An Optimal Allocation Strategy for Paper Distribution in Classroom Environments Using Mathematical Optimization Techniques

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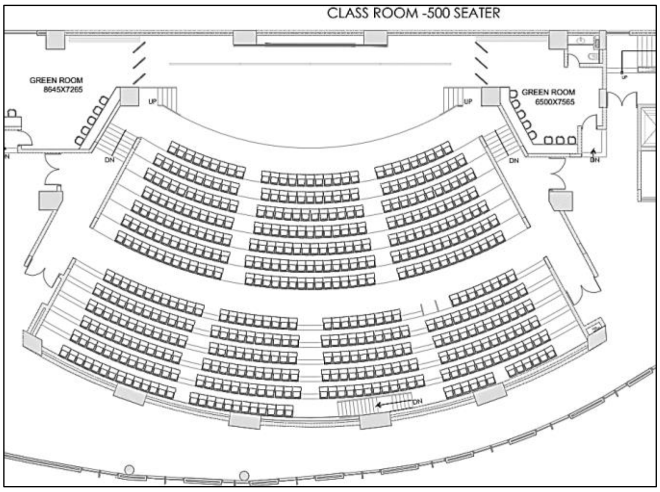
ABSTRACT

This research paper investigates the optimization of question paper distribution in exam halls, aiming to minimize resource utilization such as time and distance covered by the distributor while maximizing fairness. In this study, we propose a novel approach by modelling the exam hall as a 2D array and employing various forms of Breadth-First Search (BFS) algorithms to determine the most efficient distribution method. By systematically analysing the spatial layout of the exam hall, we aim to identify a strategy that not only streamlines the distribution process but also ensures equitable access to exam papers, contributing to an improved and fairer examination environment.

INTRODUCTION



Over the course of our inaugural year at this institution, a keen observation of various assessment instances—ranging from quizzes to midsemester and end semester examinations—has illuminated a recurrent challenge: the lack of a formalized, systematic approach to question paper distribution. This process typically unfolds from one corner of the exam hall to another, occasionally resulting in temporal disparities of up to five minutes between the receipt of question papers by students situated at different corners. This temporal variance introduces an element of unfairness into the examination experience, particularly when the presence of Teaching Assistants (TAs) is limited. In response to this observed inefficiency, this research endeavours to establish a structured methodology for the optimal distribution of question papers, focusing on Exam Hall C102 as a case study. The irregular and asymmetrical layout of C102, coupled with its circular design, poses a unique challenge that necessitates a careful mapping of the hall into a graph. By doing so, we aim to systematically apply various traversal and shortest path algorithms to derive a quantifiably efficient and fair distribution strategy, taking into account the peculiarities of C102's geometry and minimizing assumptions in our calculations. Through this technical exploration, our objective is to contribute to a more refined and equitable examination process within the complex spatial constraints of C102. There are currently 509 chairs available in C102 lecture hall.

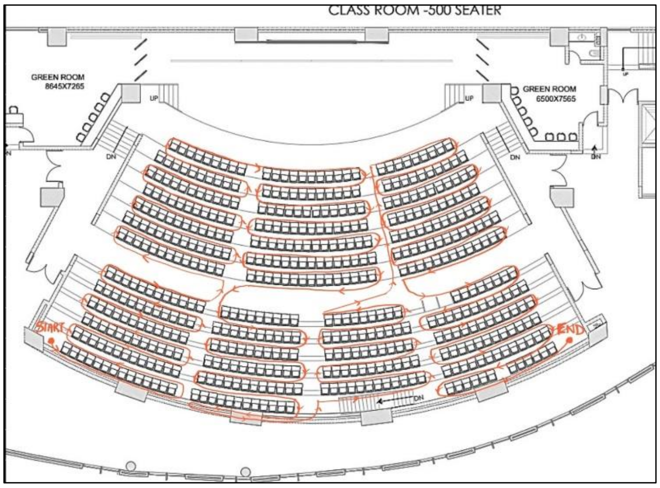
CURRENT SCENARIO

Currently two methods are most commonly used. Either the TA goes to every student and hands them their question paper, making the TA interact with all the students in the exam hall or the TA goes row-wise and gives an approximate number of sheets to first student in the row asking him to pass it on in their row.

THEORITICAL STUDY AND PROPOSED SOLUTION

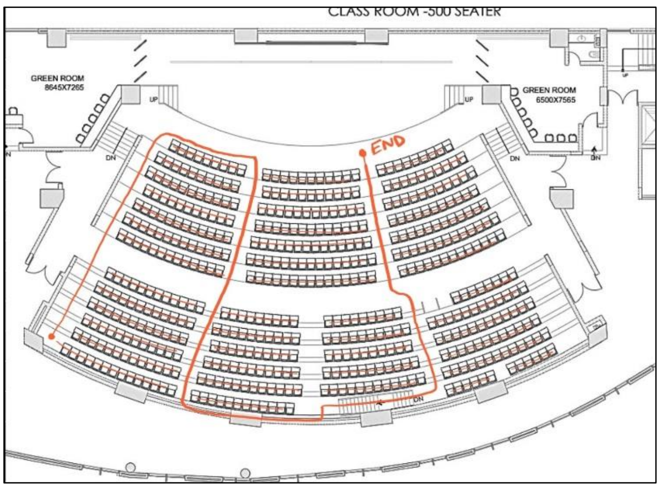
Here we will try to construct a mathematical proof of why the proposed method might show promising results and help us in optimizing the current unoptimized way.

CURRENT SCENARIO



Method – 1

In this current method, the distributor is making contact with every student and there are 509 students available. Hence, it will take at least 509 seconds plus the time it takes to walk from one block to another to finish. We will note it as 509+.

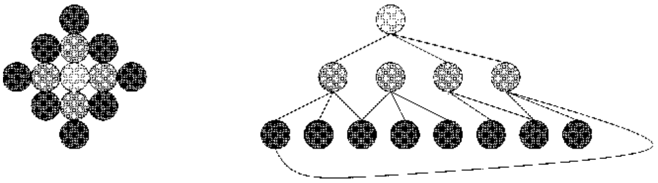


Method – 2

In this method, the walking time of the distributor comes out to be about 81 seconds.

And in these 81 seconds, the first five blocks are already covered and the sixth block is covered partially. So, we count the time taken for the last rows and we can safely approximate 90 seconds for the whole process to complete.

PROPOSED METHOD: PARALLELLY EXECUTED BREADTH FIRST TRAVERSAL



Breadth-First Traversal (BFS) is a fundamental graph traversal algorithm that explores the vertices of a graph level by level, visiting all neighbours of a given vertex before moving on to the next level.

TIME COMPLEXITY OF BREADTH FIRST TRAVERSAL

In the sequential Breadth-First Search (BFS) algorithm, the time complexity can be summarized as $O(V+E)$, where V represents the number of vertices and E denotes the number of edges in the graph. This linear growth complexity reflects the combined impact of initializing the source vertex $(O(1))$, the iterative process $(O(V+E))$, and the individual traversal of edges $(O(E))$.

THE COMPUTER VS THE REAL WORLD

A crucial distinction arises when considering the time complexity of an algorithm in the context of real-world implementation. The theoretical time complexity, $O(V+E)$, calculated based on the algorithmic code, may not perfectly align with practical execution dynamics. This complexity reflects computational steps in a simulated environment, explicitly detailing enqueuing, dequeuing, and iterative processes. However, in the tangible realm of question paper distribution in an exam hall, these computational intricacies do not directly translate.

In practice, the parallelism introduced by the movement of question papers among students contradicts the sequential nature of the algorithmic code. While the algorithm is crafted for a single processor, the reality of question papers traversing the exam hall concurrently introduces a substantial disparity. In the algorithmic representation, each neighbour is assigned the same time of arrival, implying simultaneous interactions. However, in the actual execution, a single processor can only engage with one neighbour at a time, extending the time taken to reach each neighbour individually.

In the practical domain, where parallelism facilitates the coverage of n neighbours in 1 second, the algorithm, designed for sequential processing, may require n seconds to accomplish the same task. This disjunction underscores the need to reconcile algorithmic complexities with the intricacies of real-world execution for a more accurate understanding of temporal dimensions.

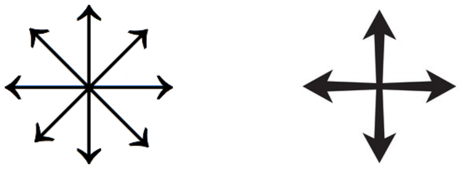
MAKING PARALLEL BFT WORK WITH ONE DISTRIBUTOR

Upon contemplating the logistics of question paper distribution in our examination hall, a novel analogy emerges—a parallel with the workings of an operating system. Initially conceiving the distributor as the CPU, the idea of one CPU core executing a sequential BFS algorithm seemed inevitable. However, envisioning each student, upon receiving a question paper, as a dynamic CPU core introduced an intriguing parallel. In this analogy, the moment a student holds a paper, they initiate a parallel process by concurrently passing it to neighbours. This imaginative leap suggests the potential for a real-world parallel BFS, where students, akin to CPU cores, collectively optimize the distribution process in Exam Hall C102.

PARALLEL BREADTH – FIRST TRAVERSAL

In the proposed parallel BFS, the graph undergoes division into subgraphs, each assigned to a parallel computing unit or processor. This strategy enables the simultaneous initiation of BFS from a designated source vertex in each subgraph. The key advantage lies in the concurrent exploration of distinct graph regions, leading to a significant reduction in overall traversal time.

GRAPH REPRESENTATION AND CONNECTIVITY

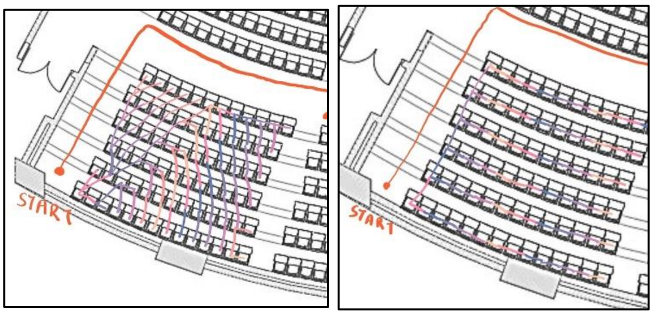


While mapping the hall into a graph, for the current student, we can have it connected with either four (up, down, left and right) or eight (up, down, left, right, upper left, upper right, lower left and lower right) students. If we consider the diagonal students as well, we must make the graph weighted where distance between up, down, left, right and the diagonals is different.

WEIGHTED GRAPH FOR SPATIAL REALISM

To introduce spatial realism, the graph can be weighted, where the distance between vertices in different directions is assigned varying weights. This accounts for the physical distance a distributor must cover while moving from one chair to another. Weighted edges contribute to a more accurate representation of the actual traversal distance within the exam hall.

DIAGRAMATIC REPRESENATION OF PARALLEL BFS

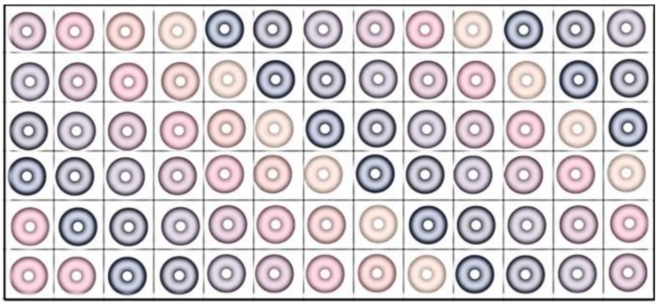


Considering Diagonals　　Not Considering Diagonals

TIME COMPLEXITY OF PARALLEL BFT FROM CORNER

First let us try with the case in which we are not considering diagonals.

We start with one processor which is the initial distributor which gives the Questions paper to first student in the corner. From then the number of processors increase as the paper is passed from student to student. The distributors at different time stamps are shown in different colours.



A block with 6 X 13 chairs

Following the BFT (diagonals not included) on above, manually the times comes out to be 17 seconds. Below is the distribution of number of processors for every second.

1 2 3 4 5 6 6 6 6 6 6 6 5 4 3 2 1

If I increase a column and make it 6 X 14 then we observe the sequence becomes

1 2 3 4 5 6 6 6 6 6 6 6 6 5 4 3 2 1

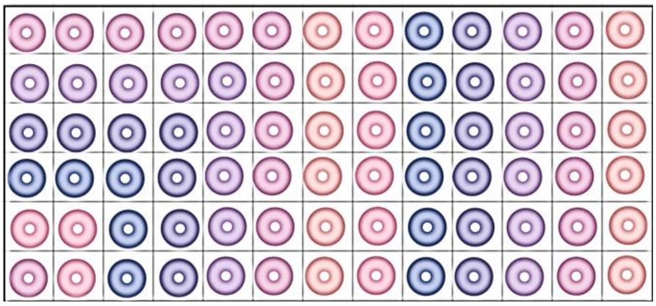
If I increase a column and make it 7 X 14 then we observe the sequence becomes

1 2 3 4 5 6 7 7 7 7 7 7 7 6 5 4 3 2 1

Hence, carefully observing we notice that processors start from 1, go up to the number of rows, and stay there for about (columns – rows) seconds and then comes back down to 1. Hence total time taken is

$$(m-1)+(n-m)+(m-1)=(m+n-2) \text{ seconds}$$

Let us look at the case when we are considering diagonals



A block with 6 X 13 chairs

Here, we observe the BFT starts from one corner and spreading over the block in a very basic pattern. The pattern won't change much if we increase the number of rows or columns. And to count the unique colours we just need to count the number of columns. Assuming the block to be $m\ x\ n$ matrix, the total time taken here is n seconds.

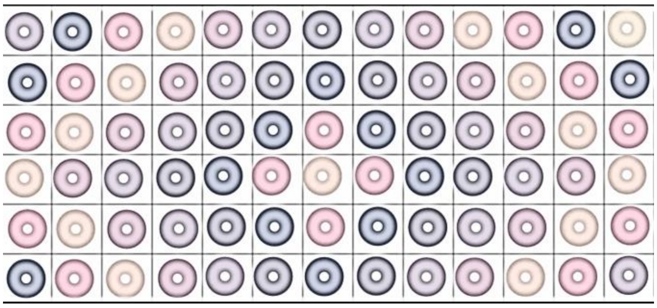
Earlier we did mention that the diagonal will take 1.5 seconds when non diagonals will take 1 second, but once the square portion is over, there are no neighbours that cannot be reached non-diagonally and by that time, the lag of diagonal neighbours also catches up and hence does not affect the time. It is the same as if the diagonal was 1 second as well.

This makes us realise not considering diagonal is like a constraint we have introduced and it might not give us a time better than the current scenario.

TIME COMPLEXITY OF PARALLEL BFT FROM CORNER

First let us try with the case in which we are not considering diagonals.

We start with one processor which is the initial distributor which gives the Questions paper to first student in the centre. From then the number of processors increase as the paper is passed from student to student. The distributors at different time stamps are shown in different colours.



A block with 6 X 13 chairs

We have to observe the simplest colour changing pattern and follow it to get a generalised formula. In above diagram, we can observe as we start from centre we can go up and then either left or right and we can count those circles to get a formula.

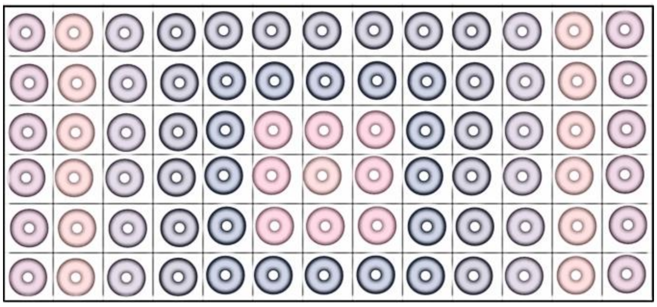
If the rows are even, then going up/down, our worst-case time will be $m/2$ and if they are odd then we $(m-1)/2$. So, in general to go up or down, we take time = $[m/2]$ where [.] represent the GIF.

In a similar way time to go left/right completely = $[n/2]$.

There is one circle that comes in calculation of both path since it is the intersection and we have not included it in above calculations hence we also add $+1$ for that. So our final formula becomes:

$$\left[\frac{m}{2}\right]+\left[\frac{n}{2}\right]+1 \; seconds$$

Now we look at the case including diagonals.



A block with 6 X 13 chairs

The same thing happens here but since diagonals are included, by the time we have gone up/down, we have also moved that much distance left/right so we subtract the time to go/up and down from the left/right time and get the final time as:

$$\left[\frac{m}{2}\right]+\left[\frac{n}{2}\right]+1-\left[\frac{m}{2}\right] = \left[\frac{n}{2}\right]+1 \; seconds$$

PRACTICALITY OF PARALLELISM

An important point to consider is how parallelism will work out in real-life scenario. The question paper can be transferred via two mechanisms:

1. The current student gives it to his neighbours.
2. The neighbours take it from the current student.

In approach 1, the student can only give away only two papers at once limited by his two hands. Hence following approach – 1 will force the process to become sequential at a point where there are more than two neighbours for the current student. But our exam hall is a connected graph in a rectangular shape and if we are starting any block from the corner, hence there will hardly come a case where there are more than two neighbours. So, we can easily neglect these cases. If starting from centre, we can think of approach 2 for our mental peace.

In approach 2, ideally infinite students or in this case four (if we consider only up, down, left and right students connected to the current student) or eight (if we also consider four more students at four diagonals also connected to the current student) students can take the question paper from the current student at once. Hence parallelism can be executed without any hiccup.

EXPLORING DEPTH FIRST TRAVERSAL

In our investigation into question paper distribution, we emphasize the critical importance of fairness. In this context, Depth-First Traversal (DFT) exhibits characteristics that may hinder our goal of achieving a fair distribution. By its nature, DFS tends to prioritize one path, potentially resulting in imbalances in distribution. Achieving both fairness and efficiency in paper distribution necessitates a more balanced exploration of the distribution space, a requirement that DFS, with its tendency for sequential exploration, may struggle to fulfil. DFS explores one path in the search tree completely before backtracking and exploring another path. This sequential nature makes it less suitable for problems that demand the consideration of multiple paths simultaneously. As we strive for a distribution strategy that ensures both fairness and efficiency, we find that DFS's inherent characteristics pose challenges in meeting our objectives.

ASSUMPTIONS AND CONSIDERED POINTS

In delineating our approach to optimizing question paper distribution within Exam Hall C102, we make several key assumptions. Firstly, we consider the involvement of a singular Teaching Assistant (TA) to oversee the distribution process. This assumption provides a baseline for our analysis. Secondly, we assume an unlimited availability of question papers, omitting the time required for determining how many question papers must be given to the next person. Lastly, we assume full hall capacity, where each seat accommodates a student. We will also assume it takes 1 second to pass the questions paper from one person to another. We also assume time taken to walk 1 chair worth of distance equals 1 second.

METHODOLOGY

MAPPING C102 INTO A GRAPH

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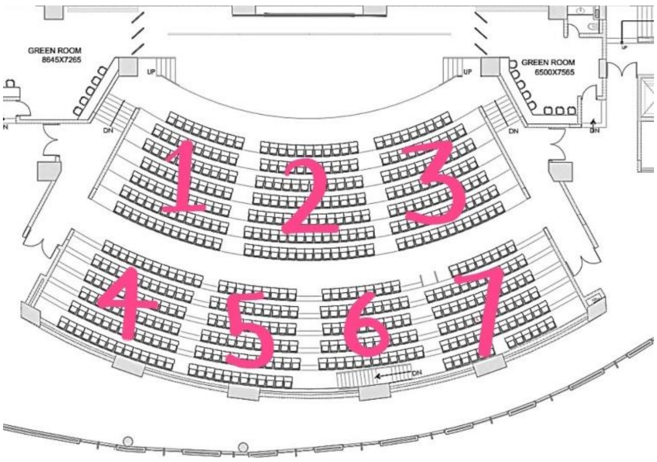
To effectively analyse the distribution dynamics within Exam Hall C102, our initial step involves representing the spatial layout of the hall as a graph. Given the intricacies of C102's design, a straightforward representation of each chair and its adjacency would result in a visually complex diagram. To address this challenge, we employ a pixel-based abstraction method, conceptualizing the entire C102 space as an array of pixels, each equivalent to the size of one chair. The presence of chairs is denoted by '1,' while empty spaces are represented by '0.' To account for wider empty spaces between blocks of chairs, we employ sequences of '0's, with the number indicating the approximate length of the empty space in terms of chairs. Additionally, in transforming the circular arrangement of chairs into a straight binary sequence, a degree of approximation is introduced. This pixel-based abstraction enables a more manageable yet representative visualization of the exam hall, laying the foundation for subsequent graph-based analyses. The 2D array above represents a 16 X 56 matrix in which 509 out of 896 entries are 1.

CHOOSING A STARTING POINT

Every block has four corner points and one centre point. We will be running our algorithm individually for each block from these five points to figure out which point in each block is the most efficient place to start distribution.

NUMBERING THE BLOCKS



CHOOSING WHICH STUDENTS TO CONNECT

We can choose one of the above cases and if we are considering the diagonals and take the weight of non diagonal connection to be 1, the weight of diagon connection can be taken as $\sqrt{2}.1 = 1.4$ by pythagoras theorem. Below we describe how the BFS will go in both the above cases.

OBSERVATIONS

CONSIDERING DIAGONALS.

Our C102 grid consisted of 16X56 points out of which 509 are 1's and rest are 0's. Since we have neglected small time delays, the answer always comes in integers. We want to find the most optimised way but a difference of one second would not do any harm to the findings hence we have negelcted small intricate details of distance and path (less than 1 second) needed to distribute the papers in C102.

X	Corner 1	Corner 2	Corner 3	Corner 4	Centre
Block 1	(0,13)	(0,21)	(6,7)	(6,19)	(2,14)
Block 2	(0,24)	(0,34)	(6,22)	(6,36)	(4,29)
Block 3	(0,37)	(0,45)	(6,39)	(6,51)	(2,44)
Block 4	(10,2)	(10,13)	(15,0)	(15,13)	(12,6)
Block 5	(10,19)	(10,27)	(15,16)	(15,27)	(12,21)
Block 6	(10,30)	(10,38)	(14,30)	(14,41)	(12,35)
Block 7	(10,48)	(10,55)	(15,45)	(15,57)	(13,50)

Time taken (seconds):

X	Corner 1	Corner 2	Corner 3	Corner 4	Centre
Block 1	8	14	14	12	7
Block 2	12	12	14	14	7
Block 3	14	8	12	14	7
Block 4	11	12	13	12	7
Block 5	8	11	11	11	6
Block 6	10	8	10	11	5
Block 7	8	11	11	13	6

Average time for the corners = 79.75 seconds
Average time for the centres = 45 seconds

NOT CONSIDERING DIAGONALS

X	Corner 1	Corner 2	Corner 3	Corner 4	Centre
Block 1	(0,13)	(0,21)	(6,7)	(6,19)	(2,14)
Block 2	(0,24)	(0,34)	(6,22)	(6,36)	(4,29)
Block 3	(0,37)	(0,45)	(6,39)	(6,51)	(2,44)
Block 4	(10,2)	(10,13)	(15,0)	(15,13)	(12,6)
Block 5	(10,19)	(10,27)	(15,16)	(15,27)	(12,21)
Block 6	(10,30)	(10,38)	(14,30)	(14,41)	(12,35)
Block 7	(10,48)	(10,55)	(15,45)	(15,57)	(13,50)

Time Taken (seconds):

X	Corner 1	Corner 2	Corner 3	Corner 4	Centre
Block 1	12	19	19	13	10
Block 2	17	17	17	17	9
Block 3	19	12	12	19	10
Block 4	15	17	17	15	9
Block 5	12	15	15	13	8
Block 6	14	11	12	14	7
Block 7	13	15	14	16	8

Average time for the corners = 105.25 seconds
Average time for the centres = 61 seconds

The values from the algoirthm match the theoritised formula. Any deviation is due to the fact that the blocks are not perfect m X n matrixes .

INFERENCE

We can clearly observe that if the blocks were standalone, the best possible distribution time is obtained when we start BFS from the centre. This is true irrespective of considering diagonals or not.

In our current scenario, the two methods used caused the time taken to be 509+ seconds and 90 seconds.

We can distribute our findings in four cases:

CASE 1: When the TA traverses the corner routes, the BFT starts from corner points and we consider diagonals, the average comes out to be 79.75 seconds. If the time delay TA causes can be kept under 10 seconds, this method is very promising.

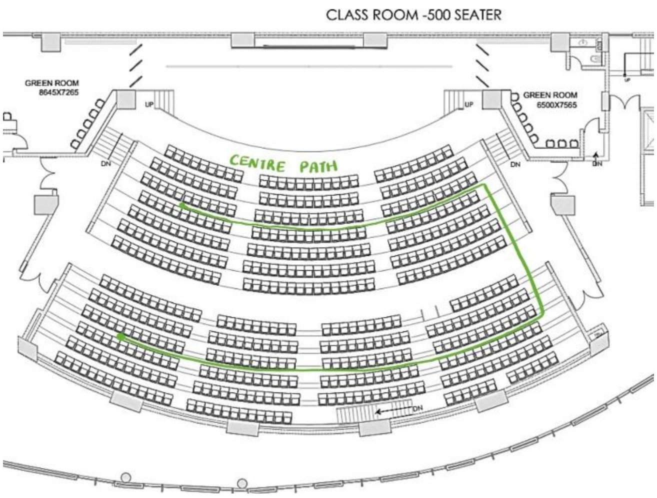
CASE 2: When the TA traverses the corner routes, the BFT starts from corner points and we don't consider diagonals, the average comes out to be 105.25 seconds. This already goes beyond our better current scenario method hence is not practical.

CASE 3: When the TA traverses the centre route, the BFT starts from centre points and we consider diagonals, the average comes out to be 45 seconds. This gives the TA 45 seconds to cover all centres and still be better than the method used these days.

CASE 4: When the TA traverses the centre route, the BFT starts from centre points and we don't consider diagonals, the average comes out to be 61 seconds. If the time delay TA causes can be kept under 29 seconds, this method is again very promising.

TIME DELAY DUE TO THE INITIAL DISTRIBUTOR

CENTRE PATH OPTIMIZATION

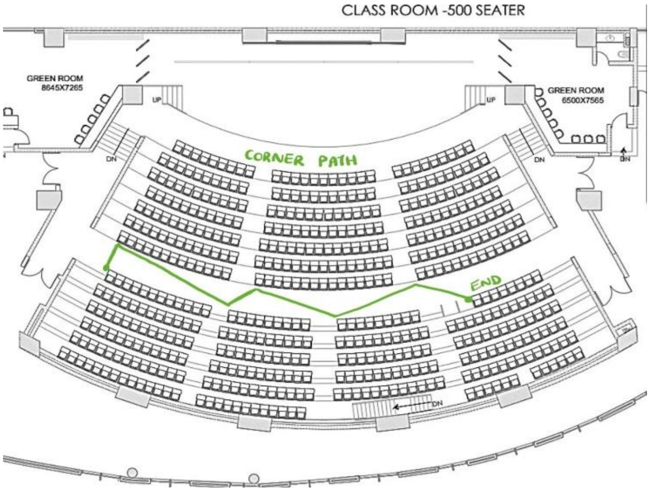


Above is the path that TA needs to take to minimize the time using BFT's with central start point and considering diagonals.

Net time TA takes to traverse this path = 62 seconds
Net time taken to cover the hall integrating TA's time delay into the BFT's = 70 seconds

CORNER PATH OPTIMIZATION

Below is the path that TA needs to take to minimize the time using BFT's with corner start point and considering diagonals.



Net time TA takes to traverse this path = 47 seconds
Net time taken to cover the hall integrating TA's time delay into the BFT's = 58 seconds

NOTE: Integrating BFT time and TA traversal time is not just simply adding them. We have to track them second by second to see what the net time is.

In observations above, we have taken average just to get an idea. In actuality, we won't be adding those times as the BFT's and TA traversal will be occuring parallely so there will be a good overlap between all those times.

INFERENCE

There was a trade-off between BFT time and TA traversal time. For centre starting points, the path of distributor got longer but BFT time was low. For corner starting points, path of distributor is shorter but BFT time is longer. Observing above data makes it clear that even though intially it looked like, centre starting points was significantly better, we realize the corner path is the best route for minimizing the resources in paper distribution process.

EXTRAPOLATING THE CONSTRAINTS

If we were to increase the number of initial distributors to equal the number of blocks, we can make the TA delay time come down to 0. And then going by the centre BFT case considering diagonals, we can complete the distribution process in just about 7 seconds which is the worst case time possible for any block. This is a huge cutdown from initial 509+ and 90 seconds.

An interesting thing to consider is, we have applied both static and dynamic parallelism in this paper. When we are deciding the number of TA's distributing the paper, we define it before runtime and it remains fixed throughout the distribution process. But the number of students who will be distributing the paper cannot be decided or defined before runtime. That will depend on the distribution of exam hall, blocks and students. That is dynamic parallelism.

CONCLUSION

This preliminary study laid the groundwork for optimizing question paper distribution in exam halls, focusing on the specific case of Exam Hall C102. While the results pointed towards Breadth-First Search (BFS) as a promising solution under certain assumptions, it is imperative to acknowledge the study's inherent simplifications and constraints. The assumptions of one Teaching Assistant (TA), infinite question papers, and full hall capacity were made to isolate and analyze specific scenarios. Moving forward, the practicality and generalizability of the proposed algorithm demand further exploration in real-life scenarios and across diverse exam hall configurations.

The study's focus on finding an optimal method led to the identification of BFS, yet it acknowledges the need for a more nuanced examination. Exploring various BFS instances from different starting points and prioritization strategies could unveil more efficient paths for both distributors and students, potentially reducing resource utilization. However, the transition from theoretical efficiency to practical applicability requires careful consideration, as real-world constraints may introduce complexities not captured in our simplified model.

Future avenues for research include conducting larger-scale studies that consider the intricacies of different exam halls, refining the algorithm to handle various scenarios, and developing a generalized framework applicable across diverse environments. Additionally, an empirical validation of the proposed methods in real-life settings would bridge the gap between theoretical optimality and pragmatic effectiveness.

In essence, while this study offers a promising step towards optimizing question paper distribution, it serves as a foundation for more comprehensive investigations. The intersection of theoretical efficacy and practical viability forms a critical juncture for future research, paving the way for enhanced strategies in exam hall logistics.