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Break chains, visible or invisible - build up from ruins, personal or social.

Digital Logic Design

FRANTZ FANON UNIVERSITY

COLLAGE OF COMPUTING AND IT

DEPARTMENT OF ICT

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Course Contents

- **Chapter 1** – Number Systems
- Chapter 2 - Binary Arithmetic
- Chapter 3 - Logic Gates
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Digital Logic Design

Chapter 1 – Number Systems

DLD?

- At some point in our education or even our daily lives, we have all heard the terms "*analog devices*" and "*digital devices*". Where analog devices use analog signals, digital devices use *digital logic design* to enter, interpret, and display data.
- In a digital system, values are discrete and there are only two states- it is either 1 or 0, or it is either '**on**' or '**off**'.
- This digital, binary representation of input & output data's basically what digital logic design is.

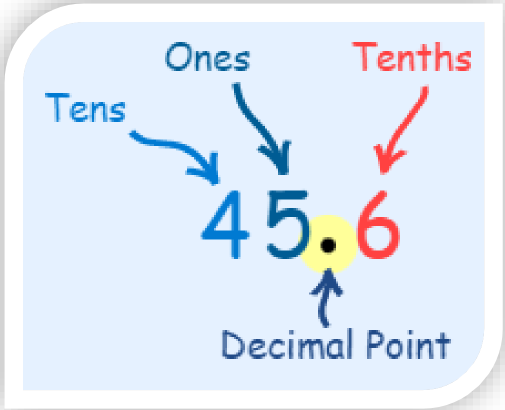
1.1 DECIMAL NUMBERS

- The word "Decimal" really means "based on 10" (From Latin decima: a tenth part). We sometimes say "decimal" when we mean anything to do with our numbering system, but a "Decimal Number" usually means there is a Decimal Point.
- The numbers we use in everyday life are decimal numbers, because they are based on 10 digits (0,1,2,3,4,5,6,7,8 and 9).
- "Decimal number" is often used to mean a number that uses a decimal point followed by digits that show a value smaller than one.

Cont….

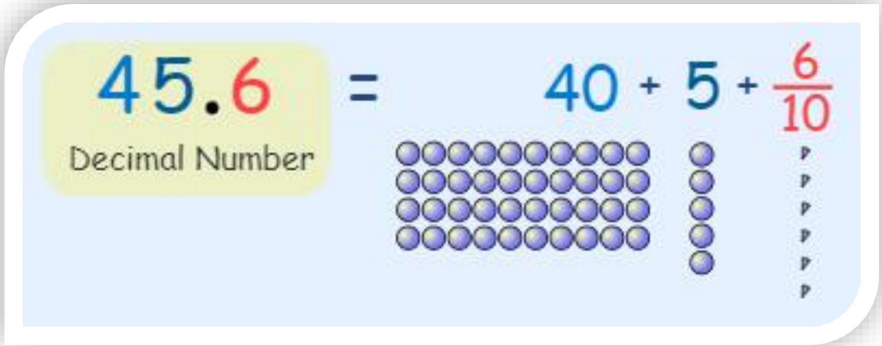
First, let's have an example:

Here is the number "forty-five and six-tenths" written as a decimal number:



The decimal point goes between Ones and Tenths.

45.6 has 4 Tens, 5 Ones and 6 Tenths, like this:



Cont….

- In the decimal number system each of the ten digits, 0 through 9, represents a certain quantity.
- As you know, the ten symbols (digits) do not limit you to expressing only ten different quantities because you use the various digits in appropriate positions within a number to indicate the magnitude of the quantity.
- if you wish to express a quantity greater than nine, you use two or more digits, and the position of each digit within the number tells you the magnitude it represents.
- If, for example, you wish to express the quantity twenty-three, you use (by their respective positions in the number) the digit 2 to represent the quantity twenty and the digit 3 to represent the quantity three, as illustrated below.

Cont….

- **The decimal number system has a base of 10.**

The position of each digit in a decimal number indicates the magnitude of the quantity represented and can be assigned a weight. The weights for whole numbers are positive powers of ten that increase from right to left, beginning with $10^0 = 1$

$$\ldots\ 10^5\ 10^4\ 10^3\ 10^2\ 10^1\ 10^0$$

For fractional numbers, the weights are negative powers of ten that decrease from left to right beginning with 10^{-1}

- **The value of a digit is determined by its position in the number.**

$$10^2\ 10^1\ 10^0\ .\ 10^{-1}\ 10^{-2}\ 10^{-3}\ \ldots$$

The value of a decimal number is the sum of the digits after each digit has been multiplied by its weight, as Examples 1 and 2 illustrated.

Cont….

- **Example 1: Express the decimal number 47 as a sum of the values of each digit?**

<u>*Solutions*</u>

The digit 4 has a weight of **10**, which is 10^1, as indicated by its position. The digit 7 has a weight of 1, which is 10^0, as indicated by its position.

$$\mathbf{47} = (4 \times 10^1) + (7 \times 10^0)$$

$$= (4 \times 10) + (7 \times 1)$$

$$= \mathbf{40} + \mathbf{7}$$

Exercise: Determine the value of each digit in 939?

Cont….

- **Example 2: Express the decimal number 568.23 as a sum of the values of each digit?**

Solutions

The whole number digit 5 has a weight of 100, which is 10^2, the digit 6 has a weight of 10, which is 10^1, the digit 8 has a weight of 1, which is 10^0, the fractional digit 2 has a weight of 0.1, which is 10^{-1}, and the fractional digit 3 has a weight of 0.01, I which is 10^{-2}.

$$\begin{aligned} 568.23 &= (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2}) \\ &= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01) \\ &= \mathbf{500} + \mathbf{60} + \mathbf{8} + \mathbf{0.2} + \mathbf{0.03} \end{aligned}$$

Exercise: Determine the value of each digit in 67.924?

1.2 BINARY NUMBERS

- The binary number system is another way to represent quantities. It is less complicated than the decimal system because it has only two digits. The decimal system with its ten digits is a base-ten system; the binary system with its two digits is a base-two system.
- The two binary digits (bits) are 1 and 0. The position of a 1 or 0 in a binary number indicates its weight, or value within the number, just as the position of a decimal digit determines the value of that digit. The weights in a binary number are based on powers of two.

Cont….

- **The binary number system has two digits (bits).**

To learn to count in the binary system, first look at how you count in the decimal system. You start at zero and count up to nine before you run out of digits. You then start another digit position (to the left) and continue counting 10 through 99.

You have only two digits, called bits.

Begin counting: 0, 1. At this point you have used both digits, so include another digit position and continue: 10, 11. You have now exhausted all combinations of two digits, so a third position is required. With three digit positions you can continue to count: 100, 101, 110, and 111. Now you need a fourth digit position to continue, and so on. A binary count of zero through fifteen is shown in Table 1.1. Notice the patterns with which the 1*s* and 0*s* alternate in each column .

Cont….

Table 1.1

The value of a bit is determined by its position in the number.

As you have seen in Table 1.1, four bits are required to count from zero to 15. In general, with n bits you can count up to a number equal to $2^n - 1$.

For example, with five bits (n = 5) you can count from zero to thirty-one.

$2^5 - 1 = 32 - 1 = \mathbf{31}$

DECIMAL NUMBER	BINARY NUMBER			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

Cont….

- **The Weighting Structure of Binary Numbers**

The weight or value of a bit increases from right to left in a binary number.

A binary number is a weighted number. The right-most bit is the LSB (least significant bit) in a binary whole number and has a weight of $2^0 = 1$. The weights increase from right to left by a power of two for each bit. The left-most bit is the MSB (most significant bit); its weight depends on the size of the binary number.

Fractional numbers can also be represented in binary by placing bits to the right of the binary point, just as fractional decimal digits are placed to the right of the decimal point. The left-most bit is the MSB in a binary fractional number and has a weight of 2^{-1} = 0.5. The fractional weights decrease from left to right by a negative power of two for each bit.

Cont….

The Weighting Structure of Binary Numbers

The weight structure of a binary number is

$$2^{n-1} \ldots 2^3\, 2^2\, 2^1\, 2^0 \,.\, 2^{-1}\, 2^{-2} \ldots 2^{-n}$$

Binary point

Where n is the number of bits from the binary point. Thus, all the bits to the left of the binary point have weights that are positive powers of two, as previously discussed for whole numbers. All bits to the right of the binary point have weights that are negative powers of two, or fractional weights.

The powers of two and their equivalent decimal weights for an 8-bit binary whole number and a 6-bit binary fractional number are shown in Table 1.2. Notice that the weight doubles for each positive power of two and that the weight is halved for each negative power of two. You can easily extend the table by doubling the weight of the most significant positive power of two and halving the weight of the least significant negative power of two; for

Example, $2^9 = 512$ and $2^{-7} = 0.0078125$.

Cont….

Table 1.2: Binary Weight

POSITIVE POWERS OF TWO (WHOLE NUMBERS)								
2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
256	128	64	32	16	8	4	2	1

NEGATIVE POWERS OF TWO (FRACTIONAL NUMBER)					
2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
1/2	1/4	1/8	1/16	1/32	1/64
0.5	0.25	0.125	0.0625	0.03125	0.015625

Binary to Decimal Conversion

- The decimal value of any binary number can be found by adding the weights of all bits that are 1 and discarding the weights of all bits that are 0.
- Add the weights of all 1s in a binary number to get the decimal value.

Example:

Convert the binary whole number 1101101 to decimal.

Solution Determine the weight of each bit that is a 1, and then find the sum of the weights to get the decimal number.

Weight:	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Binary number:	1	1	0	1	1	0	1

$$1101101 = 2^6 + 2^5 + 2^3 + 2^2 + 2^0$$
$$= 64 + 32 + 8 + 4 + 1 = \mathbf{109}$$

Related Problem Convert the binary number 10010001 to decimal.

Cont..

Example:

Convert the fractional binary number 0.1011 to decimal.

Solution Determine the weight of each bit that is a 1, and then sum the weights to get the decimal fraction.

Weight:	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Binary number: 0 .	1	0	1	1

$$0.1011 = 2^{-1} + 2^{-3} + 2^{-4}$$
$$= 0.5 + 0.125 + 0.0625 = \mathbf{0.6875}$$

Related Problem Convert the binary number 10.111 to decimal.

Decimal to Binary Conversion

Example:

Convert Decimal Fraction (**0.6875**) to Binary?

Solution

- ✓ 0.6875 * 2 = 1.375 -> **1**
- ✓ 0.375 * 2 = 0.75 -> **0**
- ✓ 0.75 * 2 = 1.5 -> **1**
- ✓ 0.5 * 2 = 1.0 -> **1**

So, the equivalent binary number of **0.6875** is **0.1011**

Exercise:

- Convert Decimal Fraction (**0.703125**) to Binary?
- Convert Binary Fraction (**0.101101**) to Binary?

1.3 HEXADECIMAL NUMBERS

- The hexadecimal number system has sixteen characters; it is used primarily as a compact way of displaying or writing binary numbers because it is very easy to convert between binary and hexadecimal.
- As you are probably aware, long binary numbers are difficult to read and write because it is easy drop or transpose a bit.
- Since computers and microprocessors understand only 1s and 0s, it is necessary to use these digits when you program in "Machine Language".
- The hexadecimal number system has a base of sixteen; that is, it is composed of 16 numeric and alphabetic characters. The hexadecimal number system consists of digits 0-9 and letters A-F. As the next table illustrated.

Cont..

DECIMAL	BINARY	HEXADECIMAL
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Counting in Hexadecimal

- With two hexadecimal digits, you can count up to $\mathbf{FF_{16}}$, which is decimal **255**. To count beyond this, three hexadecimal digits are needed.
- For instance, $\mathbf{100_{16}}$ is decimal **256**, $\mathbf{101_{16}}$ is decimal **257**, and so forth. The maximum 3-digit hexadecimal number is $\mathbf{FFF_{16}}$, or decimal **4095**. The maximum 4-digit hexadecimal number is $\mathbf{FFFF_{16}}$, which is decimal **65,535**.

Binary to Hexadecimal Conversion

Converting a binary number to hexadecimal is a straightforward procedure. Simple break the binary number into 4-bit groups, starting at the right-most bit and replace each 4-bit group with the equivalent hexadecimal symbol.

- Examples:

Convert the following binary numbers to hexadecimal:

(a) 1100101001010111 **(b)** 111111000101101001

Solution

(a) $\underbrace{1100}_{C}\underbrace{1010}_{A}\underbrace{0101}_{5}\underbrace{0111}_{7} = \mathbf{CA57_{16}}$

(b) $\underbrace{0011}_{3}\underbrace{1111}_{F}\underbrace{0001}_{1}\underbrace{0110}_{6}\underbrace{1001}_{9} = \mathbf{3F169_{16}}$

Two zeros have been added in part (b) to complete a 4-bit group at the left.

Related Problem Convert the binary number 1001111011110011100 to hexadecimal.

- **Hexadecimal to Binary Conversion**

To convert from a hexadecimal number to a binary number, reverse the process and replace each hexadecimal symbol with the appropriate four bits.

- Examples:

Determine the binary numbers for the following hexadecimal numbers:

(a) $10A4_{16}$ (b) $CF8E_{16}$ (c) 9742_{16}

Solution

(a) 1 0 A 4 → 1000010100100

(b) C F 8 E → 1100111110001110

(c) 9 7 4 2 → 1001011101000010

In part (a), the MSB is understood to have three zeros preceding it, thus forming a 4-bit group.

Related Problem Convert the hexadecimal number 6BD3 to binary.

Cont..

Hexadecimal to Decimal Conversion

One way to find the decimal equivalent of a hexadecimal number is to first convert the hexadecimal number to binary and then convert from binary to decimal.

- Examples:

Convert the following hexadecimal numbers to decimal:

(a) $1C_{16}$ **(b)** $A85_{16}$

Solution Remember, convert the hexadecimal number to binary first, then to decimal.

(a)

$$\overbrace{0001}^{1}\overbrace{1100}^{C} = 2^4 + 2^3 + 2^2 = 16 + 8 + 4 = \mathbf{28}_{10}$$

(b)

$$\overbrace{1010}^{A}\overbrace{1000}^{8}\overbrace{0101}^{5} = 2^{11} + 2^9 + 2^7 + 2^2 + 2^0 = 2048 + 512 + 128 + 4 + 1 = \mathbf{2693}_{10}$$

Related Problem Convert the hexadecimal number 6BD to decimal.

Cont..

❑ Decimal to Hexadecimal Conversion

Repeated division of a decimal number by 16 will produce the equivalent hexadecimal number, formed by the remainders of the divisions. The first remainder produced is the least significant digit (LSD). Each successive division by 16 yields a remainder that becomes a digit in the equivalent hexadecimal number.

❑ Examples:

Convert the decimal number 650 to hexadecimal by repeated division by 16.

Solution

Hexadecimal remainder

$$\frac{650}{16} = 40.625 \rightarrow 0.625 \times 16 = 10 = \text{A}$$

$$\frac{40}{16} = 2.5 \rightarrow 0.5 \times 16 = 8 = 8$$

$$\frac{2}{16} = 0.125 \rightarrow 0.125 \times 16 = 2 = 2$$

Stop when whole number quotient is zero.

2 8 A Hexadecimal number

MSD LSD

Related Problem Convert decimal 2591 to hexadecimal.

1.4 OCTAL NUMBERS

- Like the hexadecimal number system, the octal number system provides a convenient way to express binary numbers and codes. However, it is used less frequently than hexadecimal in conjunction with computers and microprocessors to express binary quantities for input and output purposes.
- The octal number system is composed of eight digits, which are; 0,1,2,3,4,5,6,7. The octal number system has a base of 8.

Counting in Octal

- Counting in octal is similar to counting in decimal, except that the digits 8 and 9 are not used. To distinguish octal numbers from decimal numbers or hexadecimal numbers, we will use the subscript 8 to indicate an octal number. For instance, 15_8 in octal is equivalent to 13_{10} in decimal.

Cont..

Octal to Decimal Conversion

Example: Convert 2374_8 to decimal number?

$$\text{Weight: } \quad 8^3 \; 8^2 \; 8^1 \; 8^0$$

$$\text{Octal number: } \quad 2 \;\; 3 \;\; 7 \;\; 4$$

$$\begin{aligned}2374_8 &= (2 \times 8^3) + (3 \times 8^2) + (7 \times 8^1) + (4 \times 8^0)\\ &= (2 \times 512) + (3 \times 64) + (7 \times 8) + (4 \times 1)\\ &= 1024 + 192 + 56 + 4 = 1276_{10}\end{aligned}$$

❑ Decimal to Octal Conversion

Example: Convert 359 to octal number?

Remainder

$$\frac{359}{8} = 44.875 \rightarrow 0.875 \times 8 = 7$$

$$\frac{44}{8} = 5.5 \rightarrow 0.5 \times 8 = 4$$

$$\frac{5}{8} = 0.625 \rightarrow 0.625 \times 8 = 5$$

Stop when whole number quotient is zero

5 4 7 Octal number

MSD LSD

Cont..

Octal to Binary Conversion

Because each octal digit can be represented by a 3-bit binary number, it is very easy to convert rom octal to binary. Each octal digit is represented by three bits as shown in Table below.

OCTAL DIGIT	0	1	2	3	4	5	6	7
BINARY	000	001	010	011	100	101	110	111

Examples:

Convert each of the following octal numbers to binary:

(a) 13_8 (b) 25_8 (c) 140_8 (d) 7526_8

Solution

(a) 1 3 → 001 011 → 001011

(b) 2 5 → 010 101 → 010101

(c) 1 4 0 → 001 100 000 → 001100000

(d) 7 5 2 6 → 111 101 010 110 → 111101010110

Related Problem Convert each of the binary numbers to decimal and verify that each value agrees with the decimal value of the corresponding octal number.

Cont..

❑ Binary to Octal Conversion

Conversion of binary number to an octal number is the reverse of the octal to binary conversion. The procedure is as follows: Start with the right-most group of three bits and moving from right to left, convert each 3-bit group to the equivalent octal digit.

Examples:

Convert each of the following binary numbers to octal:

(a) 110101 **(b)** 101111001 **(c)** 100110011010 **(d)** 11010000100

Solution

(a) $\underbrace{110}_{6}\,\underbrace{101}_{5} = \mathbf{65_8}$

(b) $\underbrace{101}_{5}\,\underbrace{111}_{7}\,\underbrace{001}_{1} = \mathbf{571_8}$

(c) $\underbrace{100}_{4}\,\underbrace{110}_{6}\,\underbrace{011}_{3}\,\underbrace{010}_{2} = \mathbf{4632_8}$

(d) $\underbrace{011}_{3}\,\underbrace{010}_{2}\,\underbrace{000}_{0}\,\underbrace{100}_{4} = \mathbf{3204_8}$

Related Problem Convert the binary number 1010101000111110010 to octal.

Thank You