

## Lec 1:

- TOC helps us formulate mathematical models of a computer
- These mathematical models in turns help us understand what a computer can/can't do, efficiency of it and so on.
- Automata / machine / mathematical model. → all mean the same.
- But to make a computer understand all the shinanigans we need a pattern maker which is where **GRAMMARS** come at play.
- Next we have PUSH DOWN AUTOMATA which will be used to verify Syntax.
- And at last TURING MACHINES where we learn about whether a given program will halt or not (THE Halting Problem).

## Lec 2: Mathematical Recap:

- **Set :** Collection of distinct objects ; no repetitions.
- **Cardinality :** Size of a set ./ no. of elements in the set.
- **Cartesian Product :**

If  $A = \{1, 2\}$  then  $A \times B = \{(1, a), (2, a), (1, b), (2, b), (1, c), (2, c)\}$

$B = \{a, b, c\}$

These are called ordered pairs/tuples.

- **Relation :**  $R \subseteq A \times B$  ; A relation is a subset of cartesian product.
- $R = \{(1, a), (2, a), (2, b)\}$
- implying  $R \subseteq A \times B$

- **Function :** Special type of relations.

→ It says that an element of set A can be mapped to

only one other element in B.

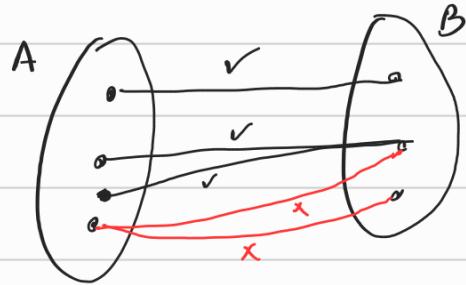


Fig: The one shown in red cannot happen.

- **Graphs :** It is a collection of vertices and edges. ( $V, E$ )



Fig: undirected graph.

- **Sequences :** It is a collection of elements (SET) where the order matters.

Ex:  $(A, B, C, D, \dots, Z)$  : seq of alphabets

$(1, 1, 2, 3, 5, 8, \dots)$  : seq of fibonacci numbers

■ One more fundamental difference of a set vs sequence is that sequences allow repetitions.

meaning:  $\{a, b, c, d, a, b, c, d, a, b, c, d\}$  is not a set but a valid sequence.

■ One other way to put is that a sequence is a function from natural numbers to objects (where order matters).

## Lec 3<sup>o</sup>: Alphabets, Strings, Languages, Grammar

**Alphabet :** It is nothing but a finite set of symbols.

Is represented using  $\Sigma$

The empty alphabet is represented using  $\Sigma_{empty} = \emptyset$

Ex:  $\Sigma_{eng} = \{a, b, c, d, \dots, z\}$

$\Sigma_{bin} = \{0, 1\}$ .

$\Sigma_{hindi} = \{ॐ, शं, शं, शं, \dots\}$

**Strings/Words :** finite sequence of alphabets ; cannot be infinite.  
length of a string = # no. of alphabets.  
Empty string  $\Rightarrow$  len = 0 ; Is represented by  $\epsilon$  (epsilon)

$$|\epsilon| = 0$$

$$\epsilon \cdot S = S \cdot \epsilon = S \quad \{ \text{concatenation} \}$$

$\Sigma^*$  = set of all strings that can be formed using  $\Sigma$  including empty strings.

**Language :** It is a subset of  $\Sigma^*$   
A symbol 'L' is used to denote a language.

**Grammar :** Set of rules to produce valid strings in a formal language.

**Substring :** Ex: abcdede 'bcd' is a substring.

Formally,  $u$  is a substring of  $v$  if  $\exists xuy = v$  where  $u, x, y, v \in \Sigma^*$

**Prefix :** Ex: 01 01 0101  $\Sigma = \{0,1\}$

$u$  is a prefix of  $v$  if  $\exists x$  s.t  $ux = v$

Similarly, we can define what a suffix is.

**Concatenation of strings :**  $u \cdot v$  such that  $u, v \in \Sigma^*$