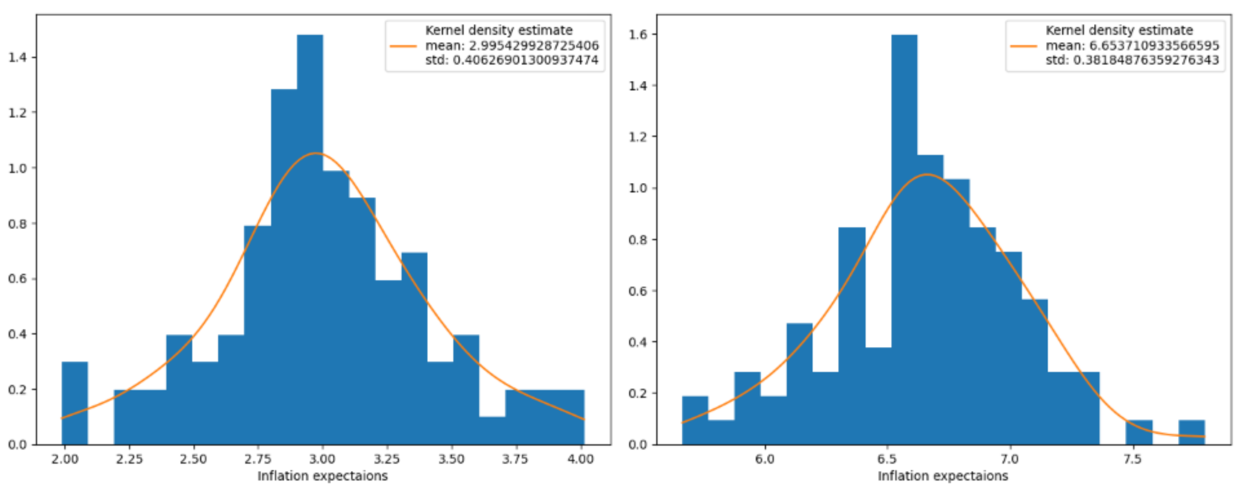


	Run 1	Run 10
Mean	2.69115897037530	1.4133814775433589
Sd	0.8848530385152594.	0.6083020301209373.

Average Inflation Expectation Per Random Graph For Each Iteration



Inflation Expectation Distribution Before (L) And after ~5000 iterations (R)



For these figures, the model was run with 100 people, 4000 edges, and 5000 iterations. So, we see in the figures that the inflation expectations do indeed follow the normal distribution. In this case for 5000 iterations, they should follow an $N(2, 0.4167)$ distribution.

Biased initial inflation expectations:

We now explore what happens to our model if the initial inflation expectations are biased. I.e., the distribution of initial inflation expectations are skewed to higher expectations for example. To accomplish this, we take $\pi_{j,i} \sim exponential(2)$ distribution $\forall i \in P$. Then we calculate 8-i,0 to skew the distribution to higher inflation expectations (8 is chosen to get inflation expectations into a reasonable range).

For our exponential initial expectations, we have:

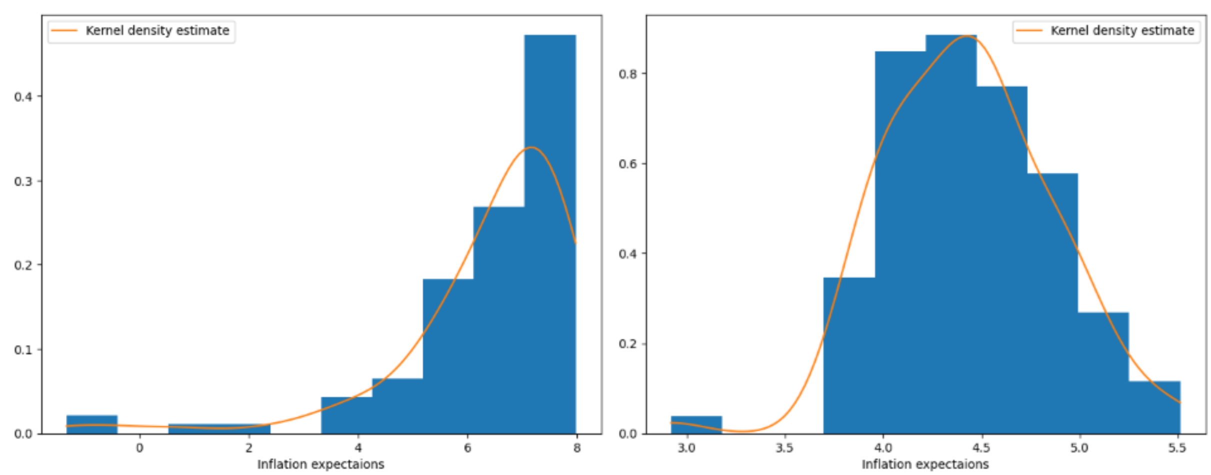
$$\pi_{j,1}^{\pm} = \frac{1}{|B_j|} \sum_{i \in |B_j|} (8 - \pi_{i,0}^{\pm}) + \varepsilon_{j,1}$$

$$= 8 - Gamma\left(\alpha = |B_j|, \beta = \frac{2}{|B_j|}\right) + Gamma\left(\alpha = \frac{1}{2}, \beta = 2(0.4115)^2\right) + 2$$

So, we should expect our inflation expectations to follow some sort of gamma distribution.

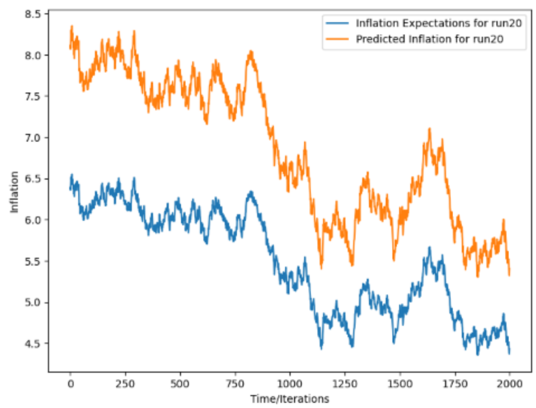
We run the voter model with population = 100, m=4000, iterations=2000, and runs=50:

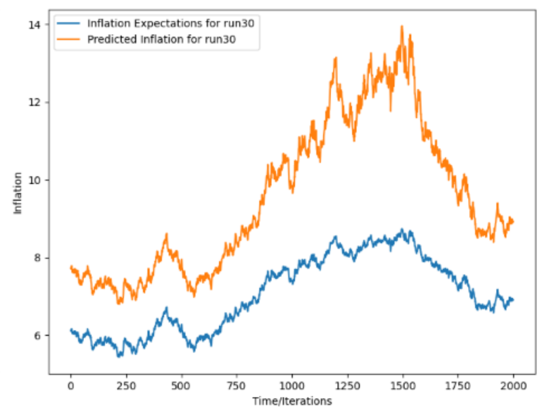
Distribution of Initial (left) And Final (right) Expectations for Graph 20



These figures show the inflation expectations distribution that is observed in a graph. As we see in the figures, inflation expectations settle to approximately a gamma distribution.

Inflation Expectations Time Series for Skewed Initial Expectations



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Mean of inflation expectations for run 20: 5.475670960710492

Standard deviation for run 20: 0.6267449076172059.
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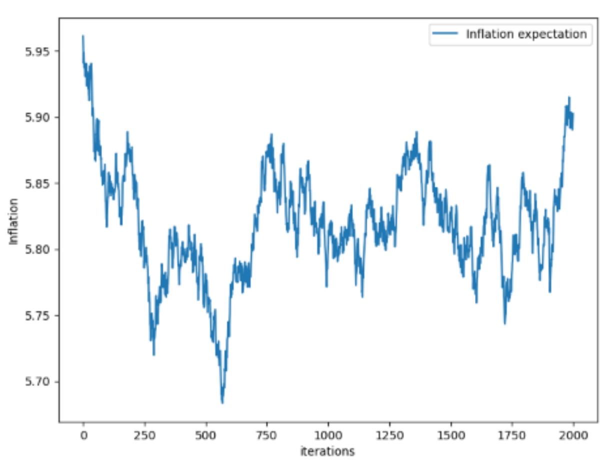
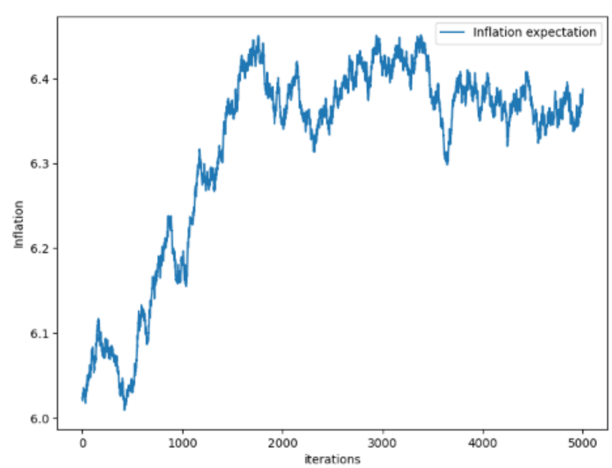
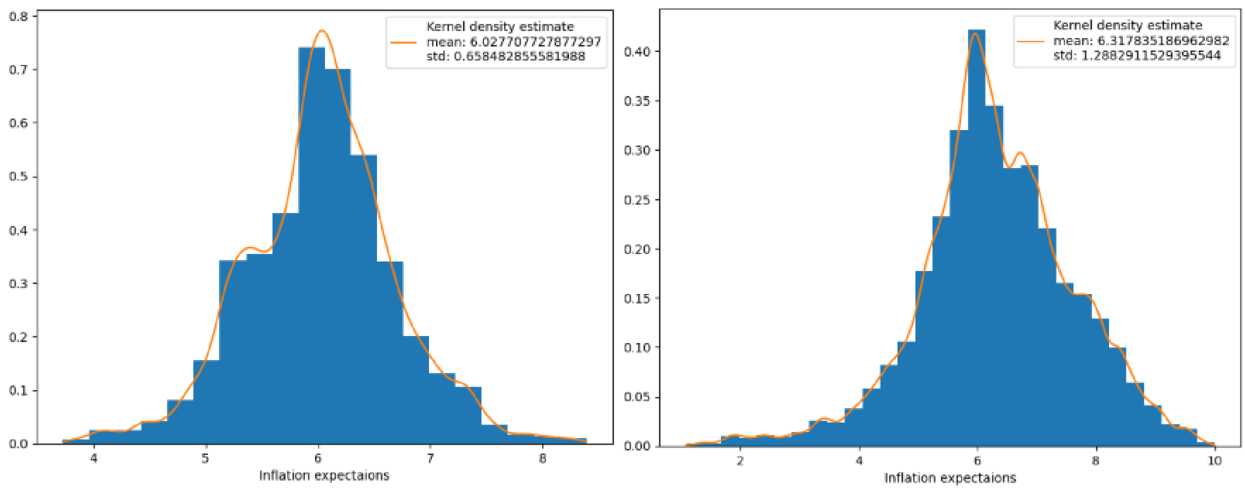
Average Inflation Expectation Per Random Graph for Each Iteration





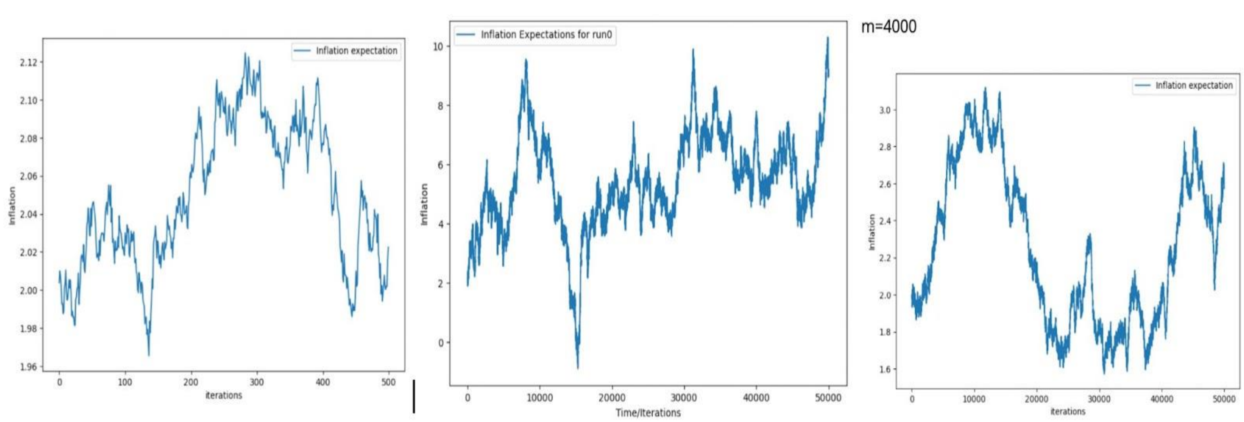
We see that the behavior of the inflation expectations look a bit different compared to the normal distributed inflation expectations.

Densities For All Inflation Expectations For 2 Runs of Runs of VM



This figure shows the distribution of all inflation expectations generated by all the random graphs which looks maybe gamma or t-distributed or possibly a mixture model due to the “humps”.

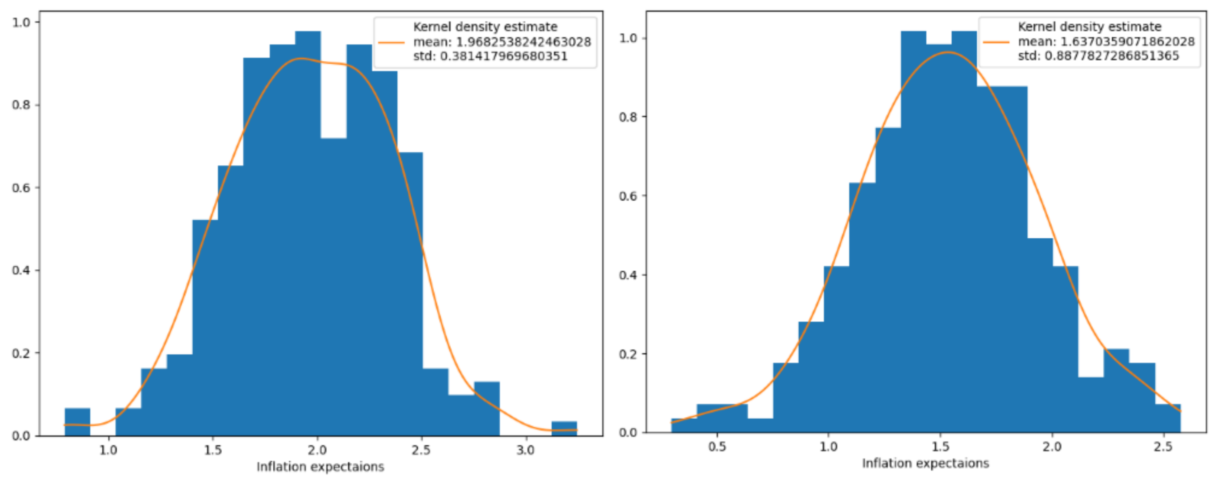
Long Run Behavior



On all three of these figures, the voter models are run using 100 nodes/people. On the left figure, voter models are run in iterations of 500, the middle is 50000-per random graph, and on the right is also 50000 iterations but we increased the number of edges between nodes to be 4000. The other two prior figures are run with only 200 edges.

As the graphs demonstrate, no matter how many runs we have, all the graphs do not follow any stationary distribution. They look like they are randomly generated, it's mostly likely because we randomize initial data which follows the normal distribution $N(\mu = 2, \sigma = 0.4115)$, and error term which is also follows the normal distribution $N(\mu = 0, \sigma = 0.4115)$.

Initial (Left) and Final (Right) Expectation Distributions After 50000 Iterations:



As we can see in the figures, the distribution of inflation expectations follows a normal distribution no matter how many iterations which was expected. Throughout time the distributions variance remains approximately the same while the mean changes causing the distributions to shift accordingly.

Analysis And Answering Our Initial Question(s)

Steady State:

Since we have observed that throughout time, the inflation expectations follow a normal distribution so we will guess our steady state is a normal distribution. With that in mind, for our model, for each node j in the random graph, we will approximate the behavior of the values the node takes with B_t^j, standard Brownian motion since the values are pure noise from a normal distribution. Since every node takes on B_t^j, there is no stationary distribution since B_t^j is pure noise which we show:

For a stationary distribution, we need to find the adjoint operator from the (L, D)-martingale problem:

$$\langle Lf(X_s), P_s \rangle = \langle f(X_s), L^* P_s \rangle, \quad X_s \sim P_s$$

If we are at a stationary solution $\pi(x)$, the martingale problem with expectations will yield:

$$\langle f, L^* \pi \rangle = 0$$

Or

$$L^* \pi(x) = 0$$

For $f \in D$ for some D.

Now for the $(\frac{\Delta}{2}, \delta_0)$ martingale problem for B_t^j,

$$f(B_t^j) - f(B_0^j) - \int_0^t \frac{\Delta}{2} f(B_s^j) ds$$

Is a $\mathcal{F}_{i,t} = \sigma(B_s, s \leq t)$ martingale where Δ is the Laplacian.

From taking expectations we get:

$$\langle f(B_t^j), P_t\rangle - \langle f(B_0^j), P_t\rangle - \int_0^t \langle \frac{\Delta}{2} f(B_s^j), P_s\rangle\, ds = 0$$

So, with $\langle \frac{\Delta}{2} f(B_s^j), P_s\rangle$, we find the adjoint operator:

$$\langle \frac{\Delta}{2} f(x), P_s\rangle = \int \frac{\partial^2}{\partial x^2}\{f\,(x)\}P_s(x)dx$$

$$= -\int \frac{\partial}{\partial x}\{f\,(x)\}\frac{\partial}{\partial x}\{P_s(x)\}dx$$

$$= \int f\,(x)\frac{\partial^2}{\partial x^2}\{P_s(x)\}dx$$

$$= \langle f(x), \frac{\Delta}{2} P_s(x)\rangle$$

The adjoint operator is:

$$L^* = \frac{\Delta}{2}$$

We will guess the stationary solution is $\pi(x) = \frac{1}{t\sqrt{2\pi}}\exp\,(\frac{(x-\mu)^2}{2t^2})$:

$$L^*\pi(x) = \frac{\Delta}{2}\pi(x) = \frac{1}{2}\frac{\partial^2 \pi}{\partial x^2} = \frac{1}{2t\sqrt{2\pi}}\exp\left(\frac{(x-\mu)^2}{2t^2}\right)\frac{(x-\mu)^2}{4t^2} + \frac{1}{2t^3\sqrt{2\pi}}\exp\left(\frac{(x-\mu)^2}{2t^2}\right)$$

$$\rightarrow \frac{1}{2t\sqrt{2\pi}}\exp\left(\frac{(x-\mu)^2}{2t^2}\right)\frac{(x-\mu)^2}{4t^4} + \frac{1}{2t^3\sqrt{2\pi}}\exp\left(\frac{(x-\mu)^2}{2t^2}\right) = 0$$

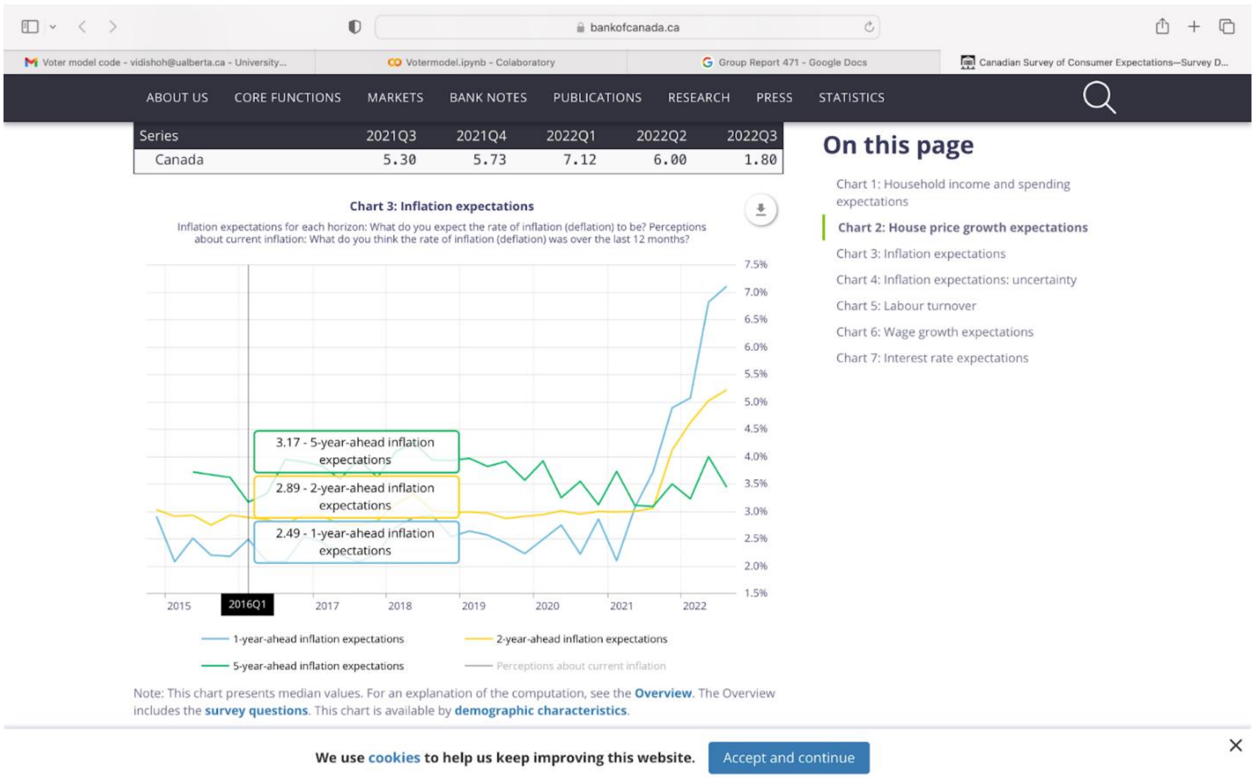
This is impossible since $\frac{1}{2t^3\sqrt{2\pi}}\exp\left(\frac{(x-\mu)^2}{2t^2}\right) > 0$ for all $t > 0$, and if $x = \mu$, we have:

$$\frac{1}{2t^3\sqrt{2\pi}} = 0$$

Which is impossible. So, we do not have a stationary distribution.

Real World Test:

We ran our code using the Bank of Canada 2-year inflation expectation for Jan 2016, to estimate the inflation expectation for Jan 2018. We used the data from their report[1], to simulate the possible inflation rate. The data used were as follows:



Our results were a normal distribution with mean of 2.89% and standard deviation of 1.349%. We then ran the code using random initial expectation generated by the distribution derived. We ended up with a mean of final expectation of 2.969529% and 0.971417%.

We drew comparison with the compounded actual inflation rate for 2018, using the actual inflation rate for 2016 and 2017:

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual

2017	2.1%	2.0%	1.6%	1.6%	1.3%	1.0%	1.2%	1.4%	1.6%	1.4%	2.1%	1.9%	1.6%
2016	2.0%	1.4%	1.3%	1.7%	1.5%	1.5%	1.3%	1.1%	1.3%	1.5%	1.2%	1.5%	1.4%

From above, we calculated our real 2-year inflation rate starting at the beginning of 2016 by taking the compounded rate of 2016 and 2017. This results in a 2-year rate of 3.0%.

Extensions Of Our Model

Given the fact that future inflation can be influenced by inflation expectations, we want to predict future inflation rates as it's crucial to understand and analyze current inflation expectations. This is what our model could accomplish.

Therefore, anyone who needs information about future inflation rates could find our model useful. This represents a significant proportion of any economy, because most financial decisions which stretch over multiple years require the consideration of future inflation rates. More value could be added to our model given that current inflation rates are soaring (e.g., the latest release of the Consumer Price Index, indicative of the rate of inflation in Canada, rose by 7.6% over the last 12 months according to StatCan), and thus increasing the level of volatility in the economy.

Usefulness towards external users such as:

• Central Banks: Such institutions need results from our model to gauge their open market operations intended at influencing the interest rate in the economy.

• Governments: Govts. can use our results to adjust their policies such as taxation and public spending.

• Insurance Underwriters: Insurance underwriters would benefit enormously from our findings because inflation is synonymous with higher cost of insurance claims.

• Real Estate and Financial Investors: These market players can also make use of our result to determine their optimal portfolio, as inflation affects stocks, bonds, real estate, and other securities.

• Regular households: Such a group can utilize our results to forecast their spending and investment. They might be interested in for e.g., by how much their mortgage payment or grocery bill might be affected by constantly rising prices.

References

1. https://www.bankofcanada.ca/publications/canadian-survey-of-consumer-expectations/canadian-survey-of-consumer-expectations-survey-data/#chart4

2. https://www.rateinflation.com/inflation-rate/canada-historical-inflation-rate/