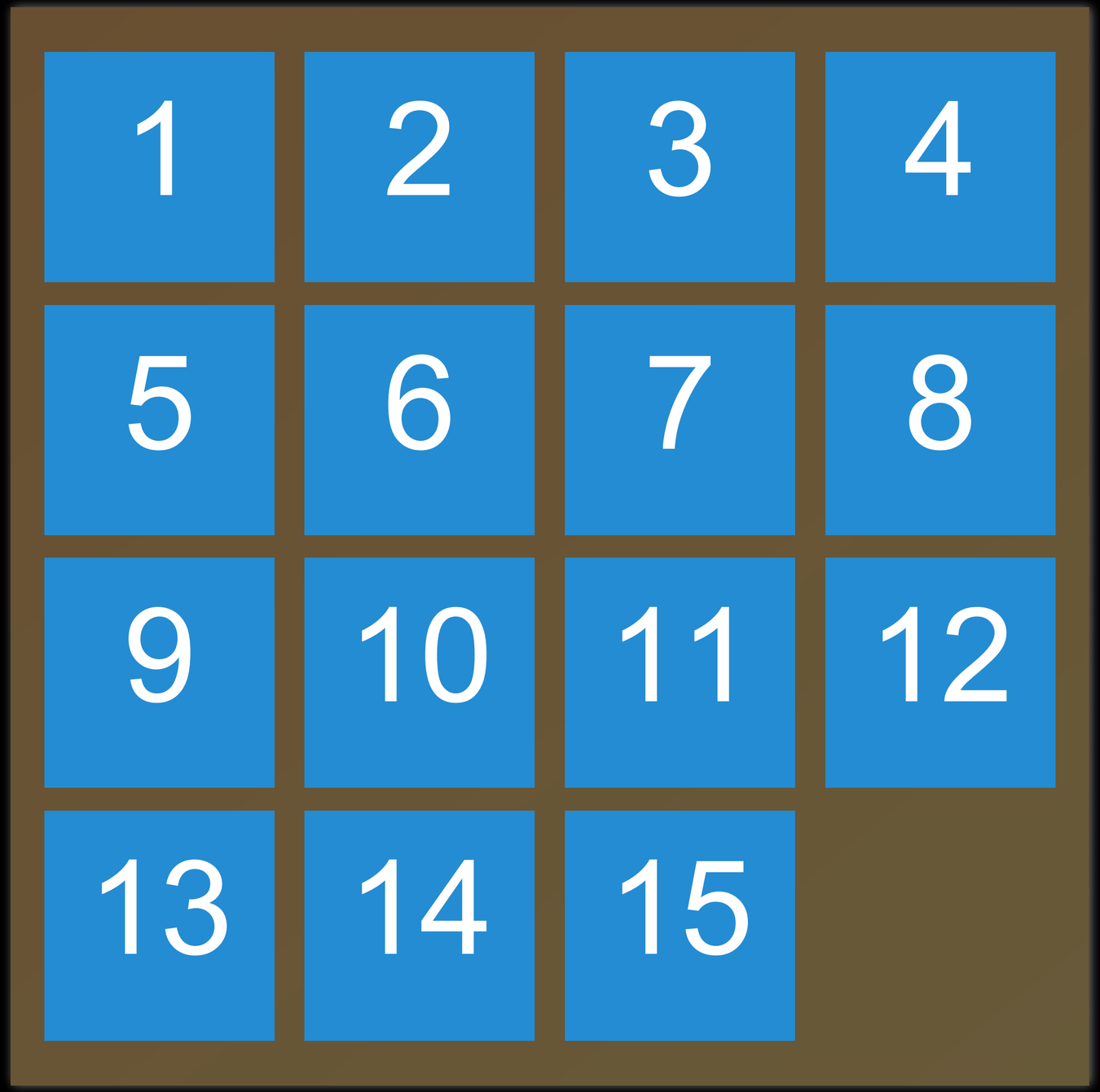
Design and Analysis of Algorithms

Chapter 3 – Analysis of Searching Algorithms

Instructor: Suleiman Gargaare

Contents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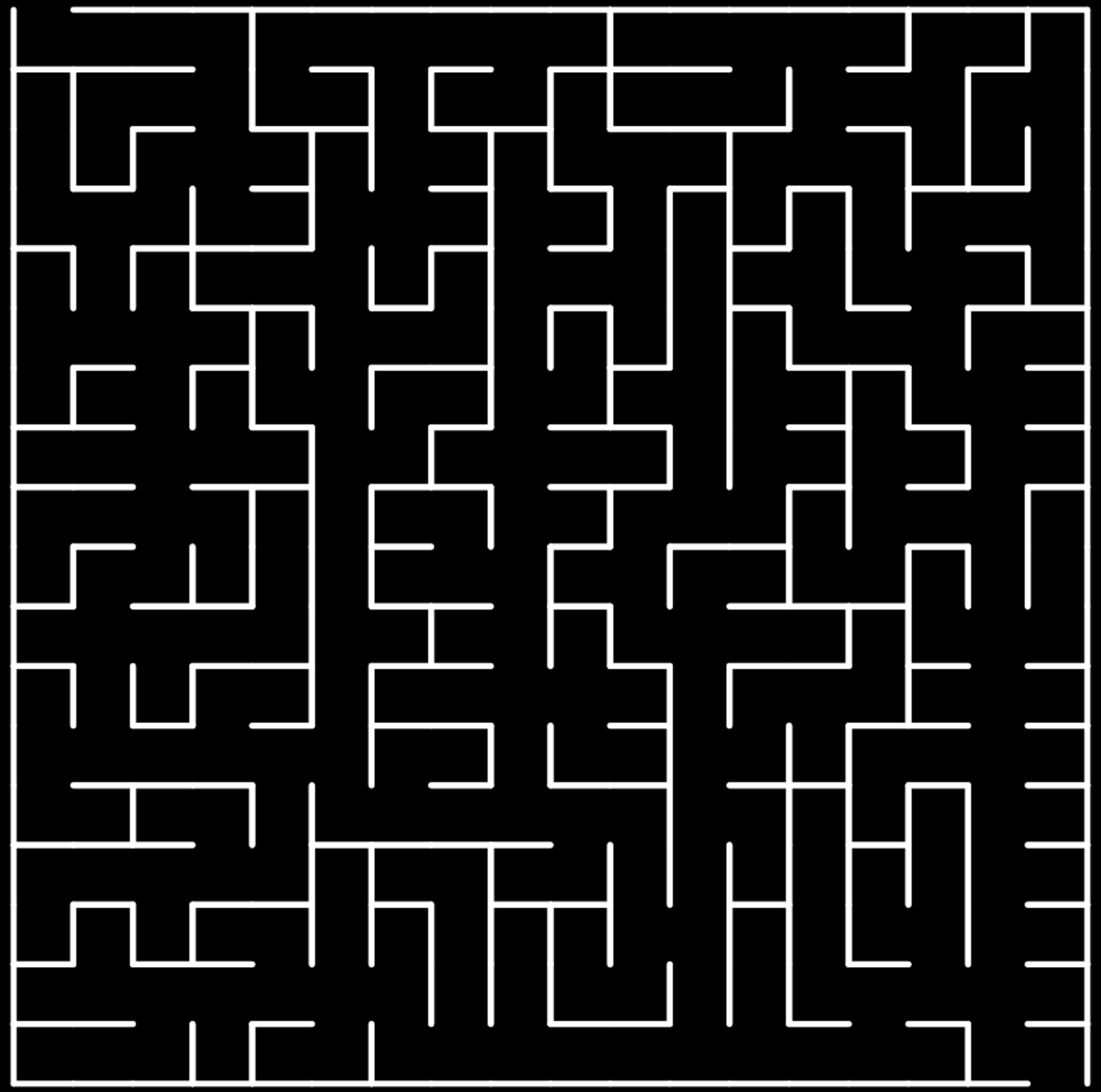

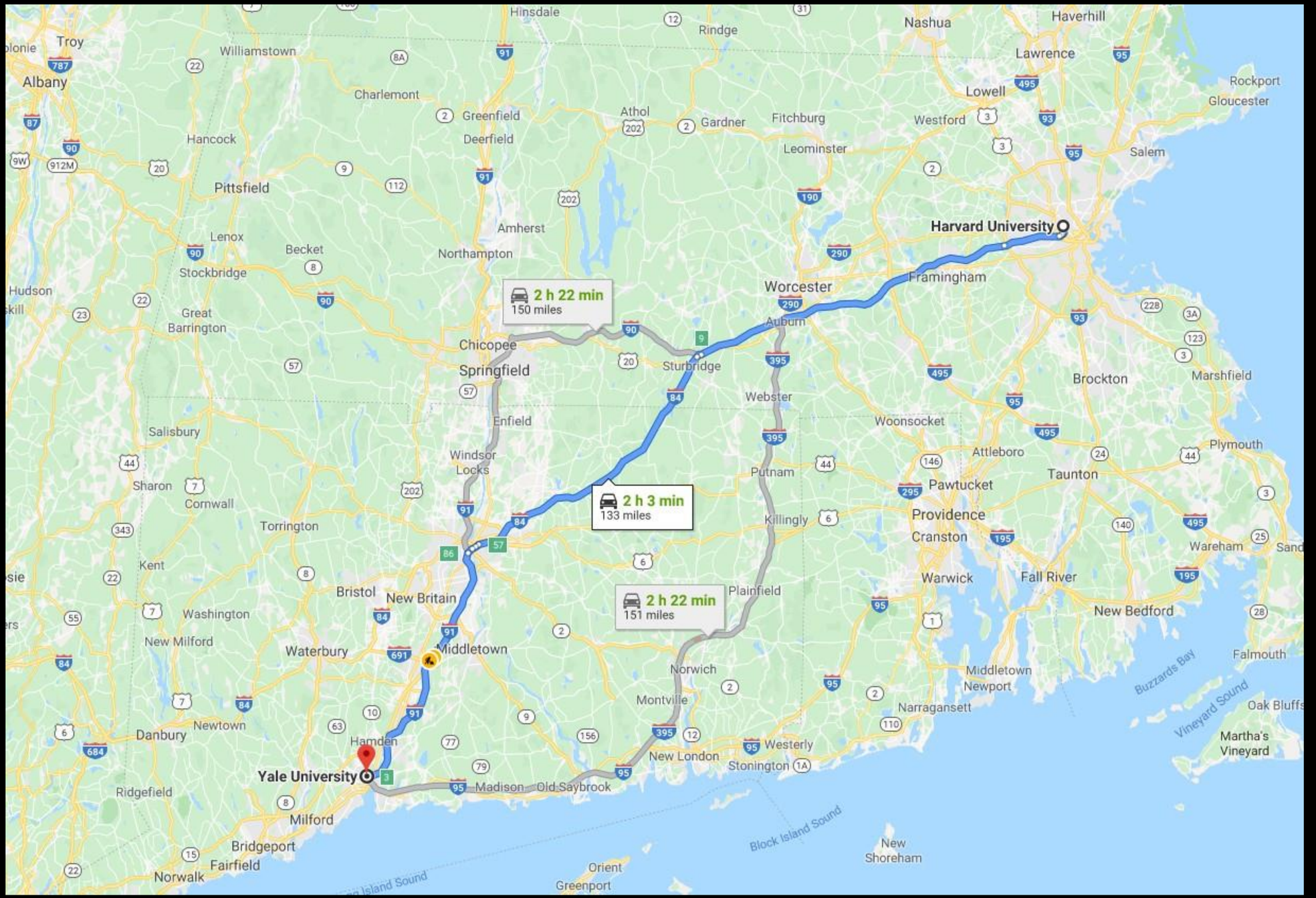
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Warwick
Fall River
Bristol
New Britain
Plainfield
Washington
New Bedford
New Milford
Waterbury
Middletown
Falmouth
Norwich
Middletown
Newport
Montville
Narragansett
Buzzards Bay
Vineyard Sound
Oak Bluffs
Newtown
Danbury
Hamden
New London
Westerly
Stonington
Martha's Vineyard
Ridgefield
Madison
Old Saybrook
Milford
Block Island Sound
Bridgeport
Fairfield
Norwalk
New Shoreham
Orient
Greenport

Lecture 1.1

Linear Search & Binary Search

Linear Search and Binary Search

- Searching algorithms are fundamental techniques for finding specific data within a collection.
- This lecture will explore the concepts, pseudocode, algorithm and time complexities of two widely used algorithms: Linear Search and Binary Search.

Introduction to Searching Algorithms

1 Efficient Data Retrieval

Searching algorithms allow for quick and accurate identification of desired information from large data sets.

2 Wide Applications

These algorithms are used across various fields, from computer science to business intelligence.

3 Algorithmic Complexity

The time and space complexity of searching algorithms directly impact their performance and scalability.

Linear Search: Definition and Concept

Sequential Approach

Linear search involves checking each element in a data structure, one by one, until the target is found.

Simplicity

This algorithm is straightforward to implement and understand, making it a popular choice for small data sets.

Unordered Data

Linear search can be used on both sorted and unsorted data, as it does not rely on any specific order.

Worst-Case Scenario

The target element may be the last one in the data structure, leading to a longer search time.

Linear Search: Pseudocode and Example

1

Initialize

Set the index to 0, the starting point of the data structure.

2

Compare

Check if the element at the current index matches the target.

3

Increment

If the target is not found, move to the next index and repeat the process.

Time Complexity of Linear Search

Best Case

If the target is the first element in the data structure, the algorithm terminates immediately, resulting in a time complexity of O(1).

Average Case

On average, the target element is found after searching half the data structure, leading to a time complexity of O(n/2) or O(n).

Worst Case

If the target is the last element or not present in the data structure, the algorithm must search the entire structure, resulting in a time complexity of O(n).

Binary Search: Definition and Concept

1 Divide and Conquer

Binary search is a more efficient algorithm that repeatedly divides the search space in half until the target is found.

2 Ordered Data

Binary search requires the data structure to be sorted in order to work effectively.

3 Logarithmic Time

The time complexity of binary search is logarithmic, making it much faster than linear search for large data sets.

Binary Search: Pseudocode and Example

1

Initialize

Set the left and right pointers to the start and end of the sorted data structure.

2

Compare

Check if the middle element matches the target. If not, compare the target to the middle element.

3

Divide

Depending on the comparison, update the left or right pointer to narrow the search space.

Binary Search: Algorithm

Example: Find the Target/key (42)

3	6	8	12	14	17	25	29	31	36	42	47	53	55	62

Solution

L	H	M = L + H / 2
1	15	1 + 15 / 2 = 16 / 2 = **8**
9	15	9 + 15 / 2 = 24 / 2 = **12**
9	11	9 + 11 / 2 = 20 / 2 = **10**
11	11	11 + 11 / 2 = 22 / 2 = **11**

Binary Search (A, n, key)

```
{
    l = 1; h = n;
  while (l ≤ h)
{
      mid = (l + h)/2;
    if (key == A [mid])
     return mid;
    if (key < A [mid])
     h = mid -1;
    else (l = mid + 1);
  }
    return 0;
}
```

Time Complexity of Binary Search

Best Case

If the target is the middle element, the algorithm terminates immediately, resulting in a time complexity of O(1).

Average Case

The target is found by repeatedly dividing the search space in half, leading to a time complexity of O(log n).

Worst Case

If the target is not present in the data structure, the algorithm must search the entire structure, resulting in a time complexity of O(log n).

Comparison: Linear Search vs. Binary Search

Search Approach

Linear search is sequential, while binary search is divide-and-conquer.

Time Complexity

Linear search is O(n), while binary search is O(log n).

Data Requirement

Linear search works with both sorted and unsorted data, while binary search requires sorted data.

Algorithmic Complexity

Linear search is simpler to implement, while binary search is more sophisticated.

Lecture 1.2

DFS and BFS

Types of Search Algorithms

Now let’s see the types of the search algorithm.

Based on the search problems, we can classify the search algorithm as

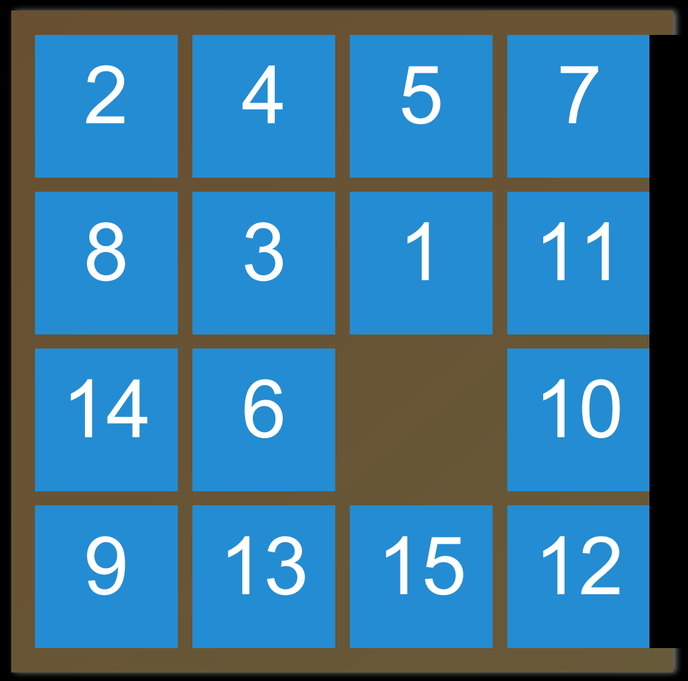
- **Uninformed search**
- **Informed search**

agent

entity that perceives its environment
and acts upon that environment

state

a configuration of the agent and its environment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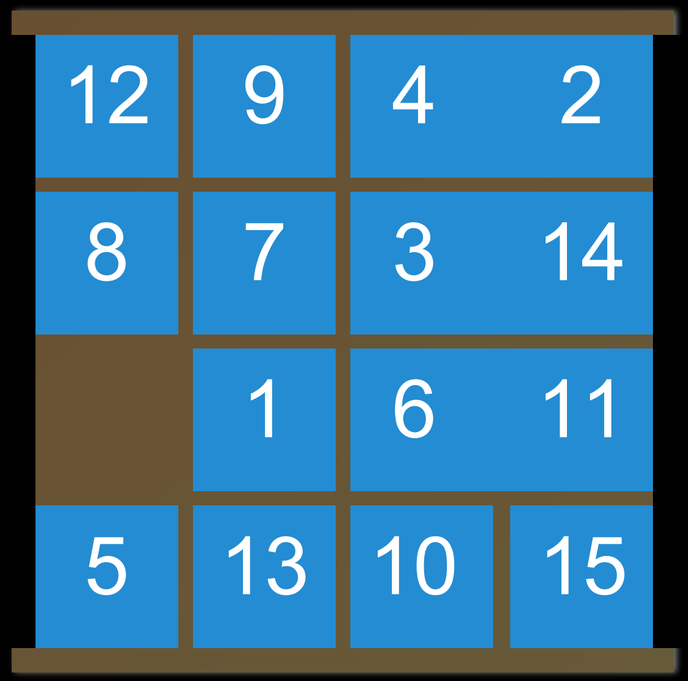
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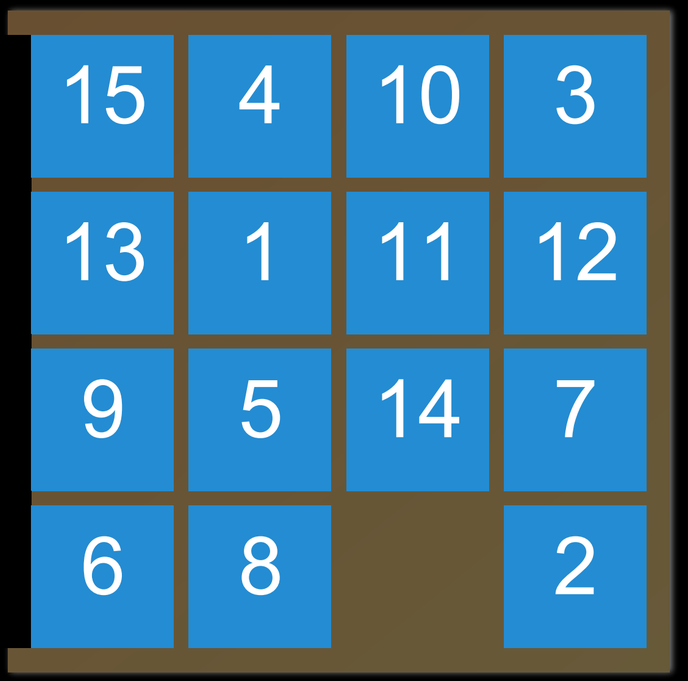
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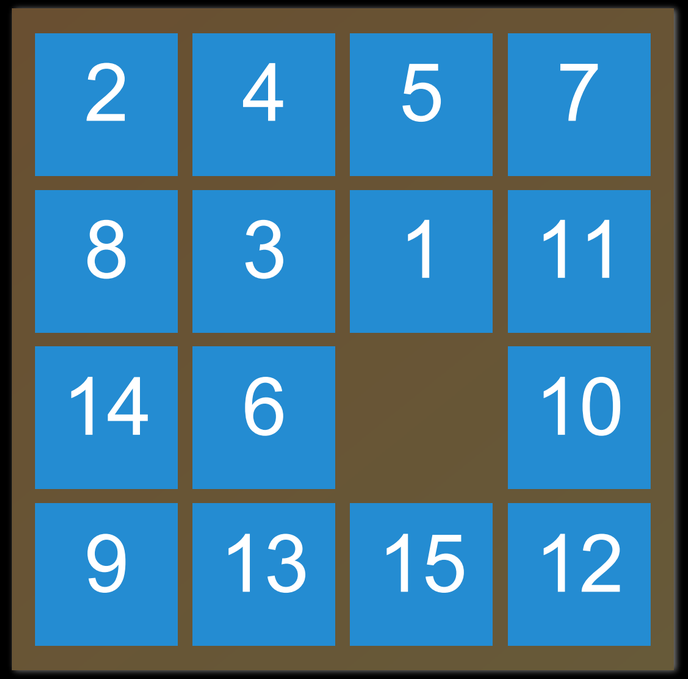
initial state

the state in which the agent begins

initial state



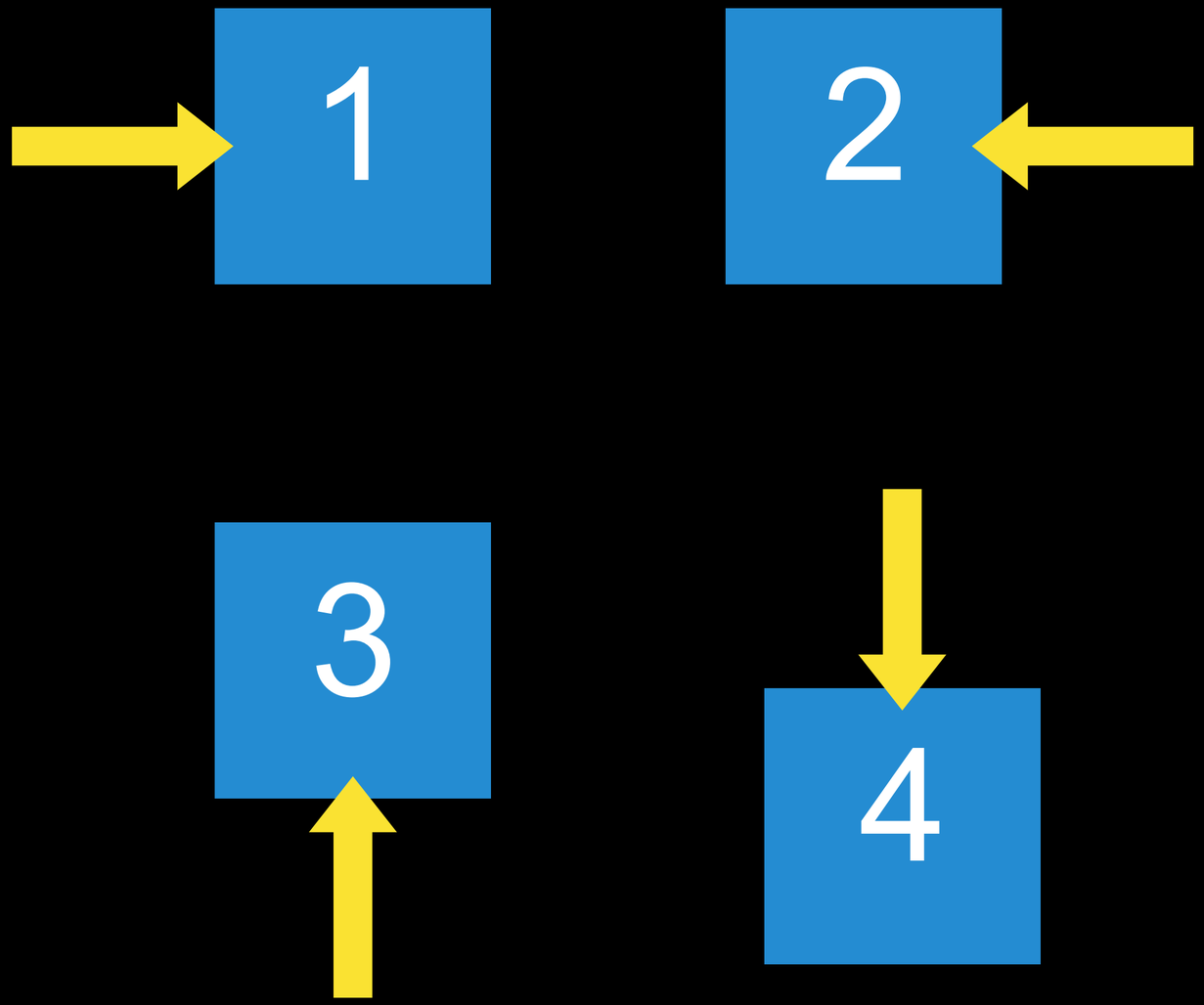
actions

choices that can be made in a state

actions

ACTIONS(s) returns the set of actions that can be executed in state s

actions



transition model

a description of what state results from performing any applicable action in any state

transition model

RESULT(s, a) returns the state resulting from performing action a in state s

RESULT(

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,

) =

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RESULT(

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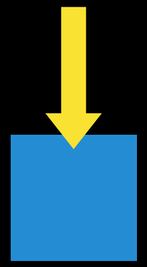

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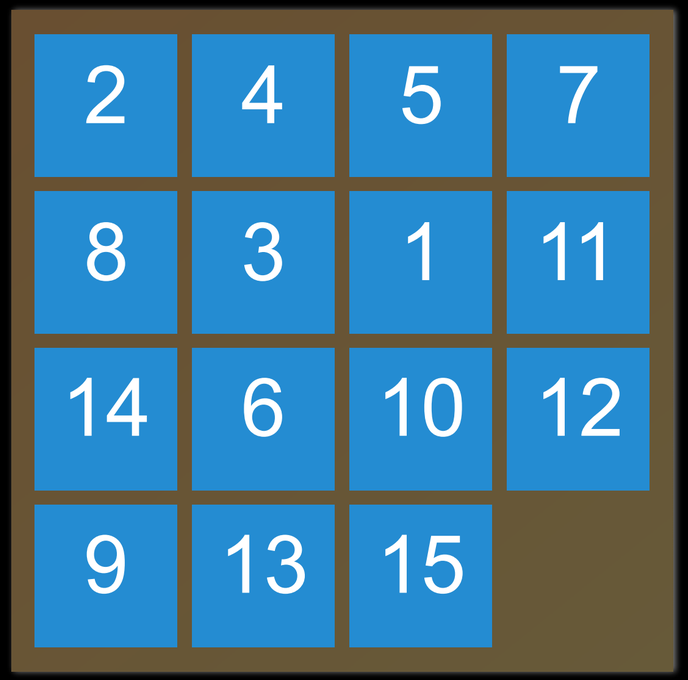
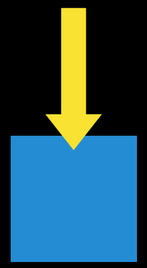
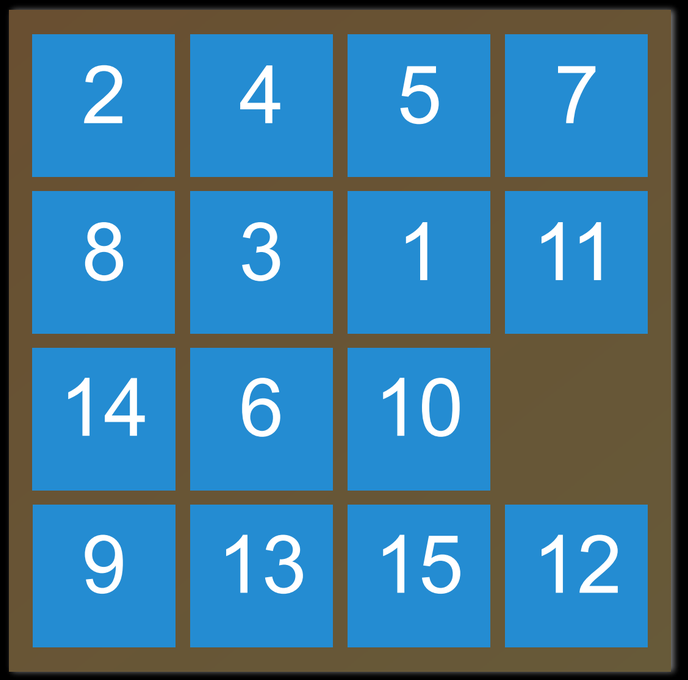
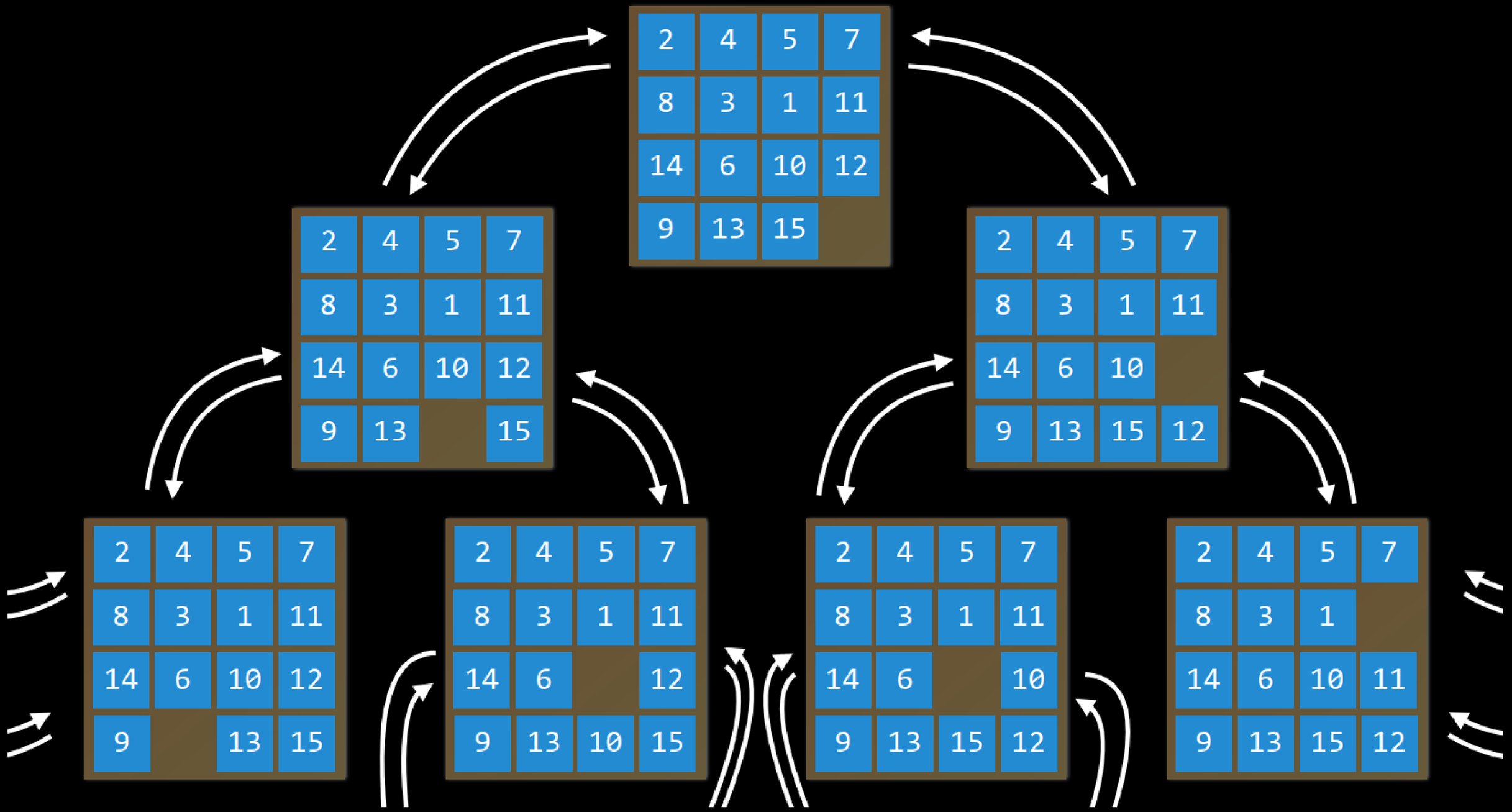
transition model

RESULT(

, ) =


state space

the set of all states reachable from the initial state by any sequence of actions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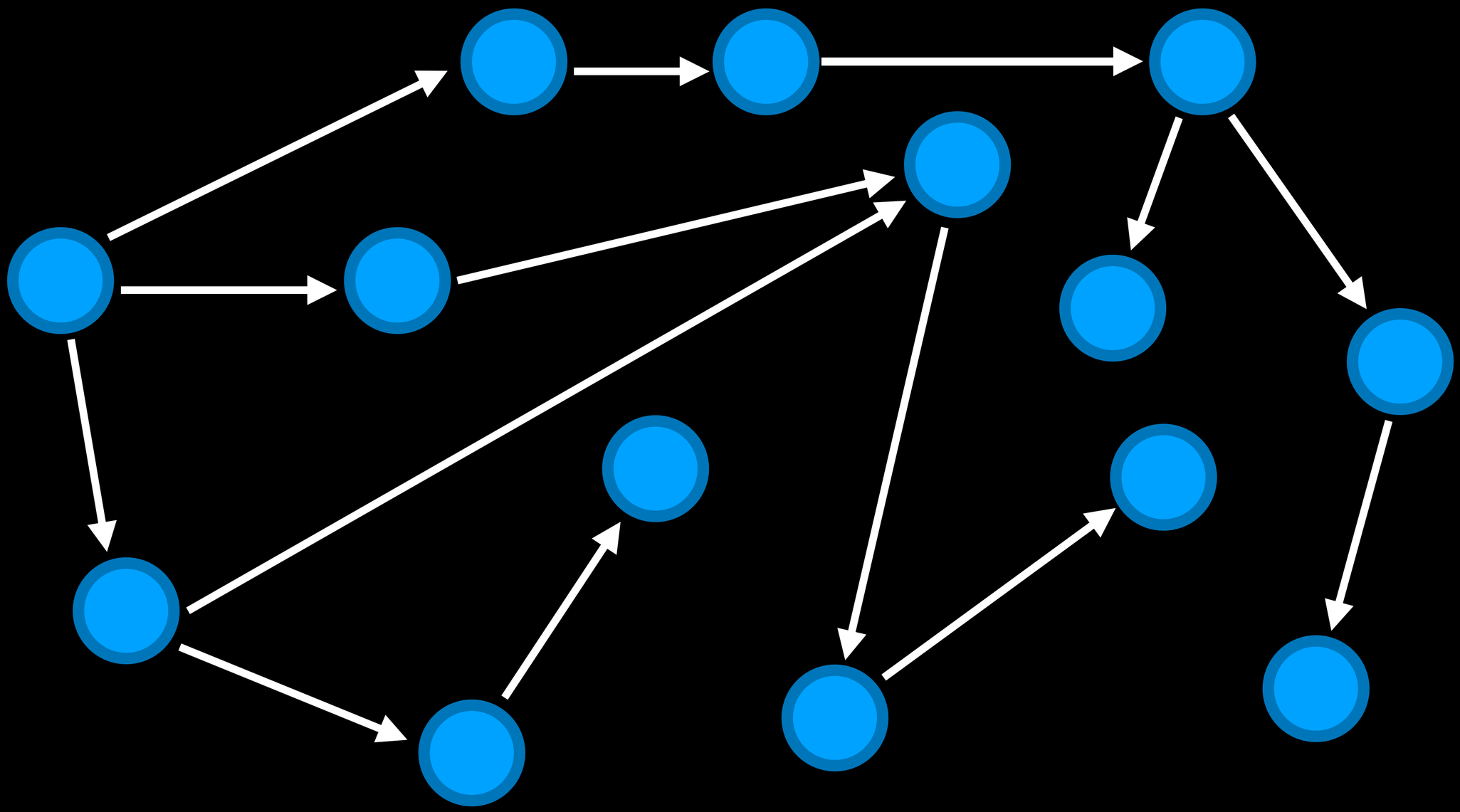
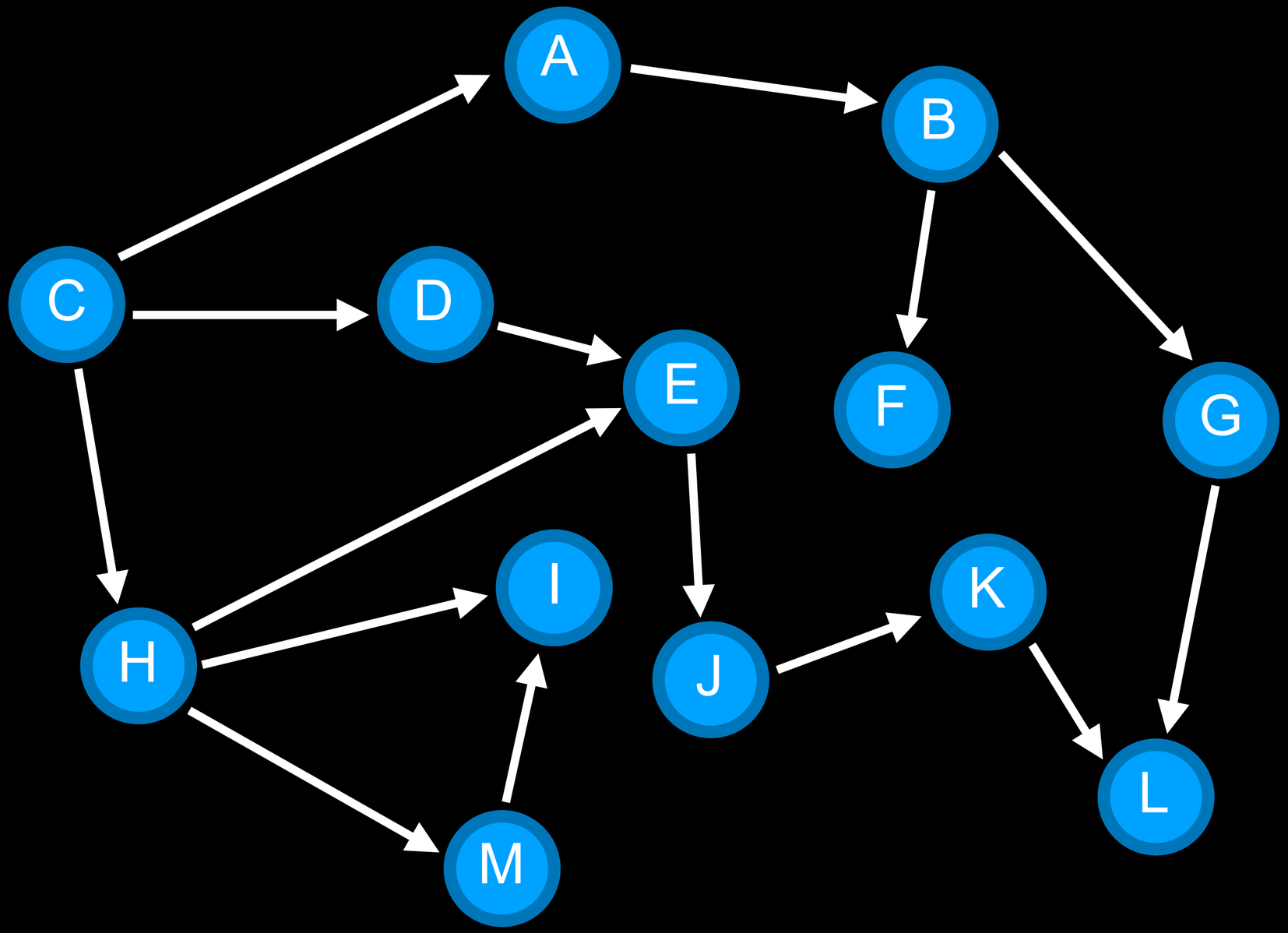


goal test

way to determine whether a given state is a goal state

path cost

numerical cost associated with a given path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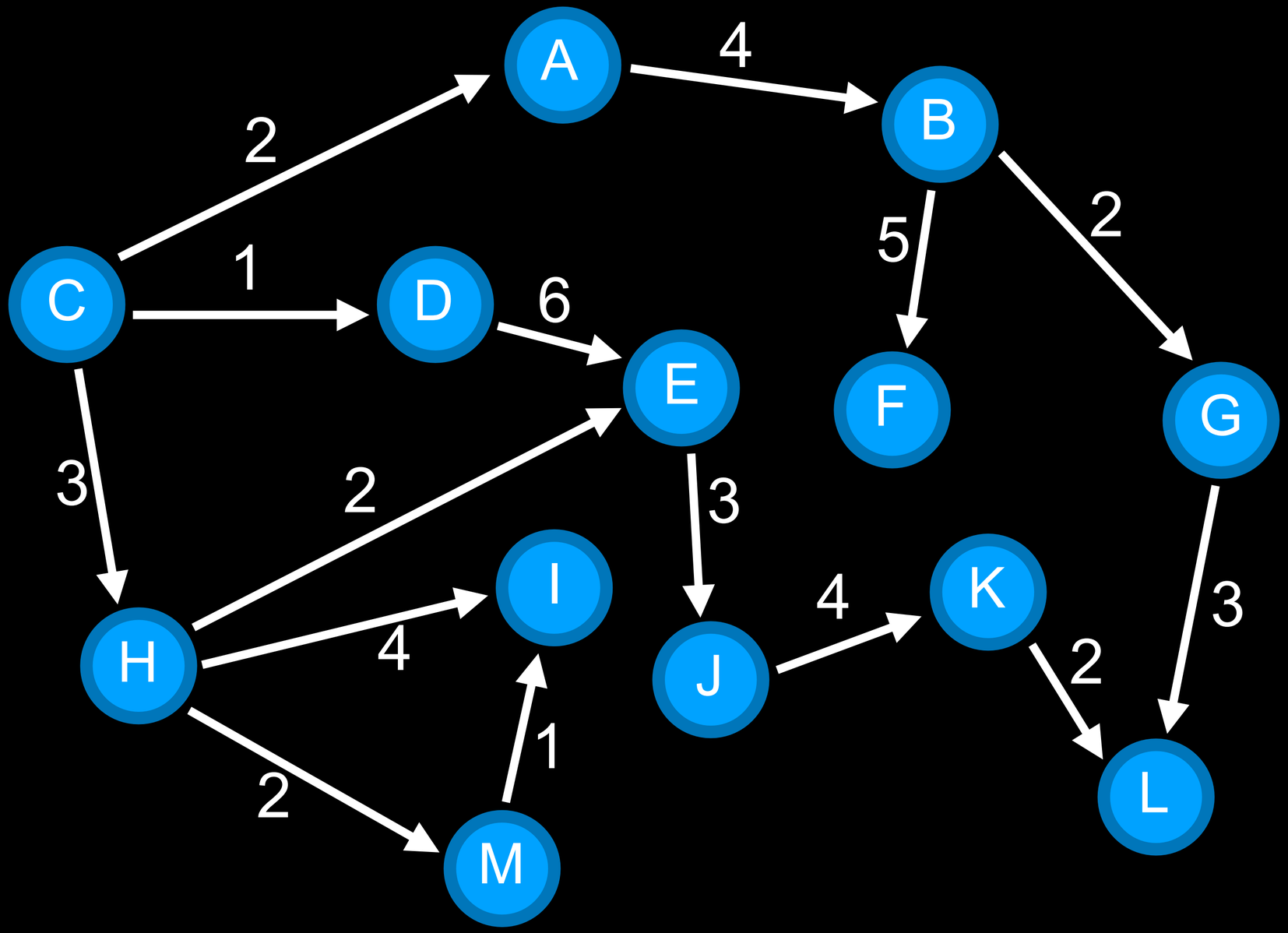
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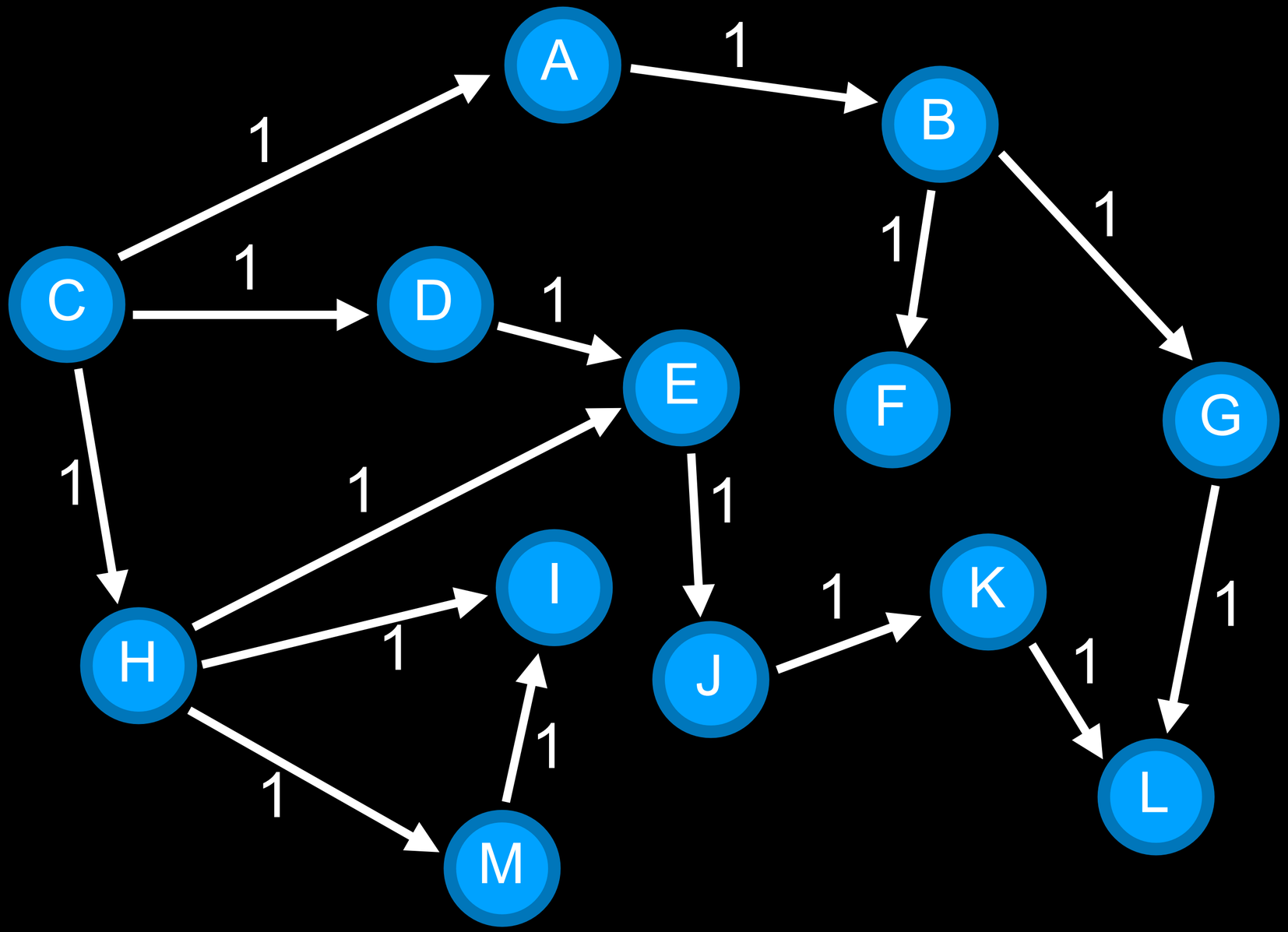
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Search Problems

- initial state
- actions
- transition model
- goal test
- path cost function

solution

a sequence of actions that leads from the initial state to a goal state

optimal solution

a solution that has the lowest path cost among all solutions

node

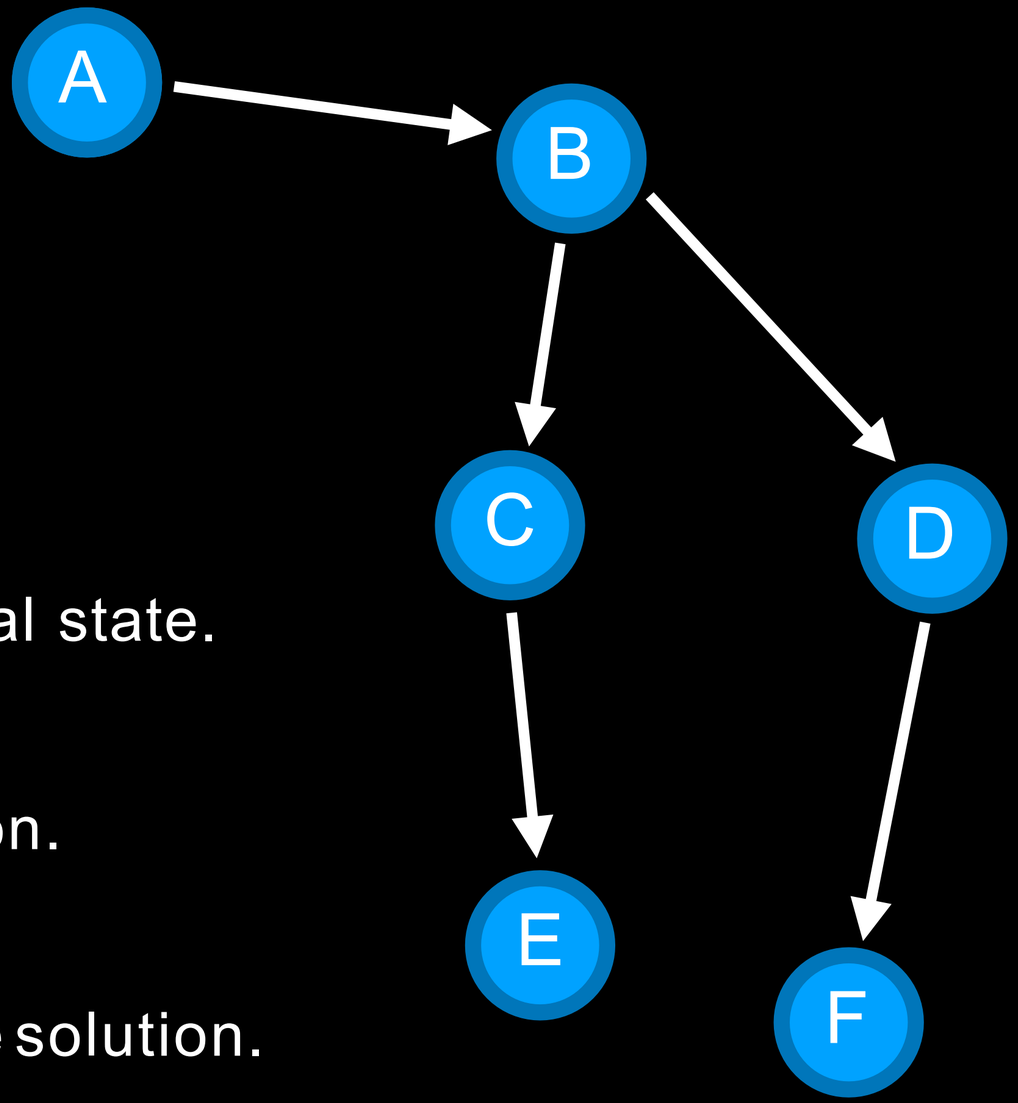
a data structure that keeps track of
- a **state**
- a **parent** (node that generated this node)
- an **action** (action applied to parent to get node)
- a **path cost** (from initial state to node)

Approach

- Start with a **frontier** that contains the initial state.
- Repeat:
  - If the frontier is empty, then no solution.
  - Remove a node from the frontier.
  - If node contains goal state, return the solution.
  - **Expand** node, add resulting nodes to the frontier.

Find a path from A to E.

Frontier



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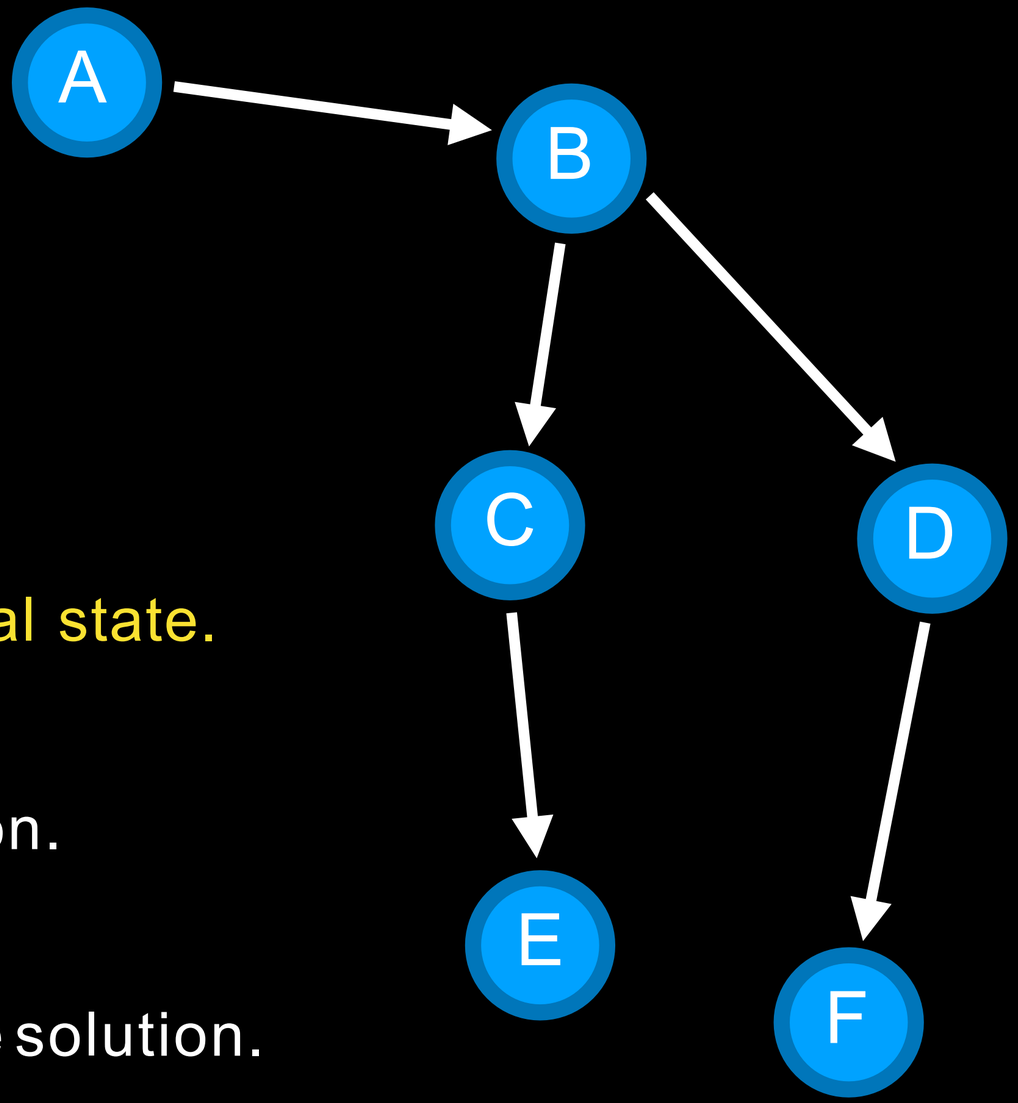

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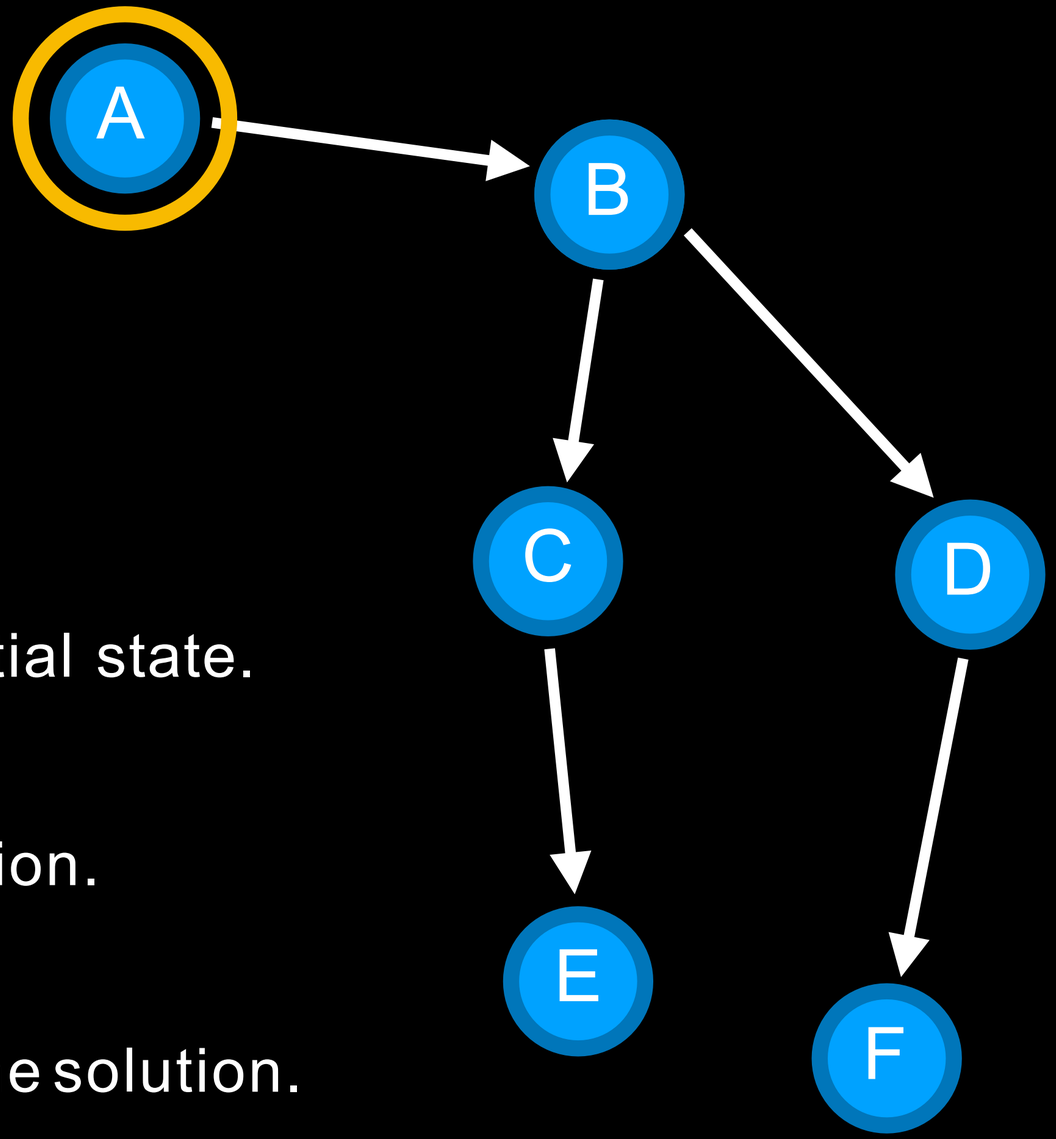

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Frontier

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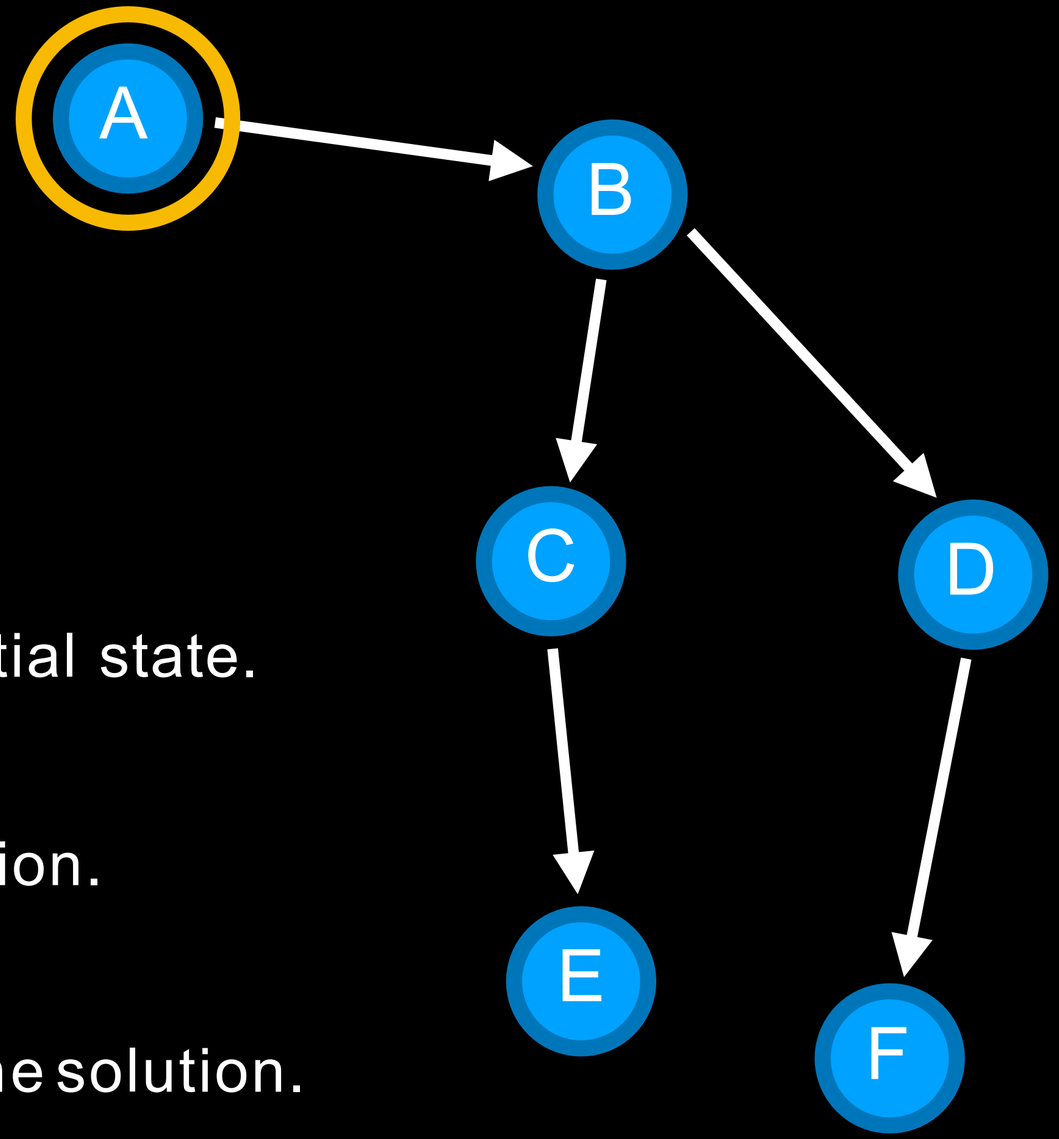

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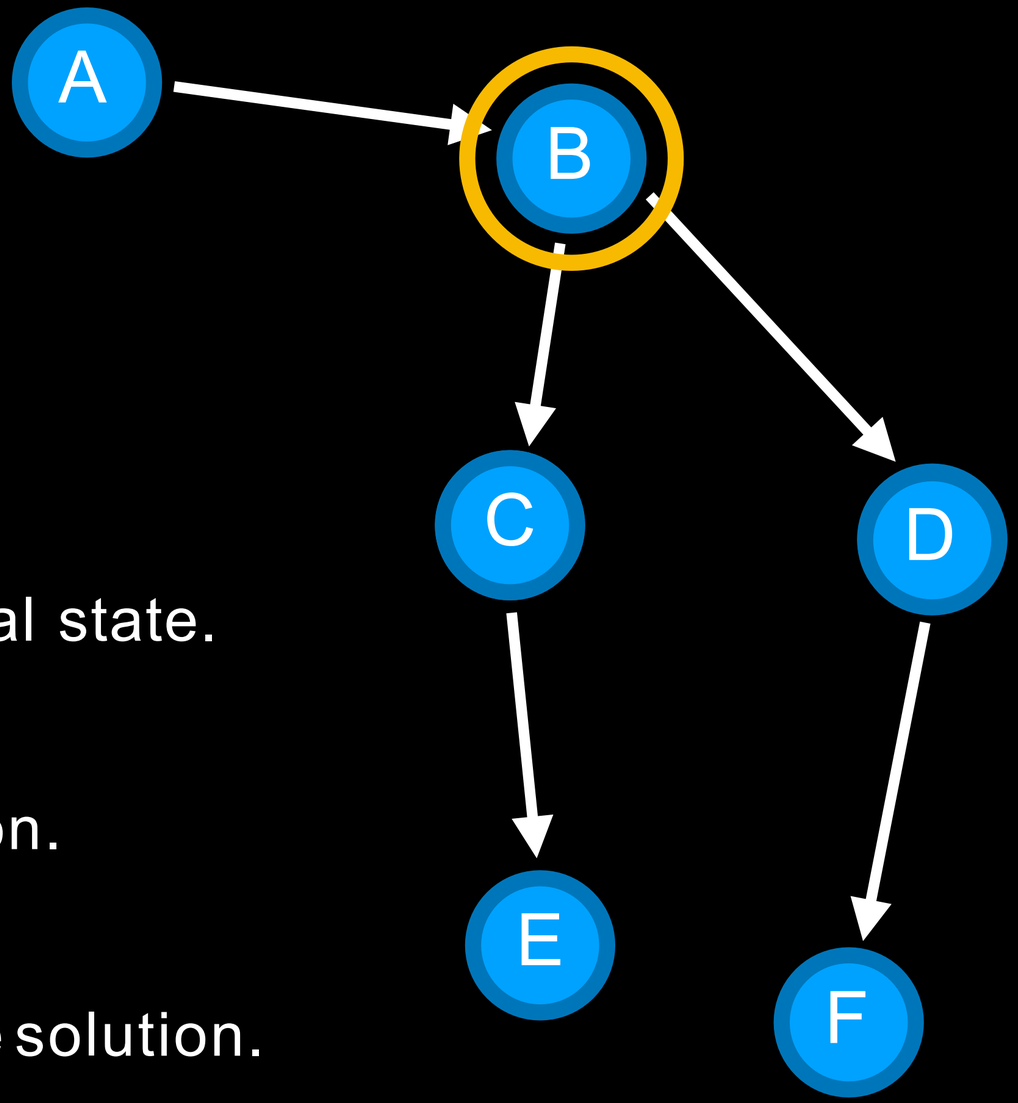

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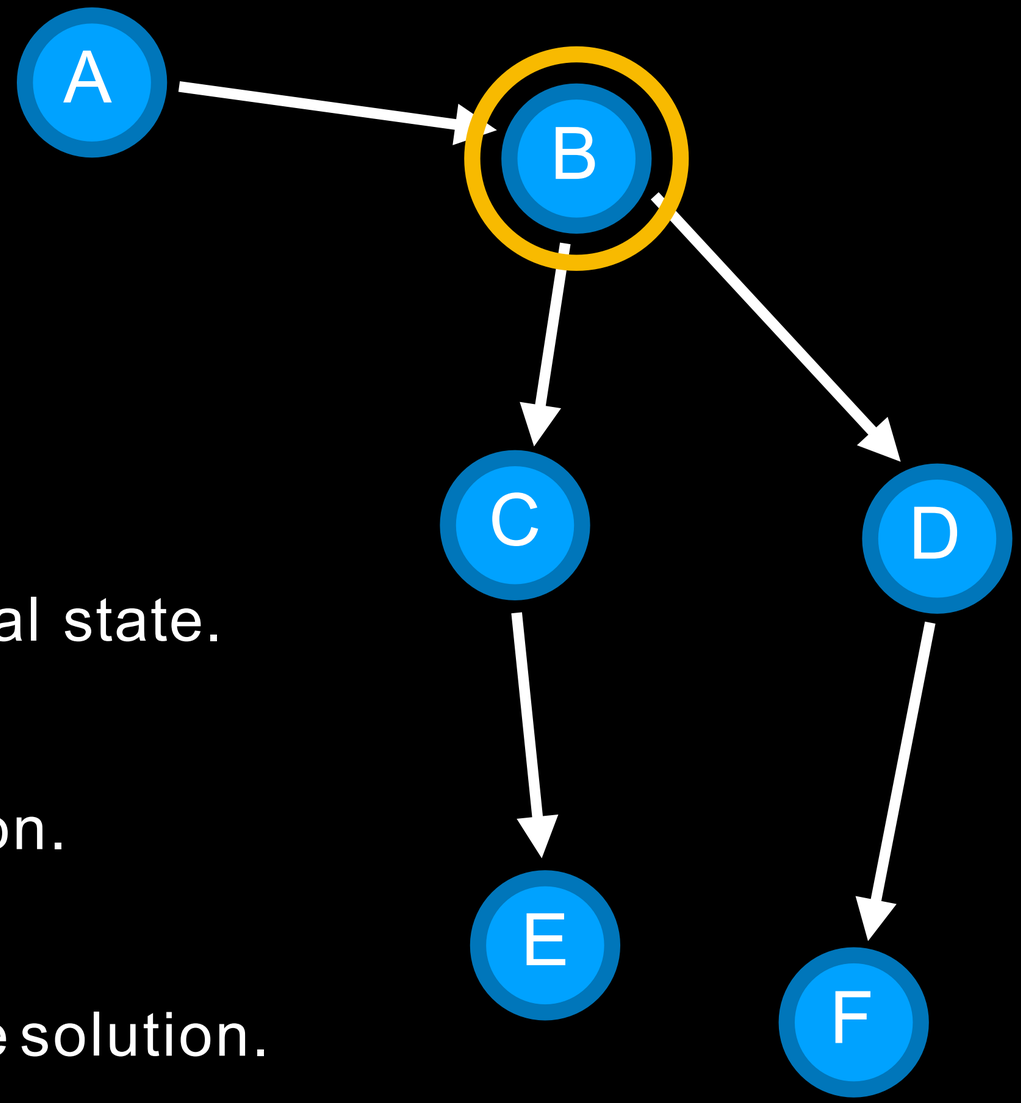
Frontier

C D



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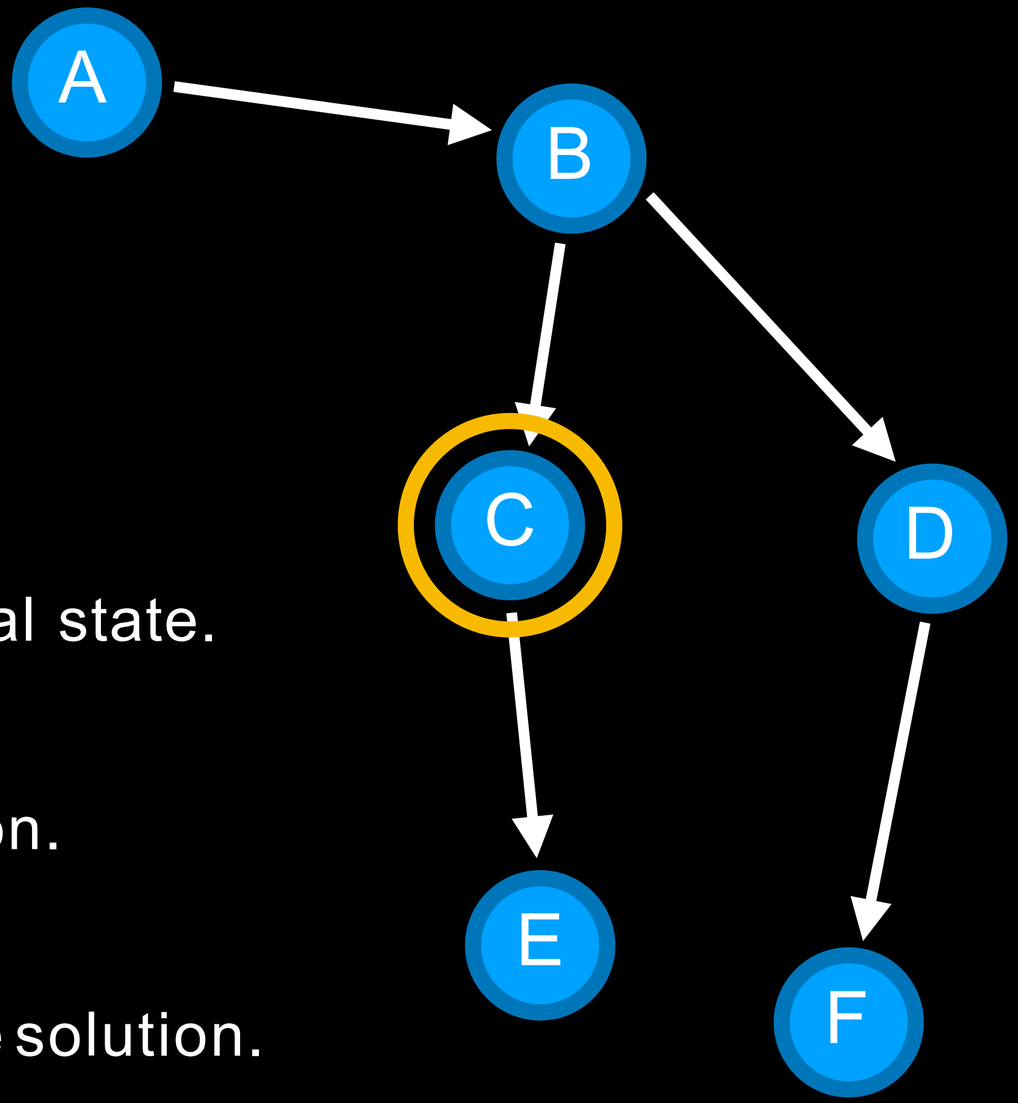
Frontier

D



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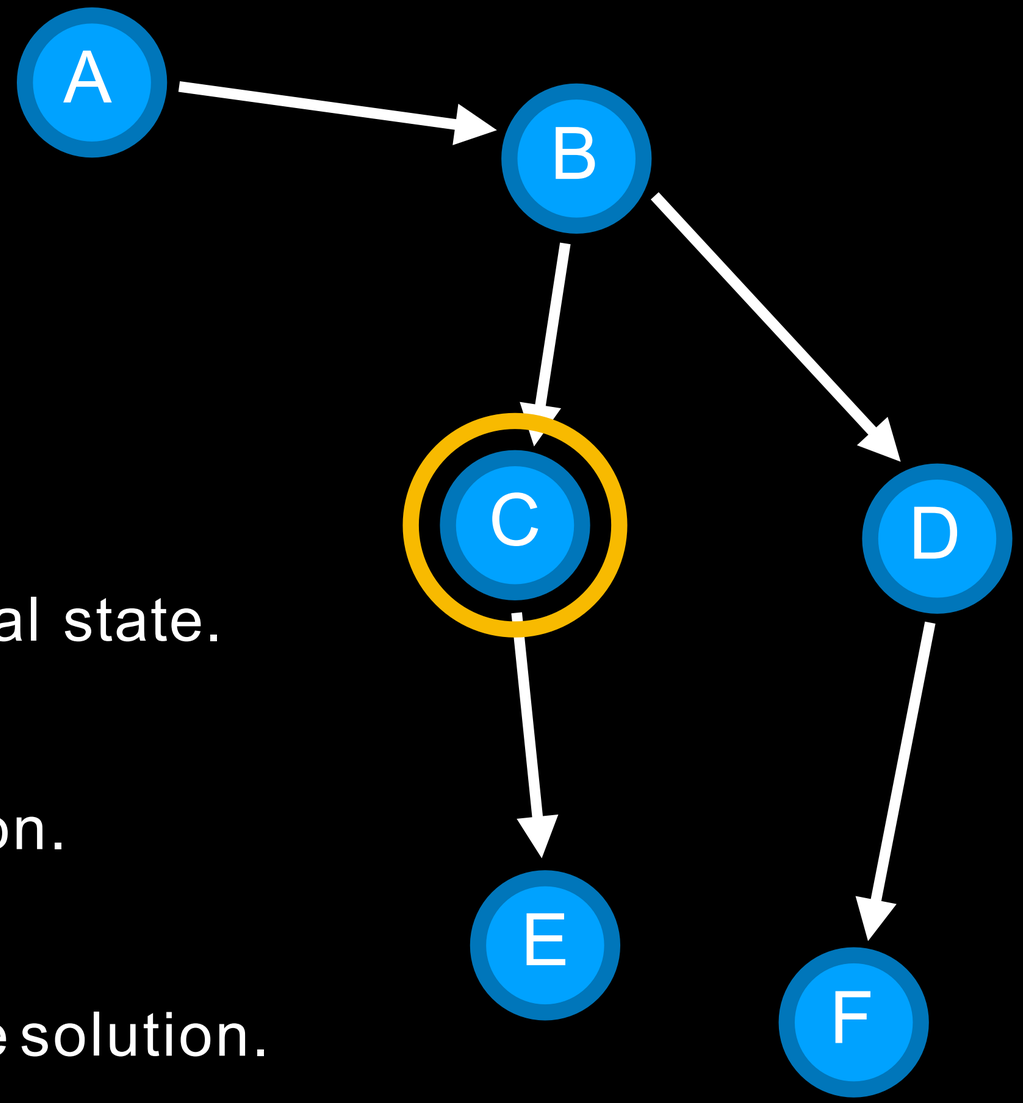

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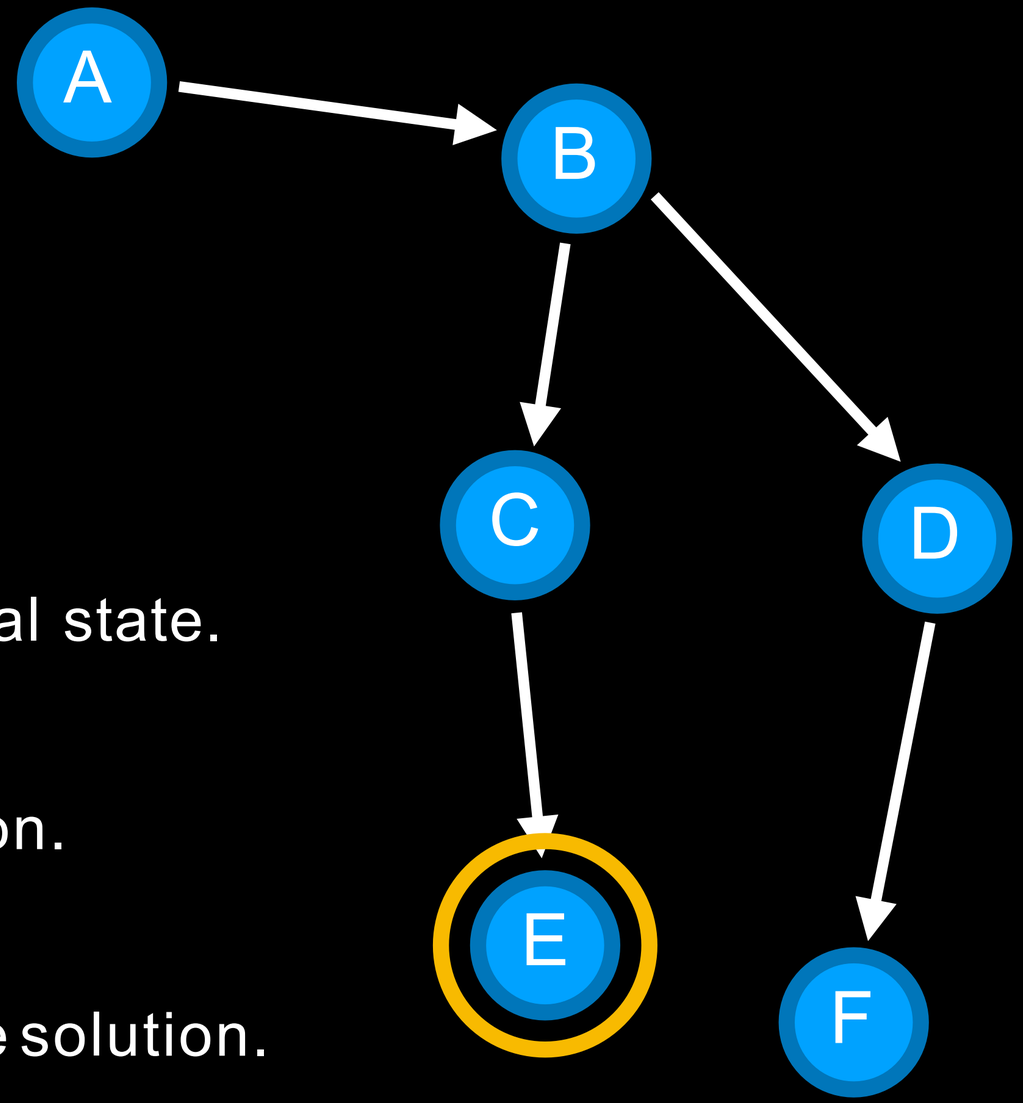

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Find a path from A to E.

Frontier

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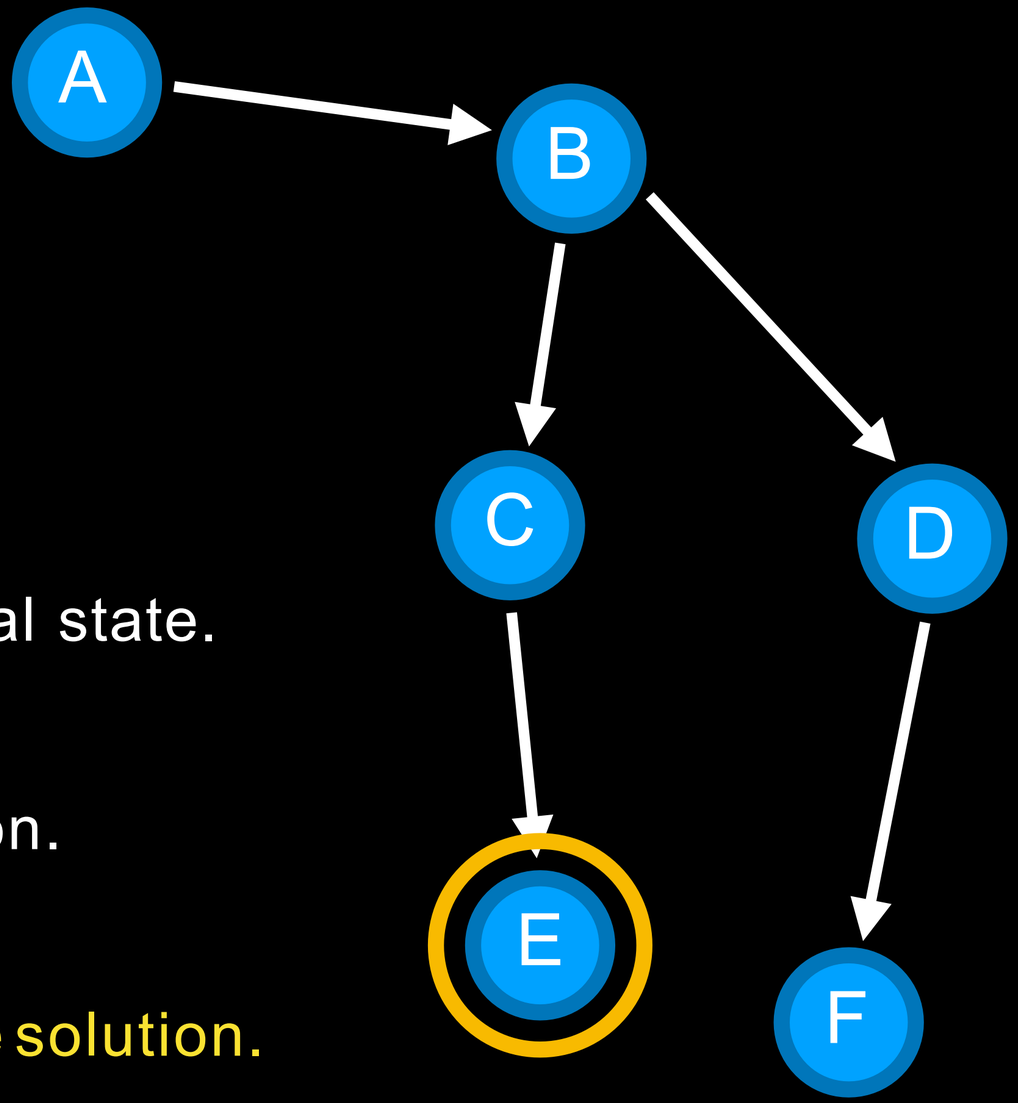

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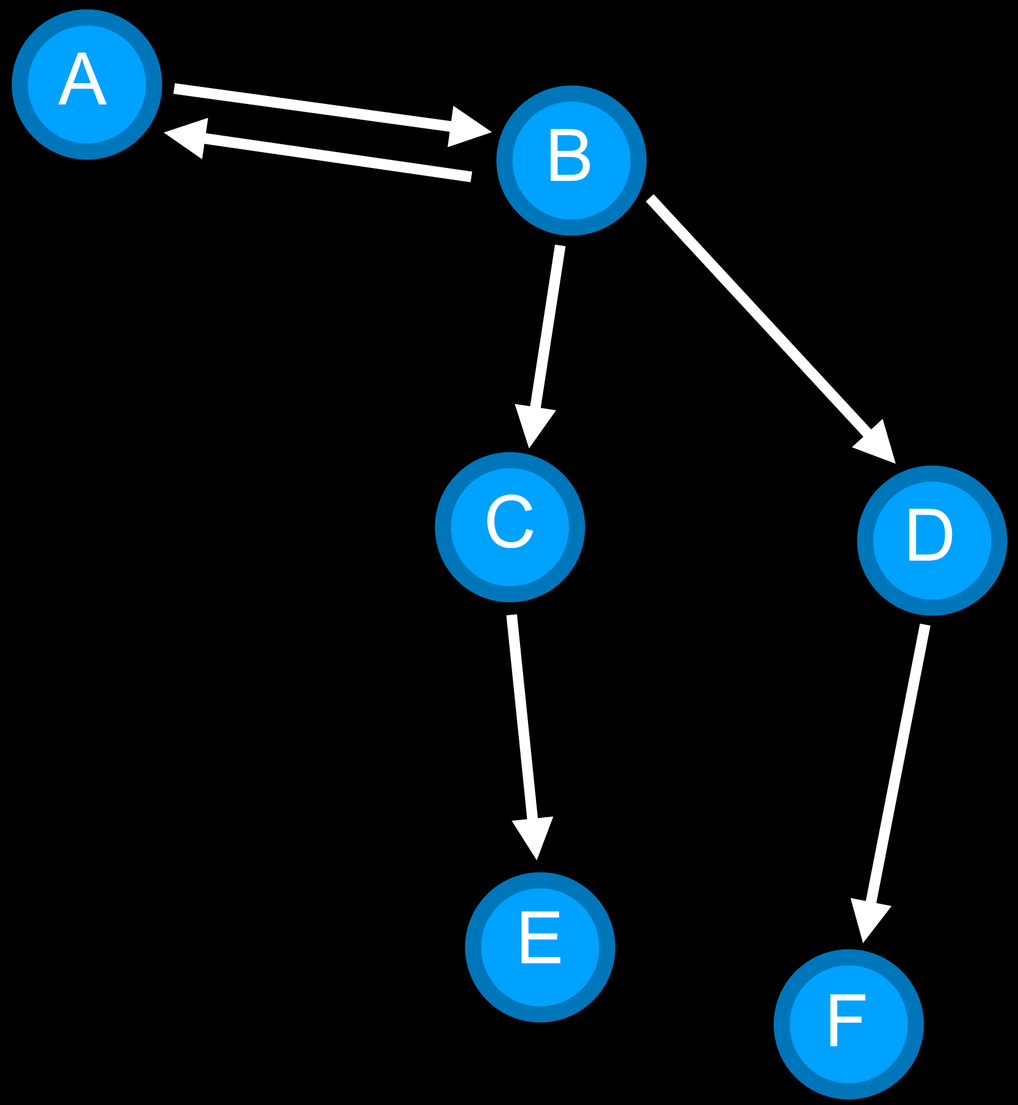
What could go wrong?

Find a path from A to E.

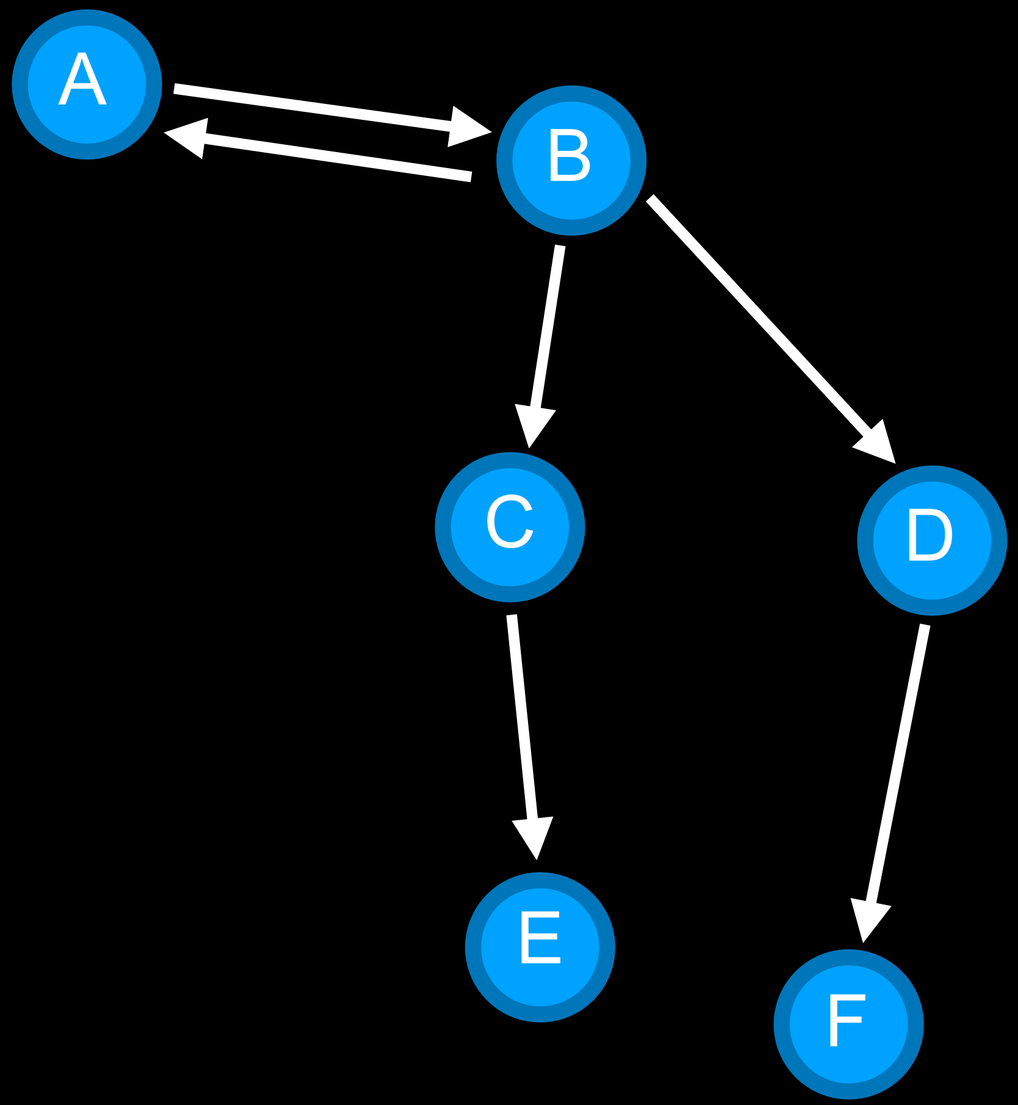
Frontier



Find a path from A to E.

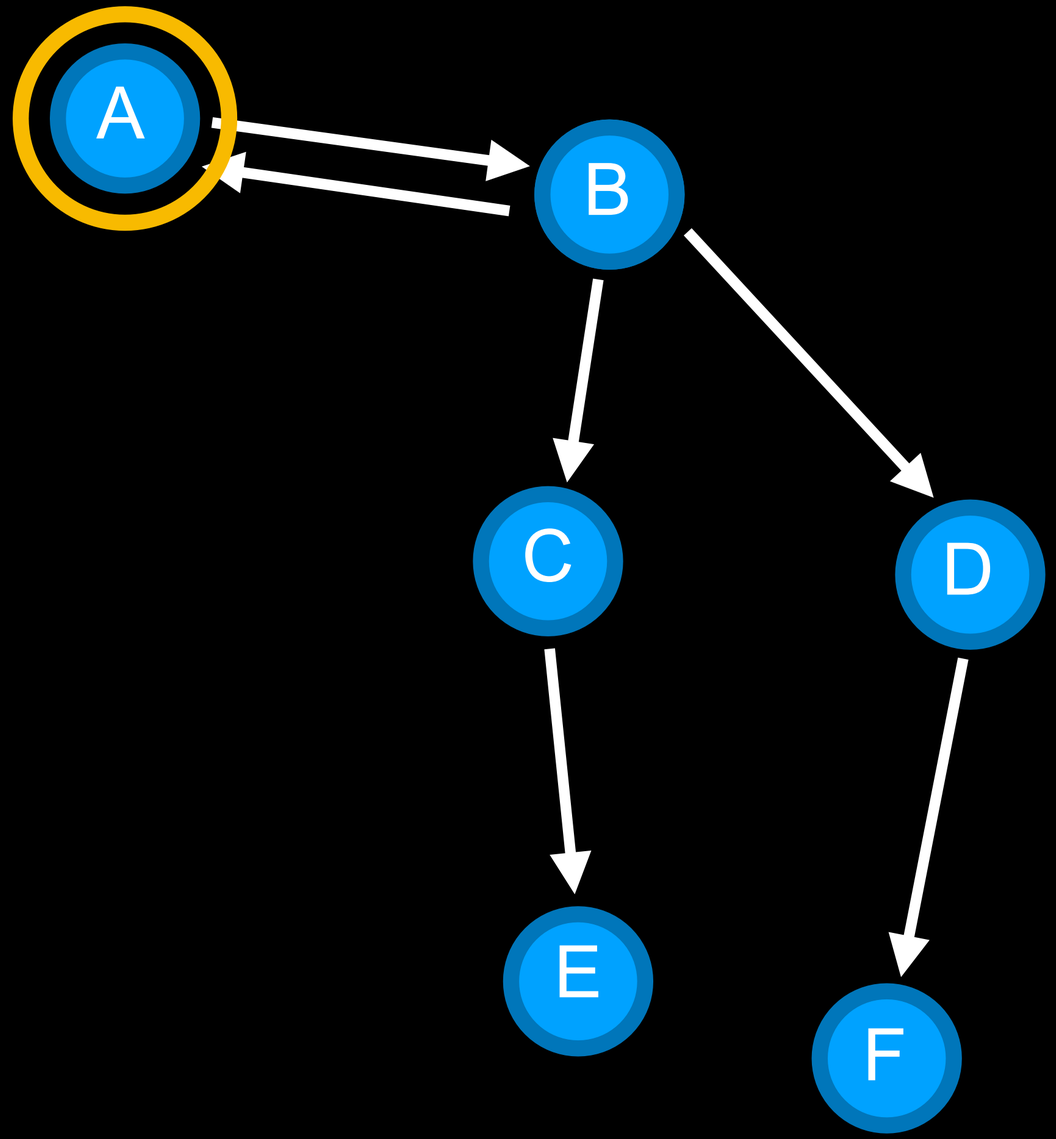
Frontier

A



Find a path from A to E.

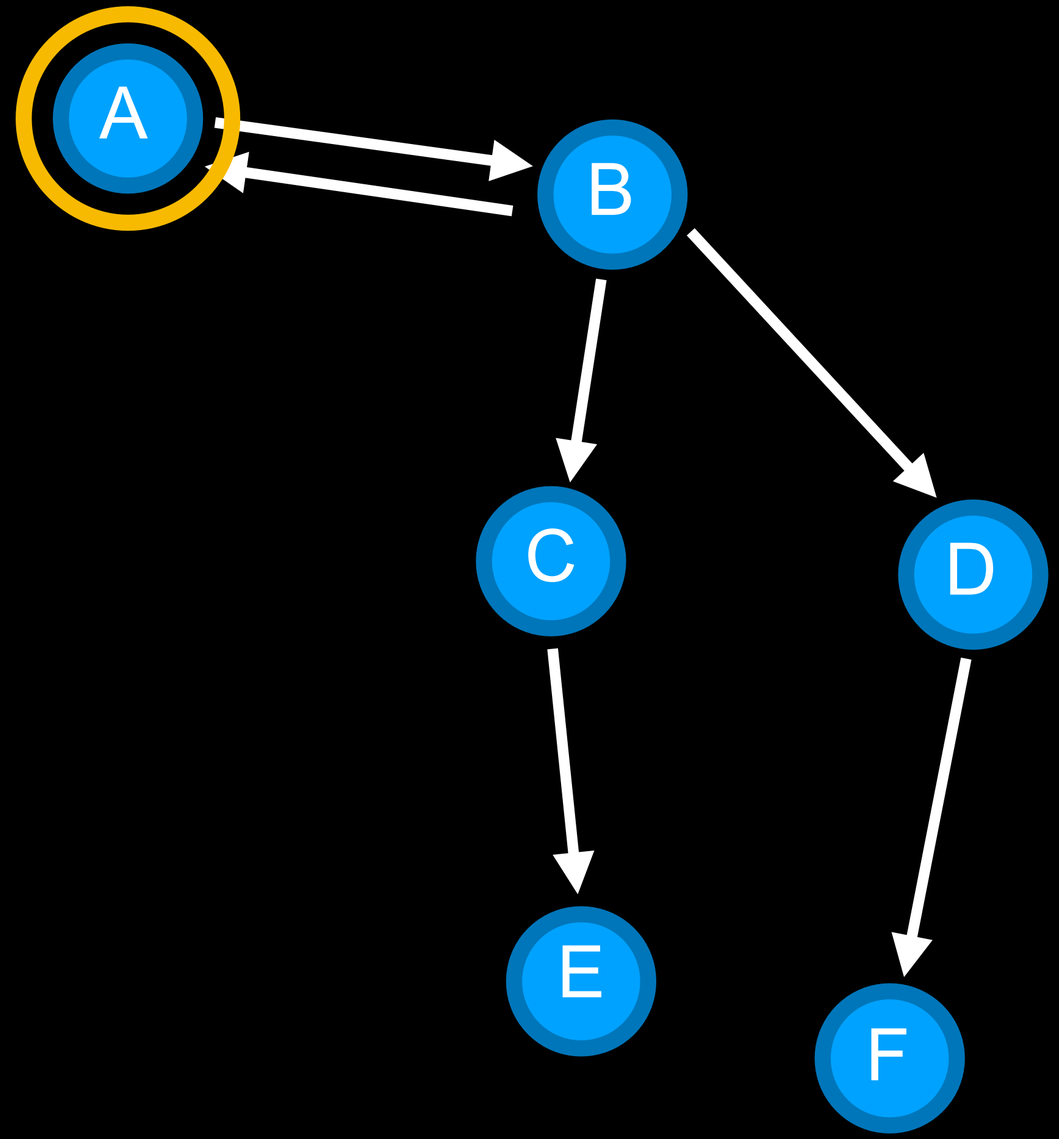
Frontier



Find a path from A to E.

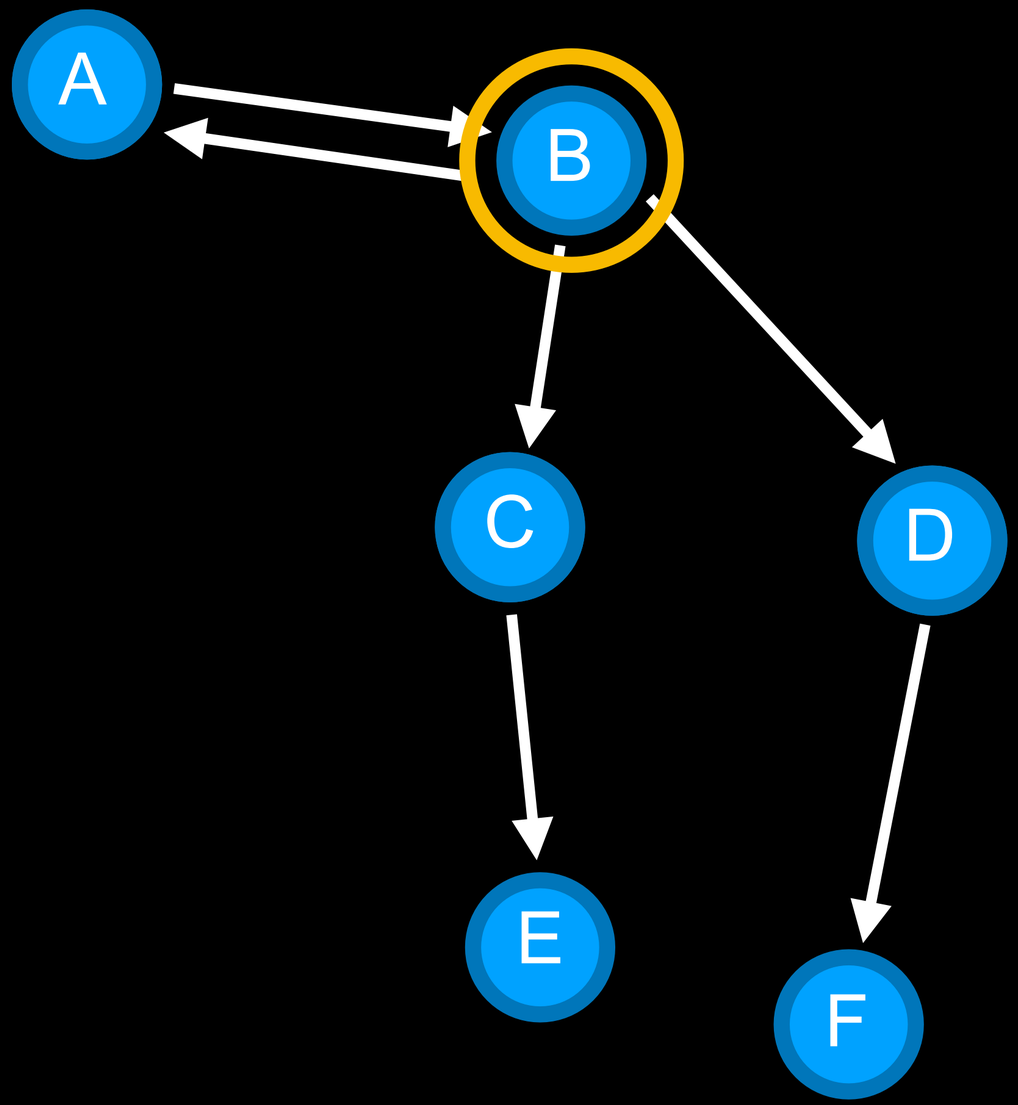
Frontier

B



Find a path from A to E.

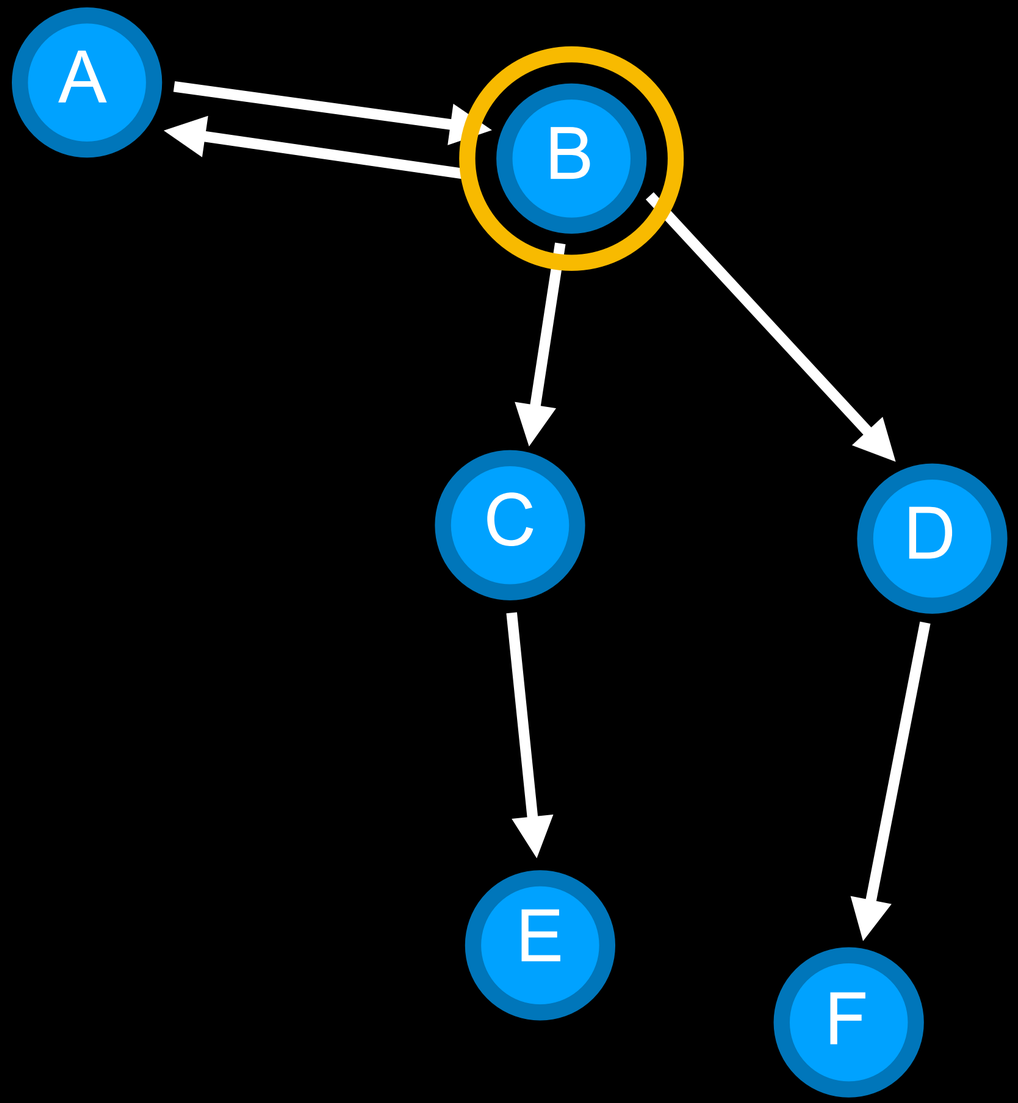
Frontier



Find a path from A to E.

Frontier

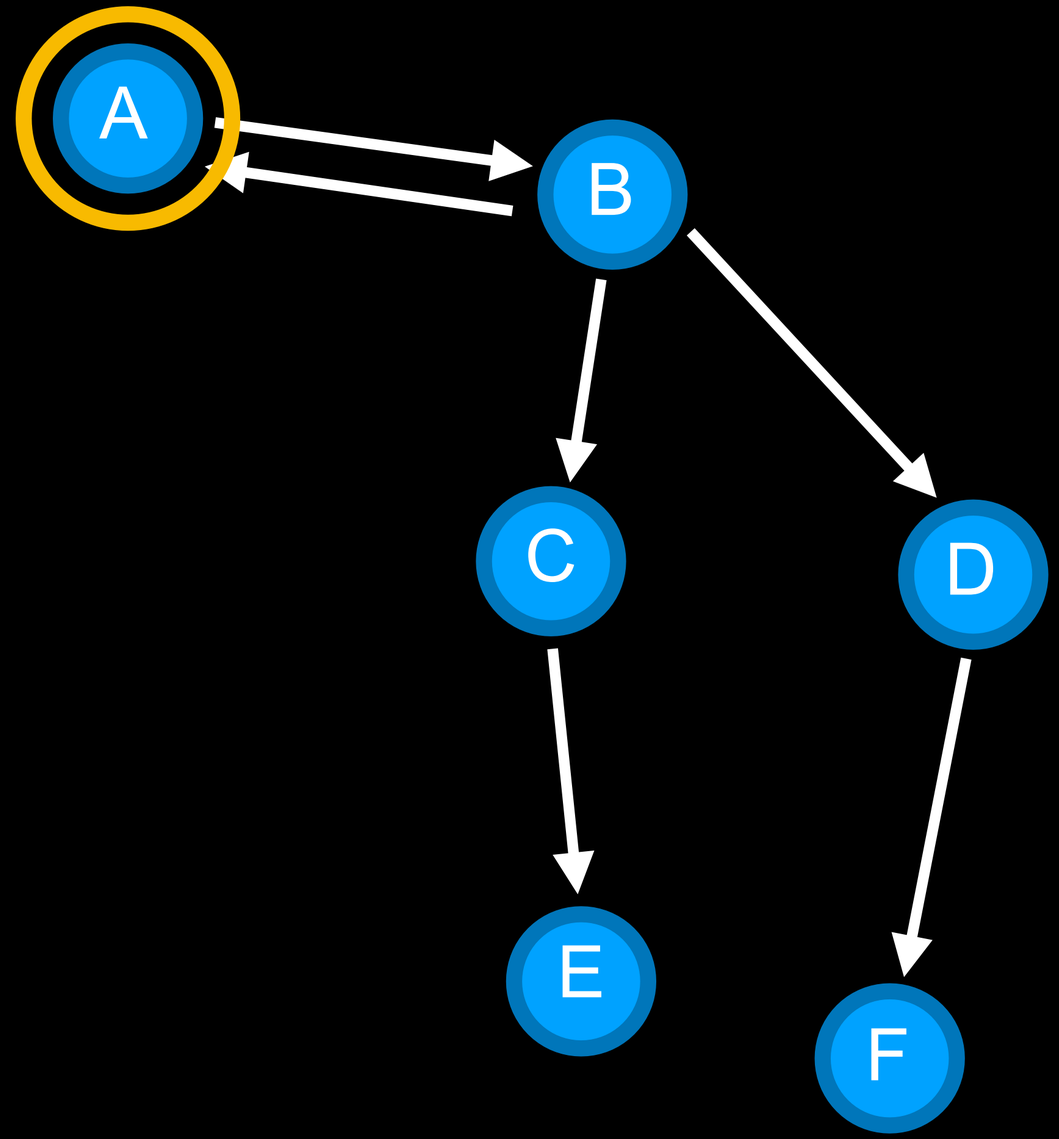
A C D



Find a path from A to E.

Frontier

C D



Revised Approach

- Start with a **frontier** that contains the initial state.
- Start with an empty **explored set**.
- Repeat:
  - If the frontier is empty, then no solution.
  - Remove a node from the frontier.
  - If node contains goal state, return the solution.
  - Add the node to the explored set.
  - **Expand** node, add resulting nodes to the frontier if they aren't already in the frontier or the explored set.

Revised Approach

- Start with a **frontier** that contains the initial state.
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  - **Expand** node, add resulting nodes to the frontier if they aren't already in the frontier or the explored set.

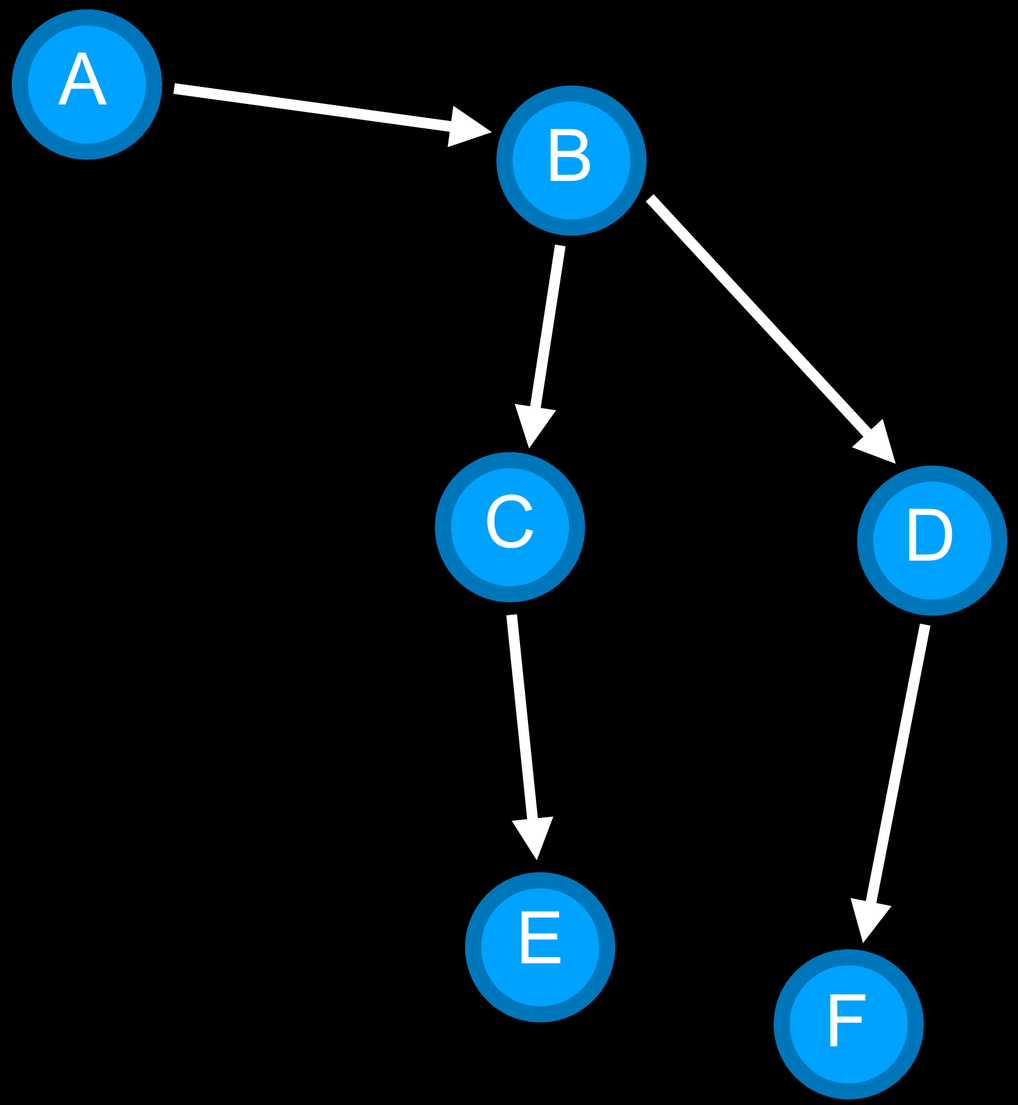
stack

last-in first-out data type

Find a path from A to E.

Frontier

Explored Set

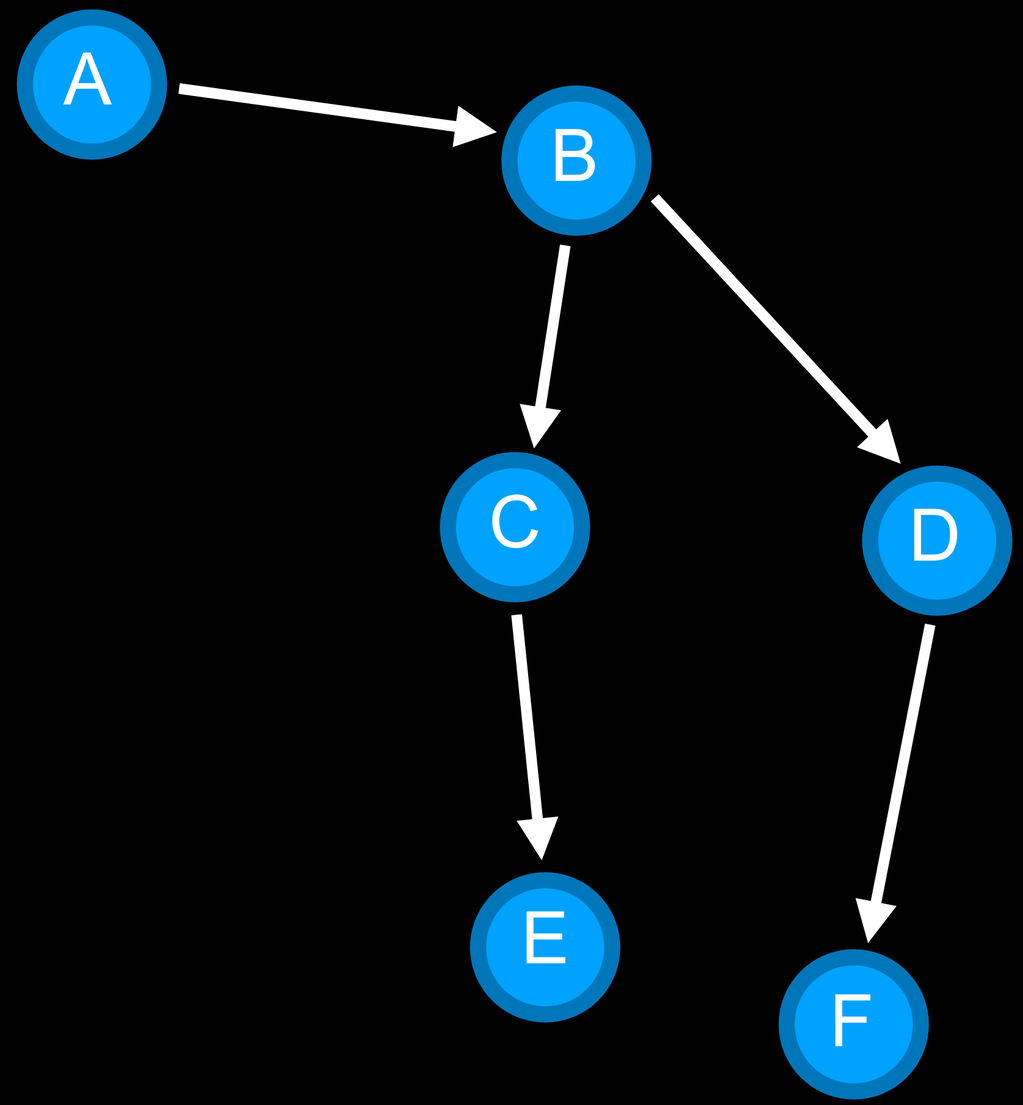


Find a path from A to E.

Frontier

A

Explored Set

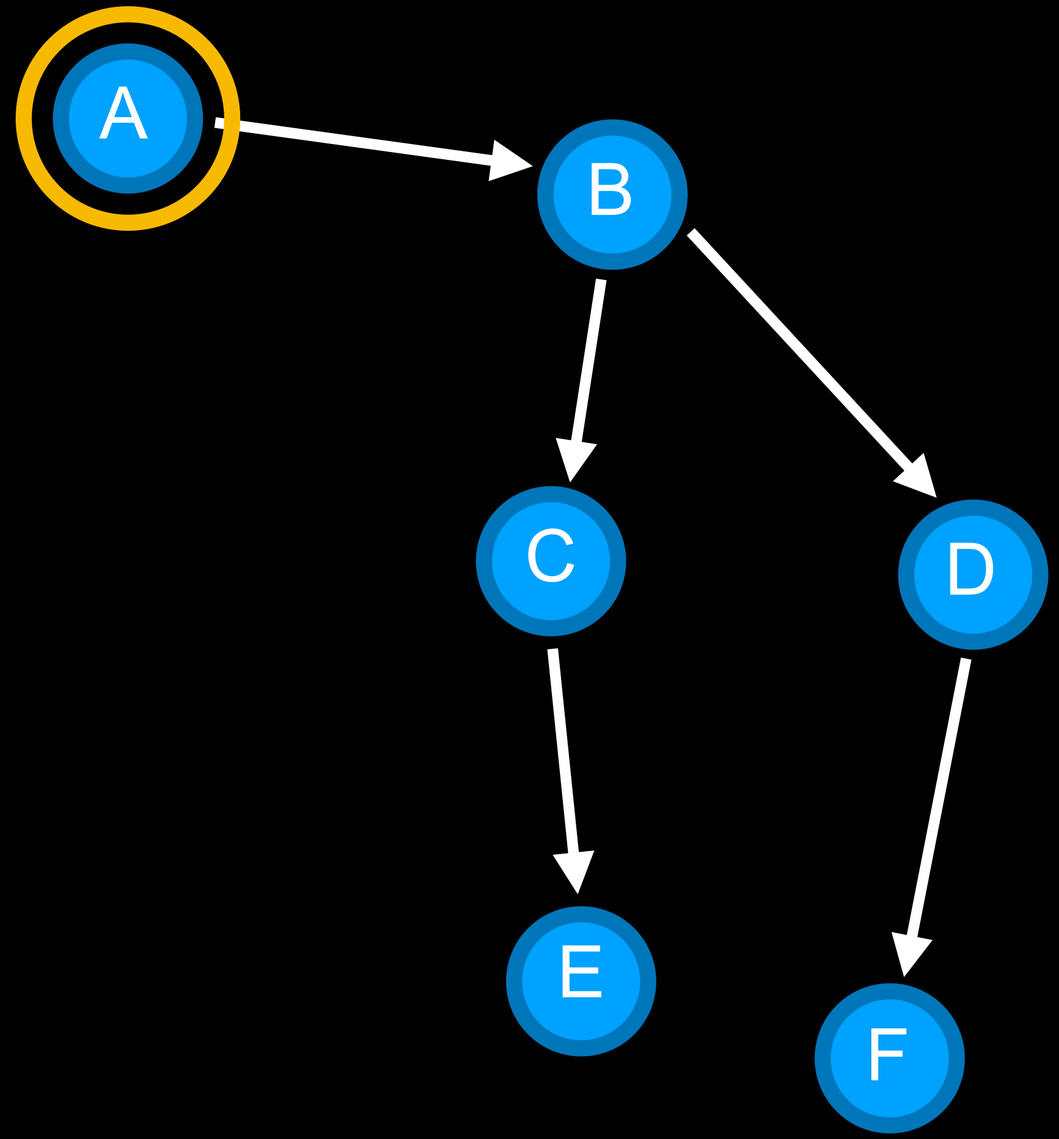


Find a path from A to E.

Frontier

Explored Set

A



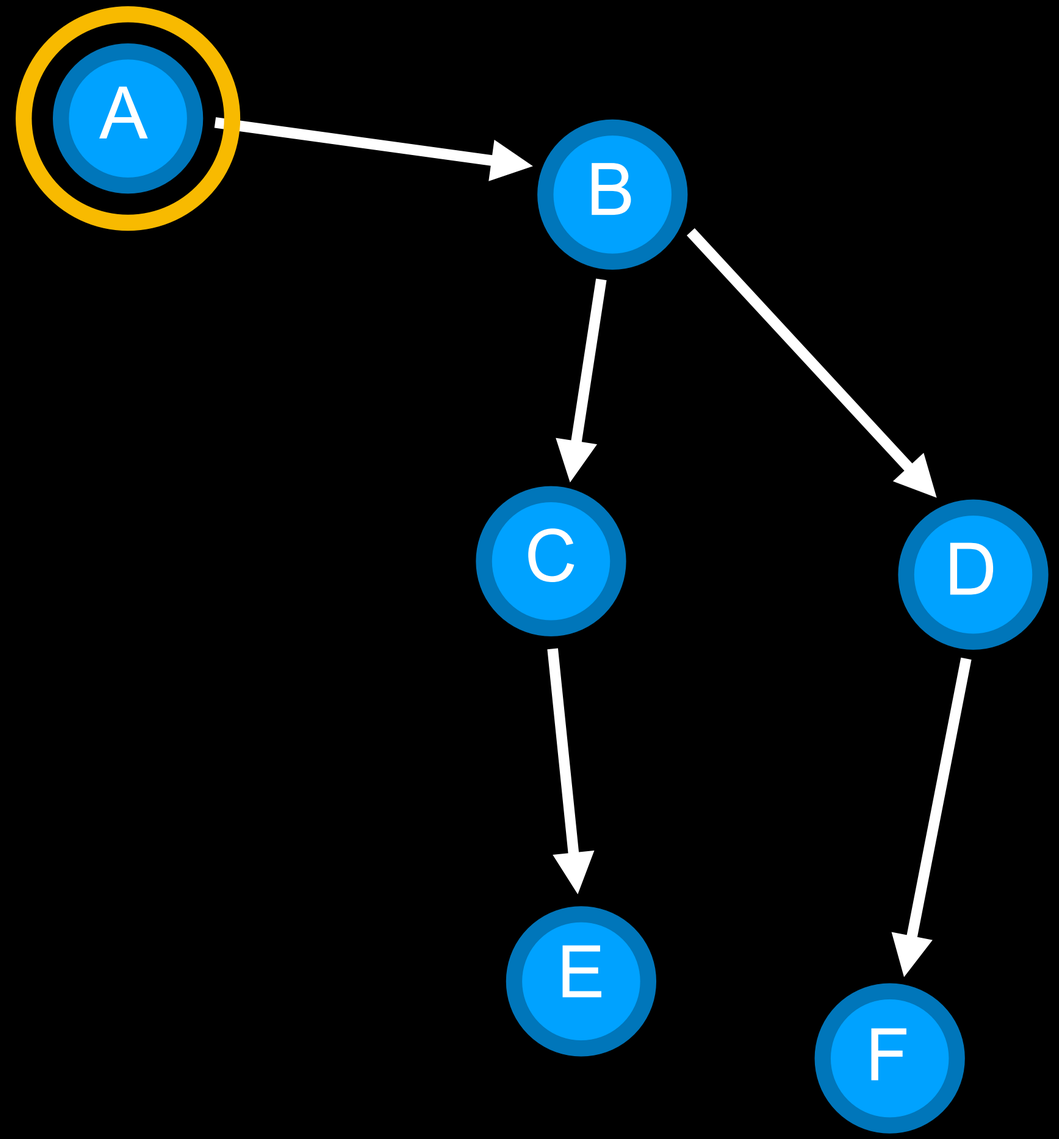
Find a path from A to E.

Frontier

B

Explored Set

A

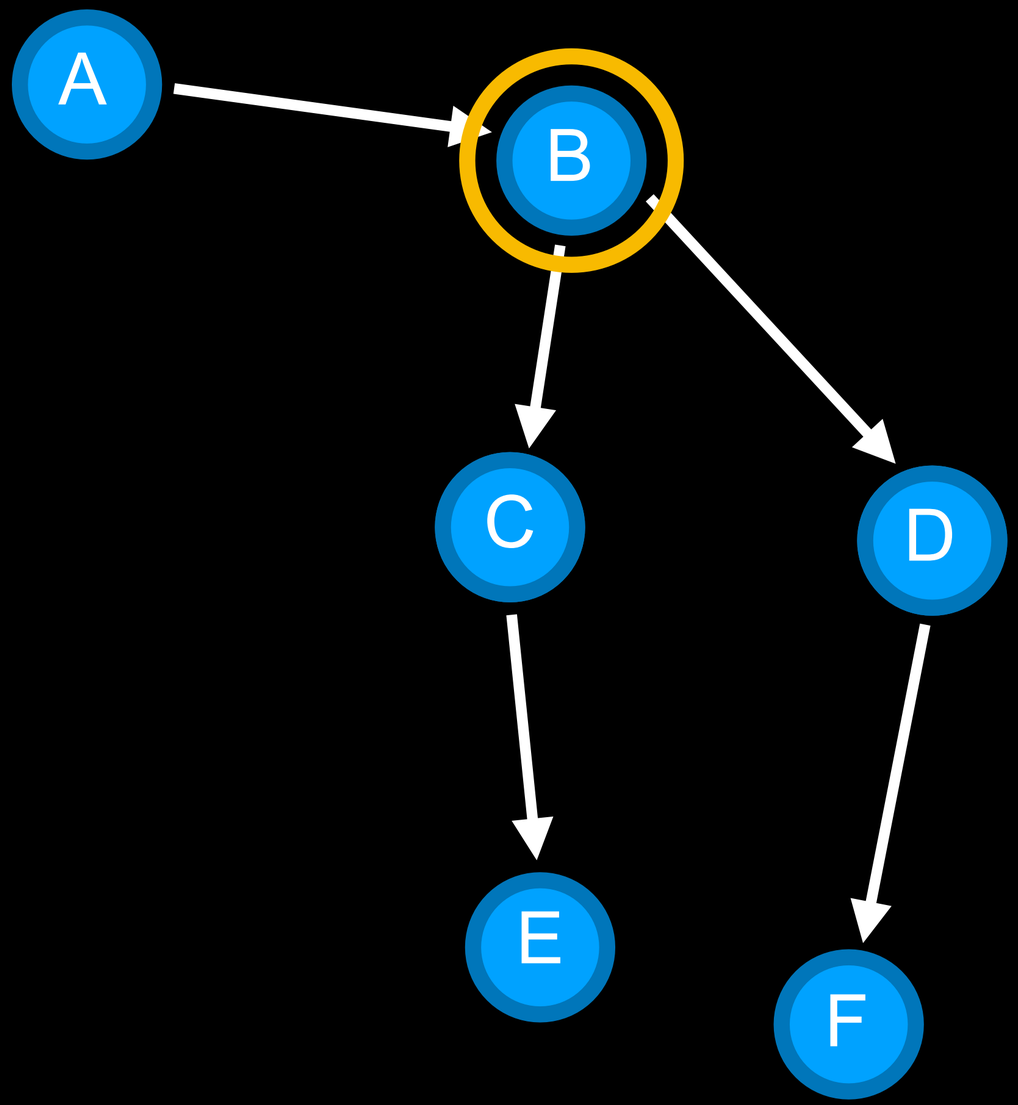


Find a path from A to E.

Frontier

Explored Set

A B



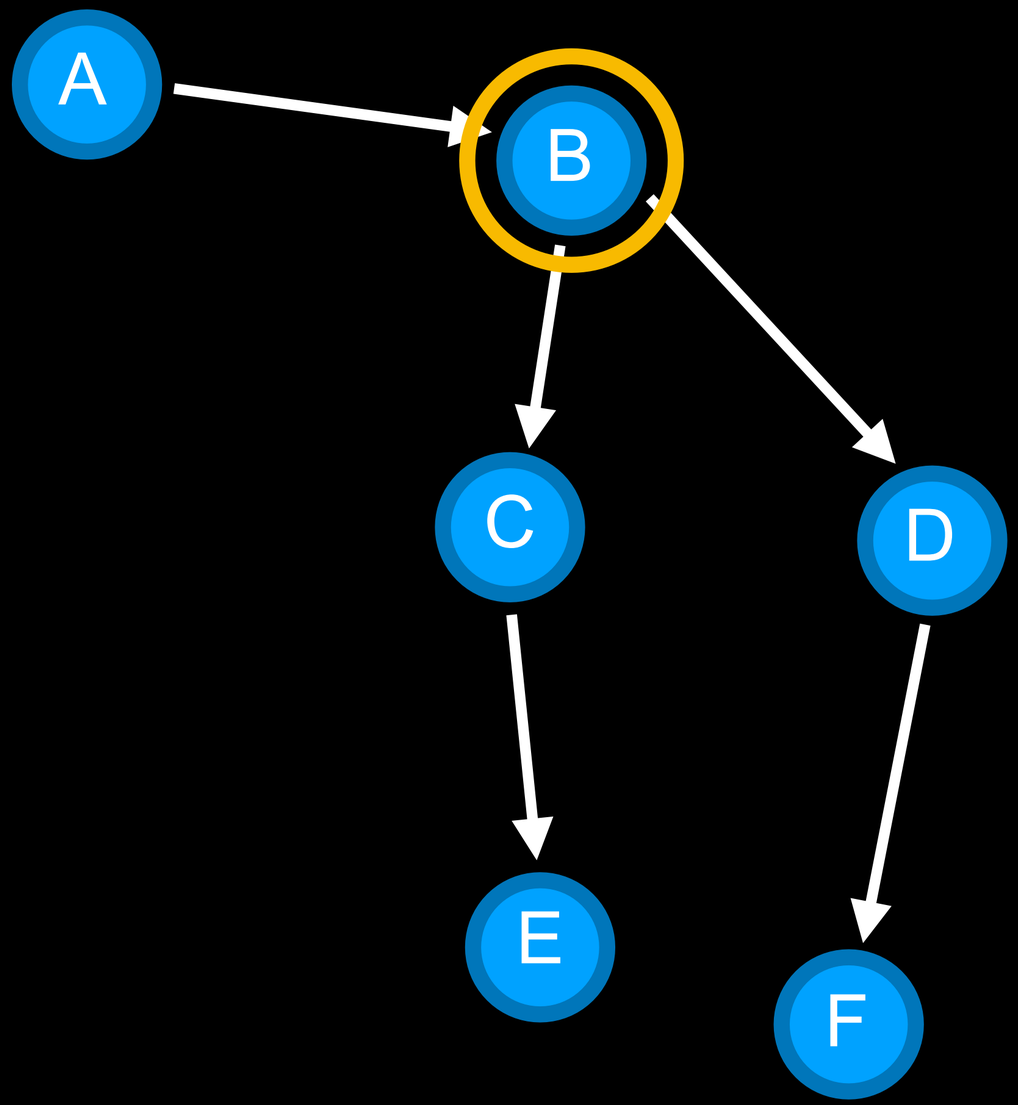
Find a path from A to E.

Frontier

C D

Explored Set

A B



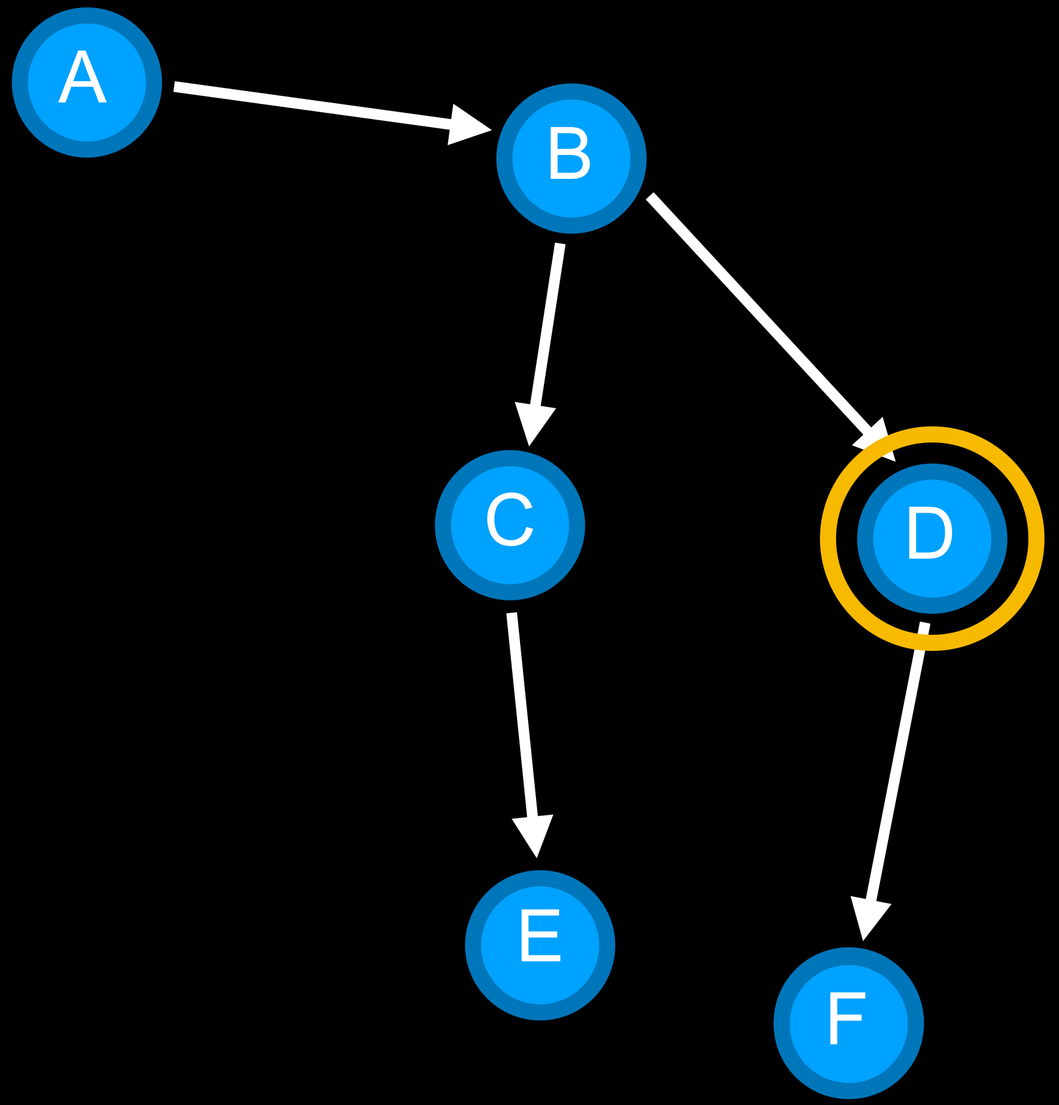
Find a path from A to E.

Frontier

C

Explored Set

A B D



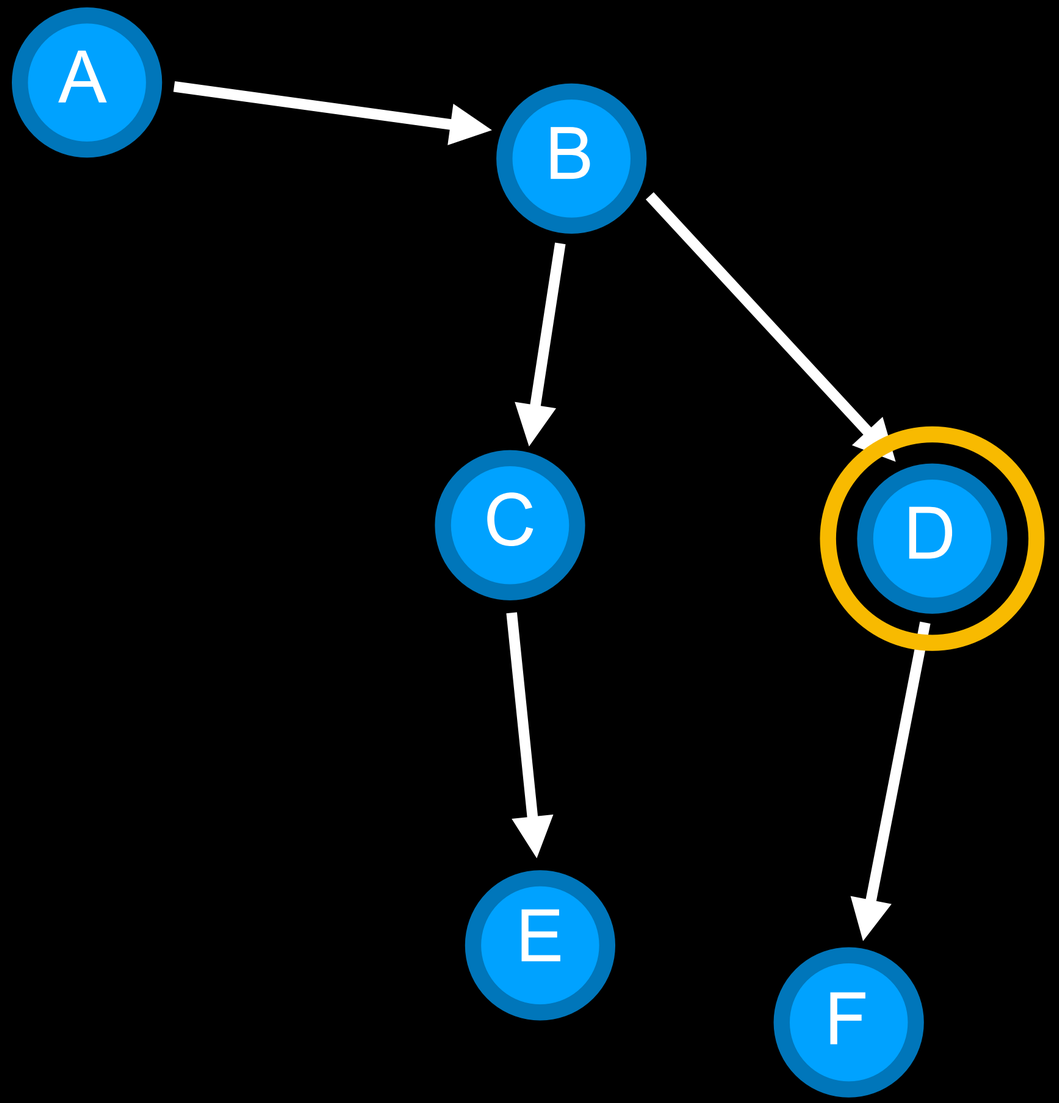
Find a path from A to E.

Frontier

C F

Explored Set

A B D



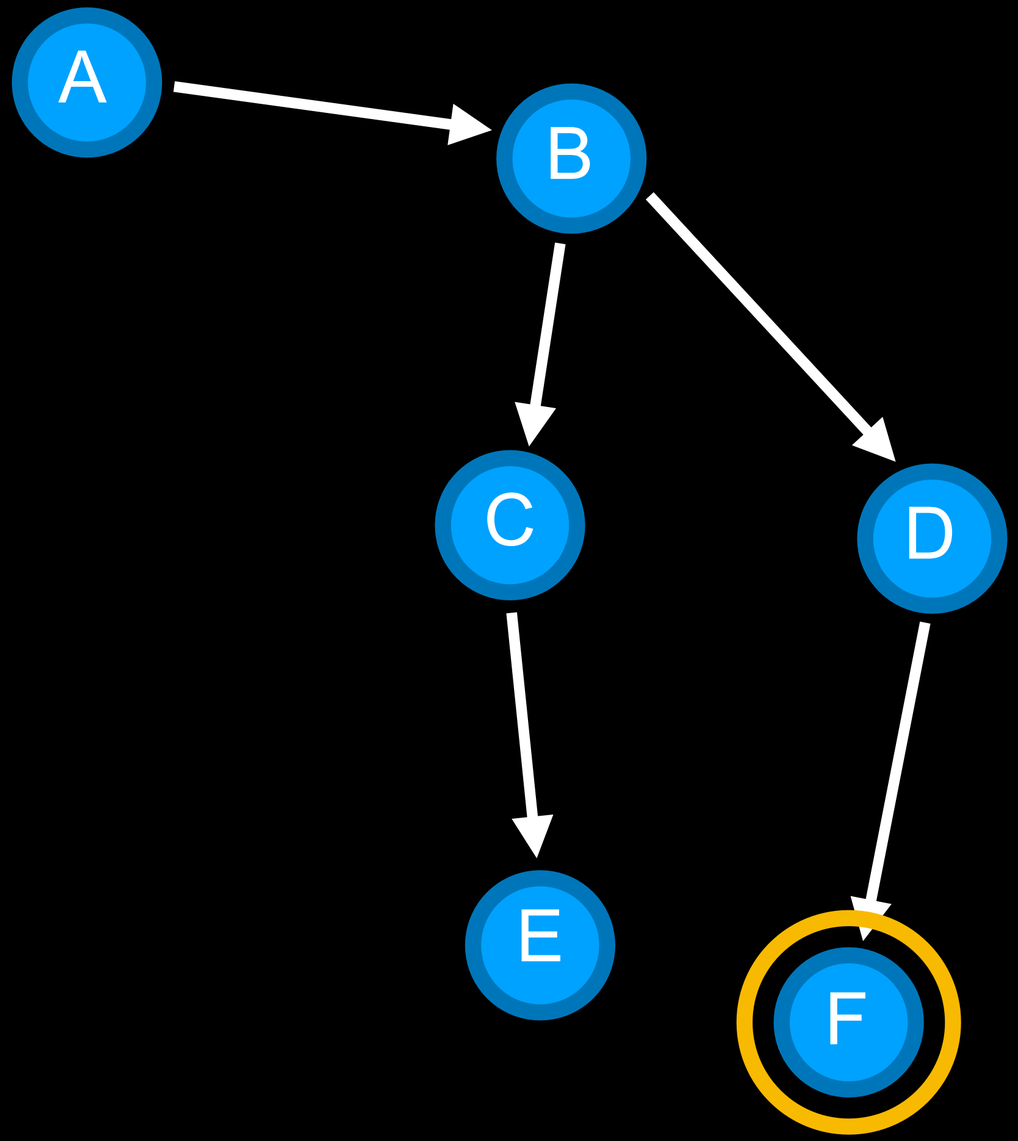
Find a path from A to E.

Frontier

C

Explored Set

A B D F

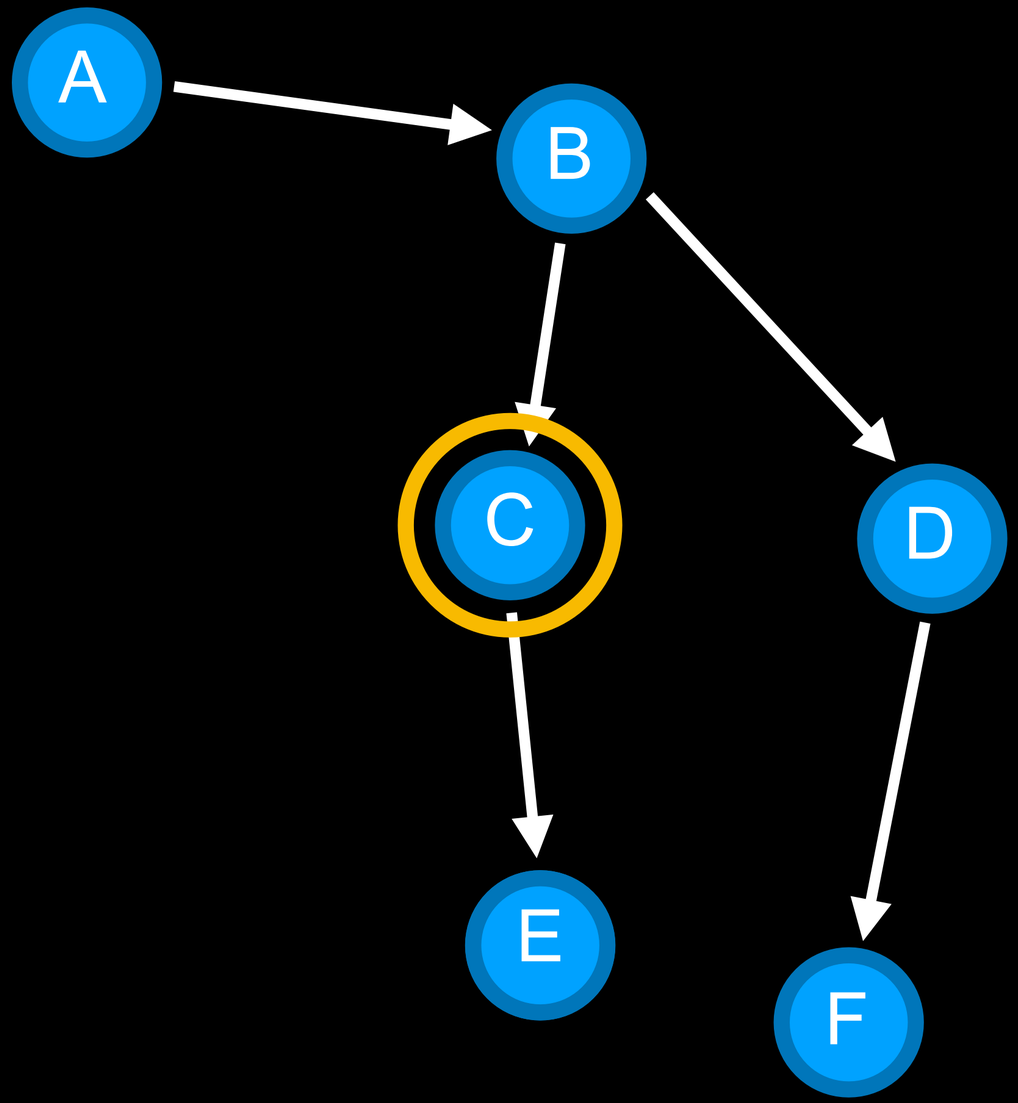


Find a path from A to E.

Frontier

Explored Set

A B D F C



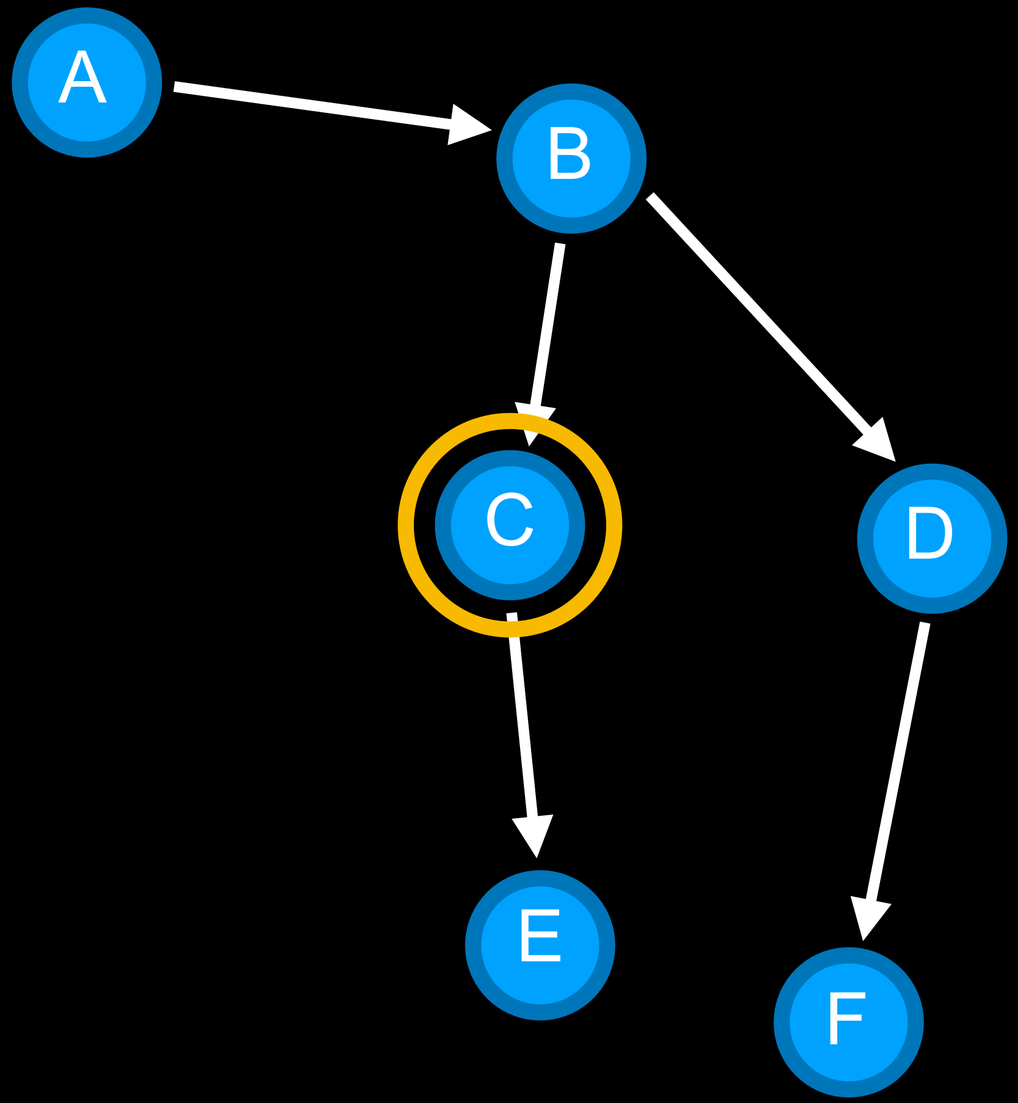
Find a path from A to E.

Frontier

E

Explored Set

A B D F C

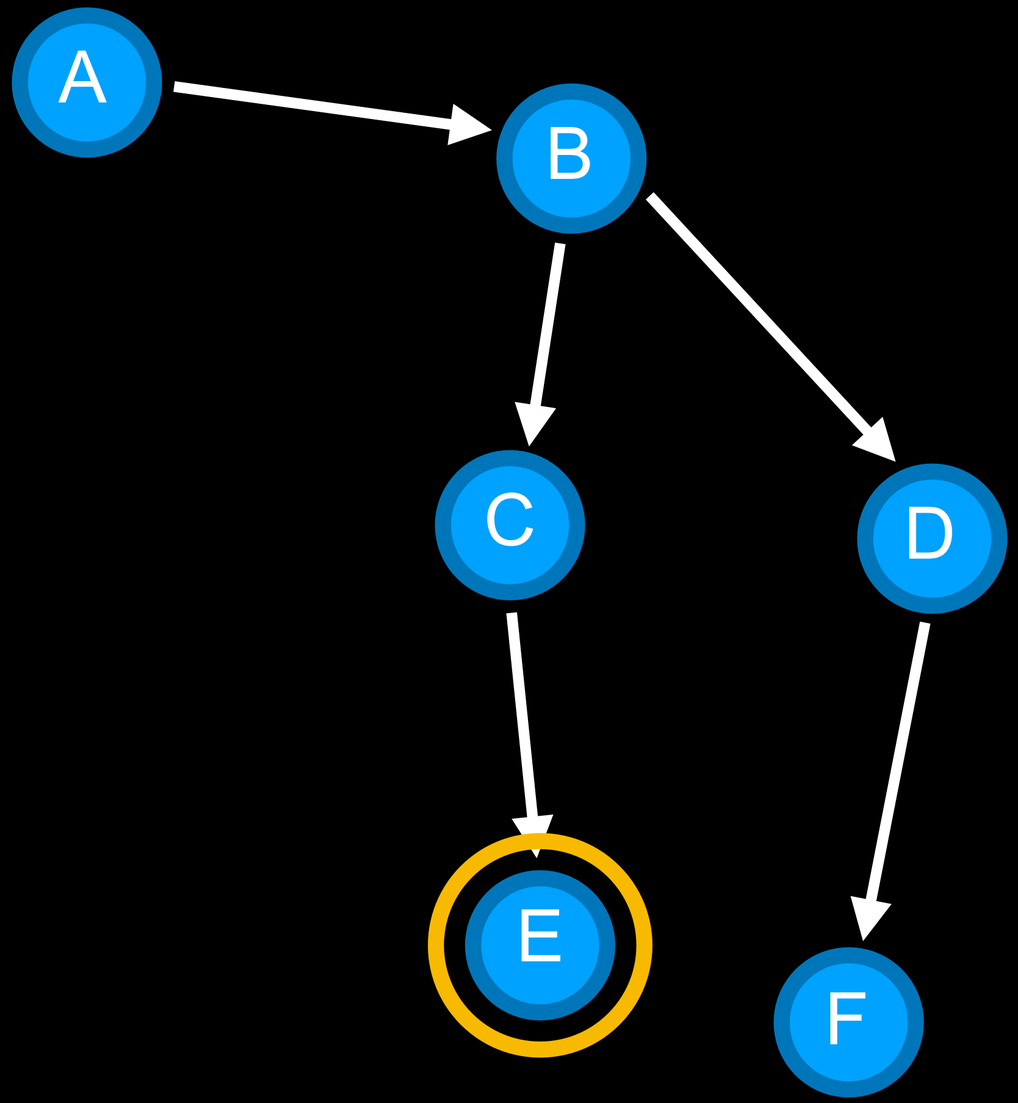


Find a path from A to E.

Frontier

Explored Set

A B D F C



Depth-First Search

depth-first search

search algorithm that always expands the deepest node in the frontier

Breadth-First Search

breadth-first search

search algorithm that always expands the shallowest node in the frontier

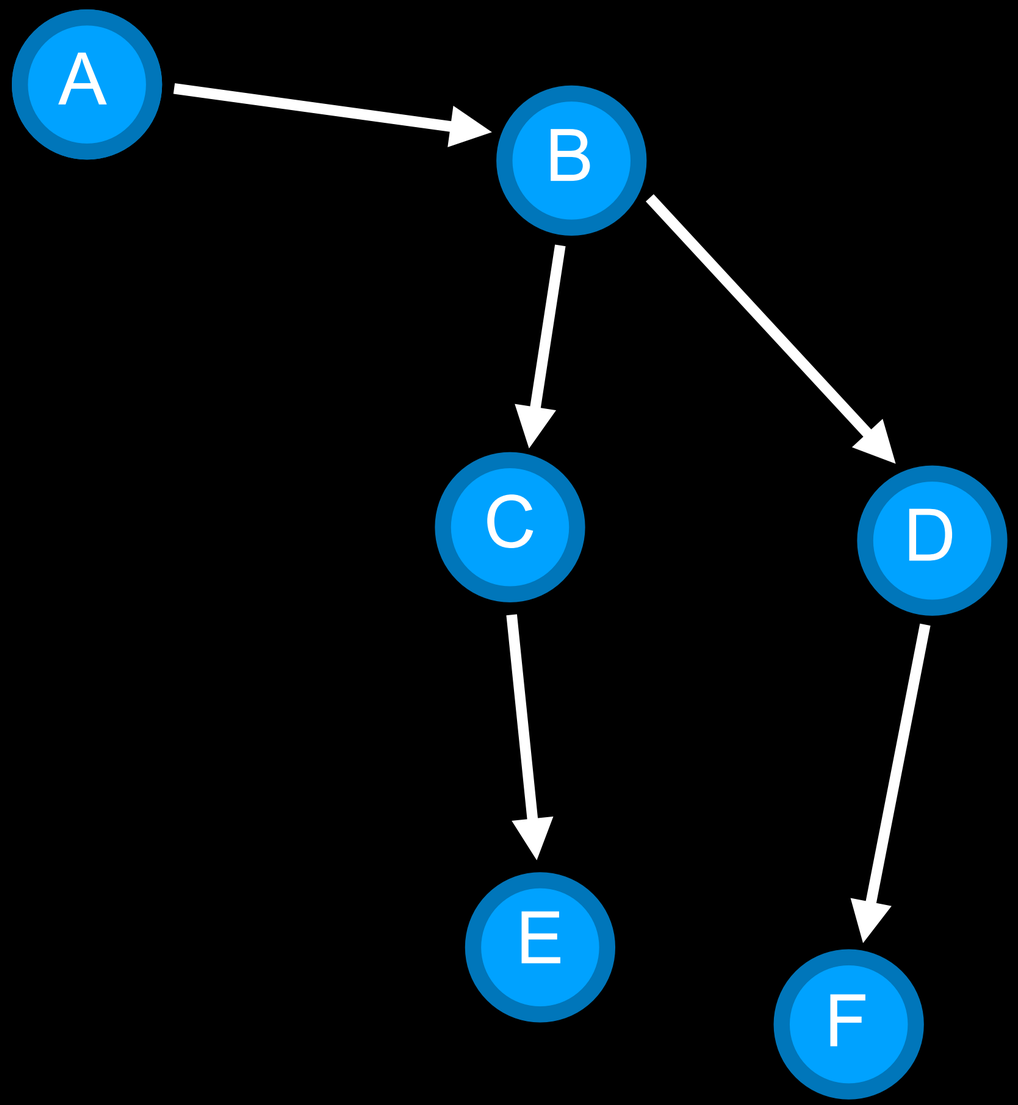
queue

first-in first-out data type

Find a path from A to E.

Frontier

Explored Set

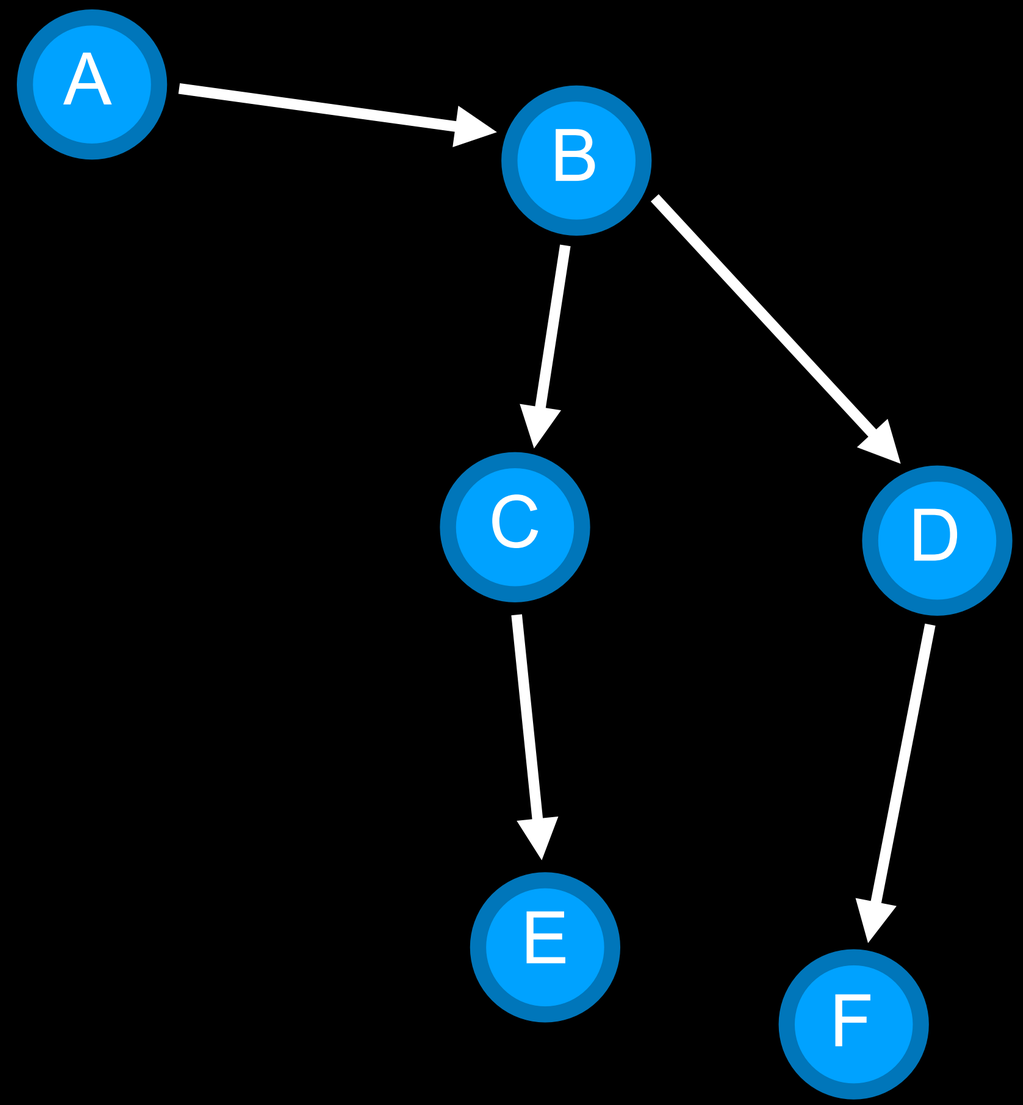


Find a path from A to E.

Frontier

A

Explored Set

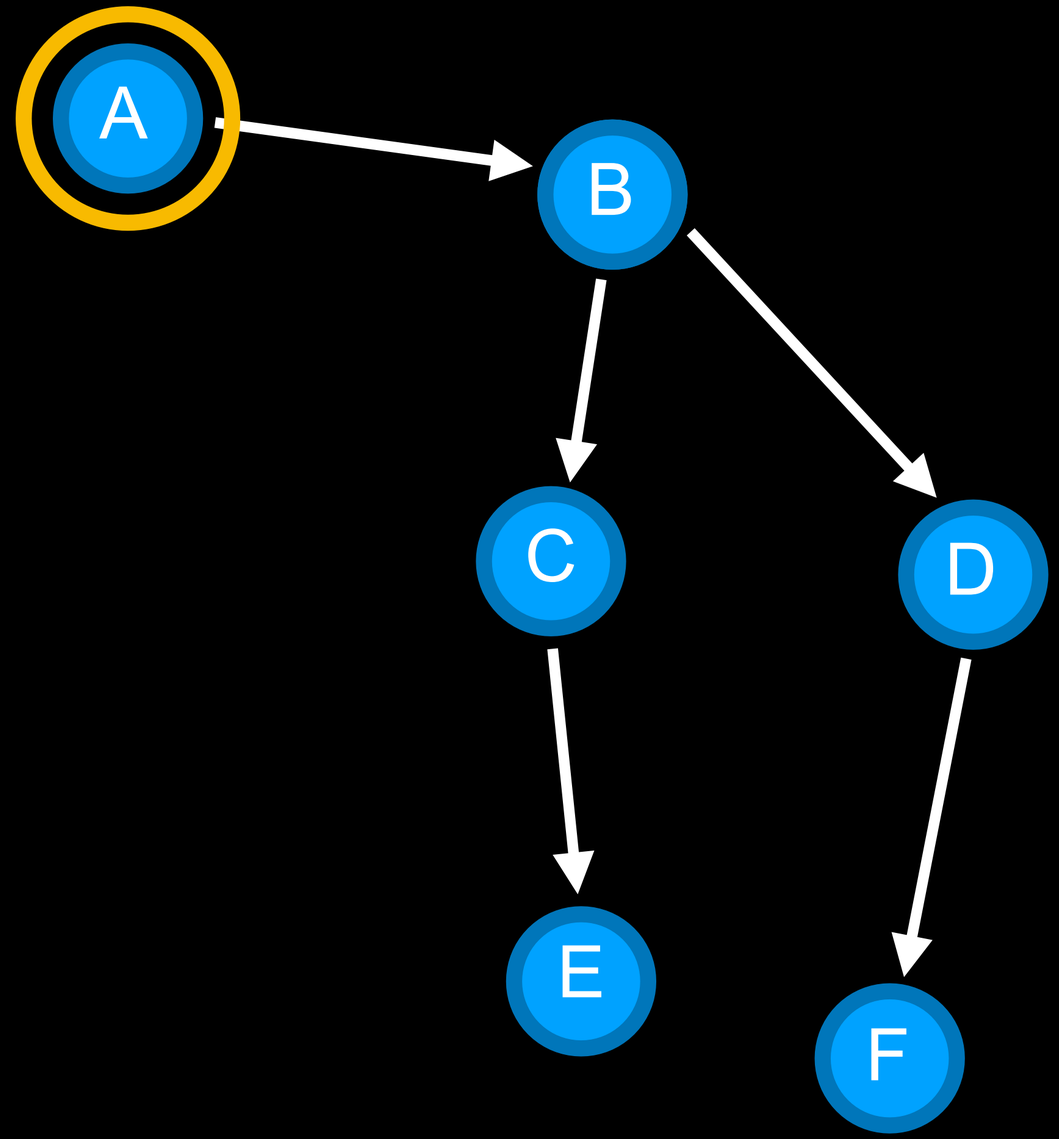


Find a path from A to E.

Frontier

Explored Set

A



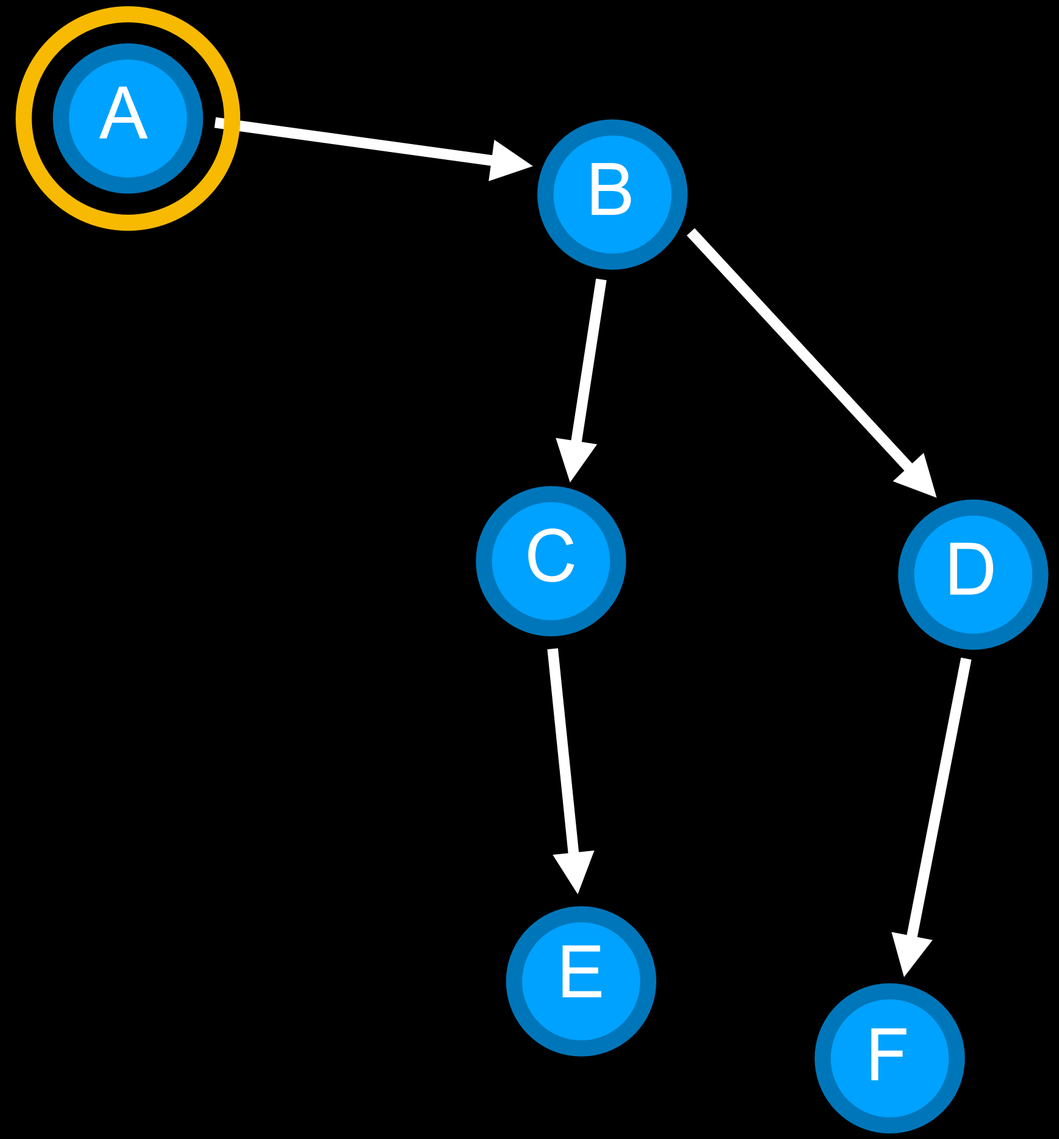
Find a path from A to E.

Frontier

B

Explored Set

A

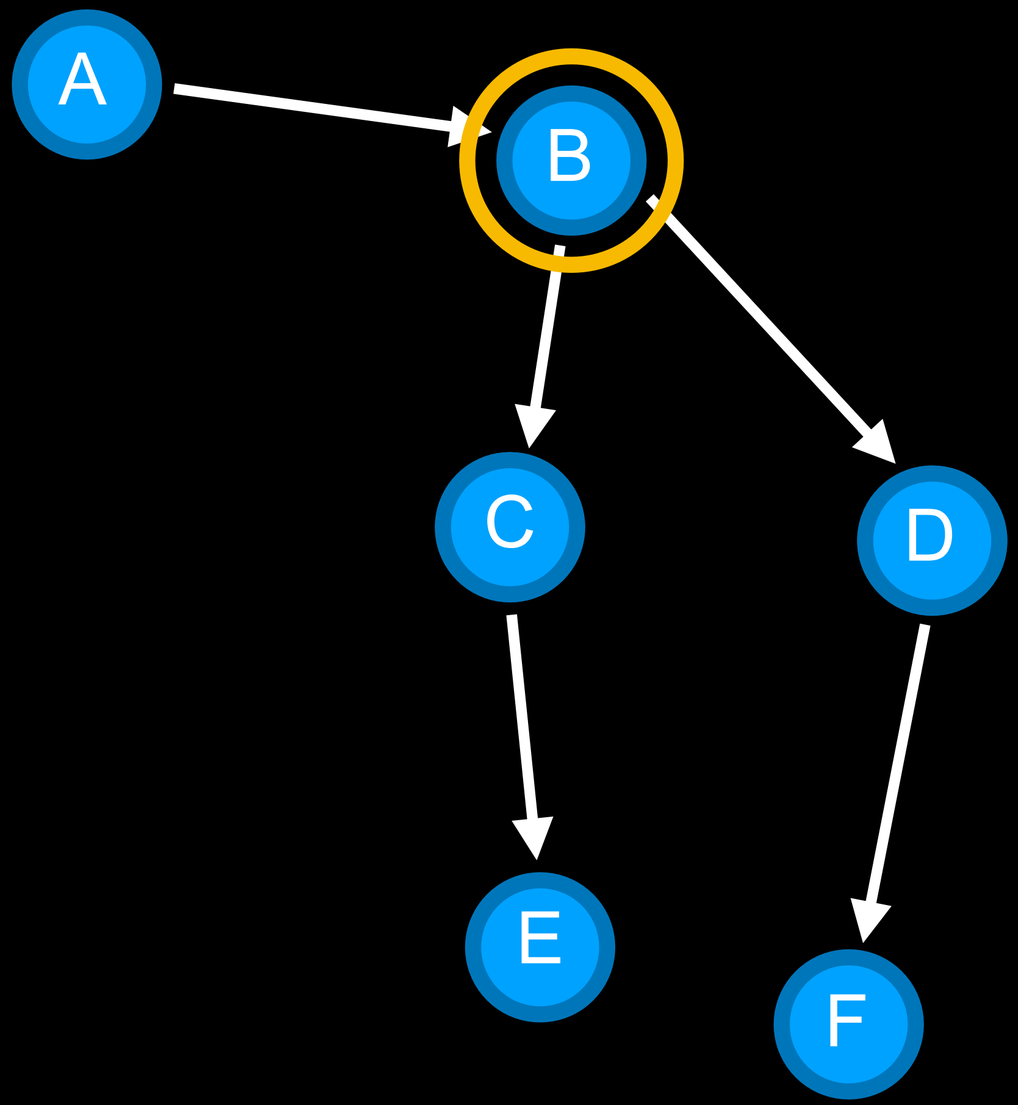


Find a path from A to E.

Frontier

Explored Set

A B



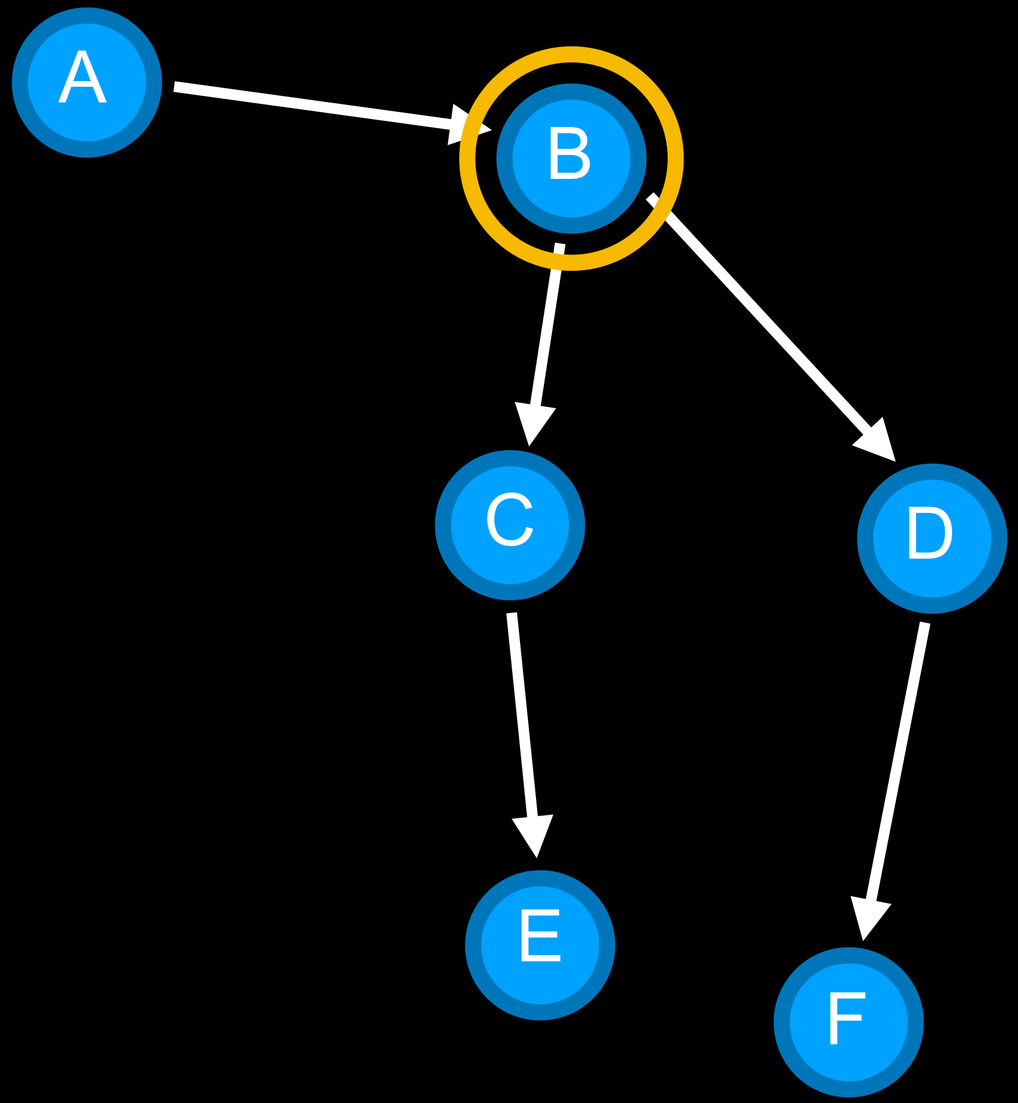
Find a path from A to E.

Frontier

C D

Explored Set

A B



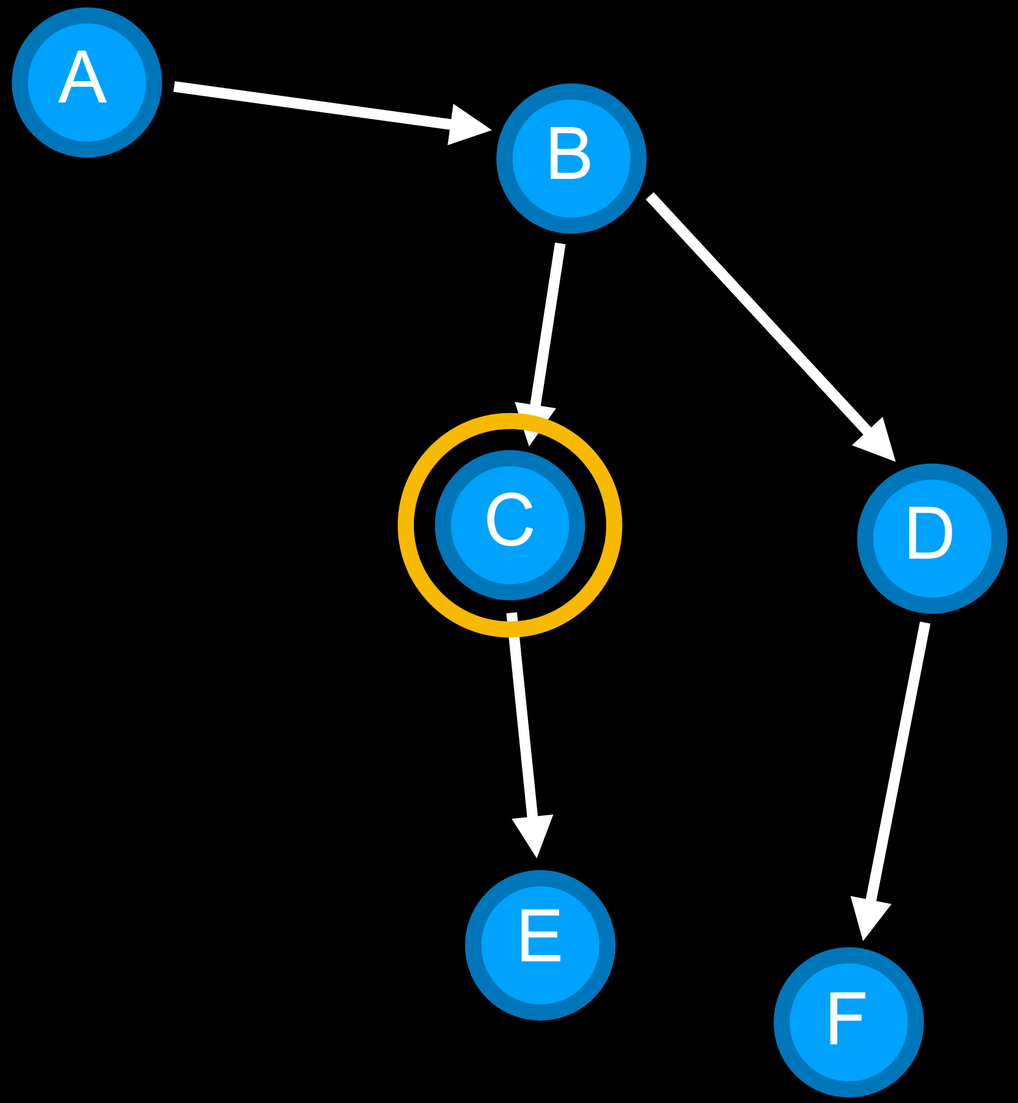
Find a path from A to E.

Frontier

D

Explored Set

A B C



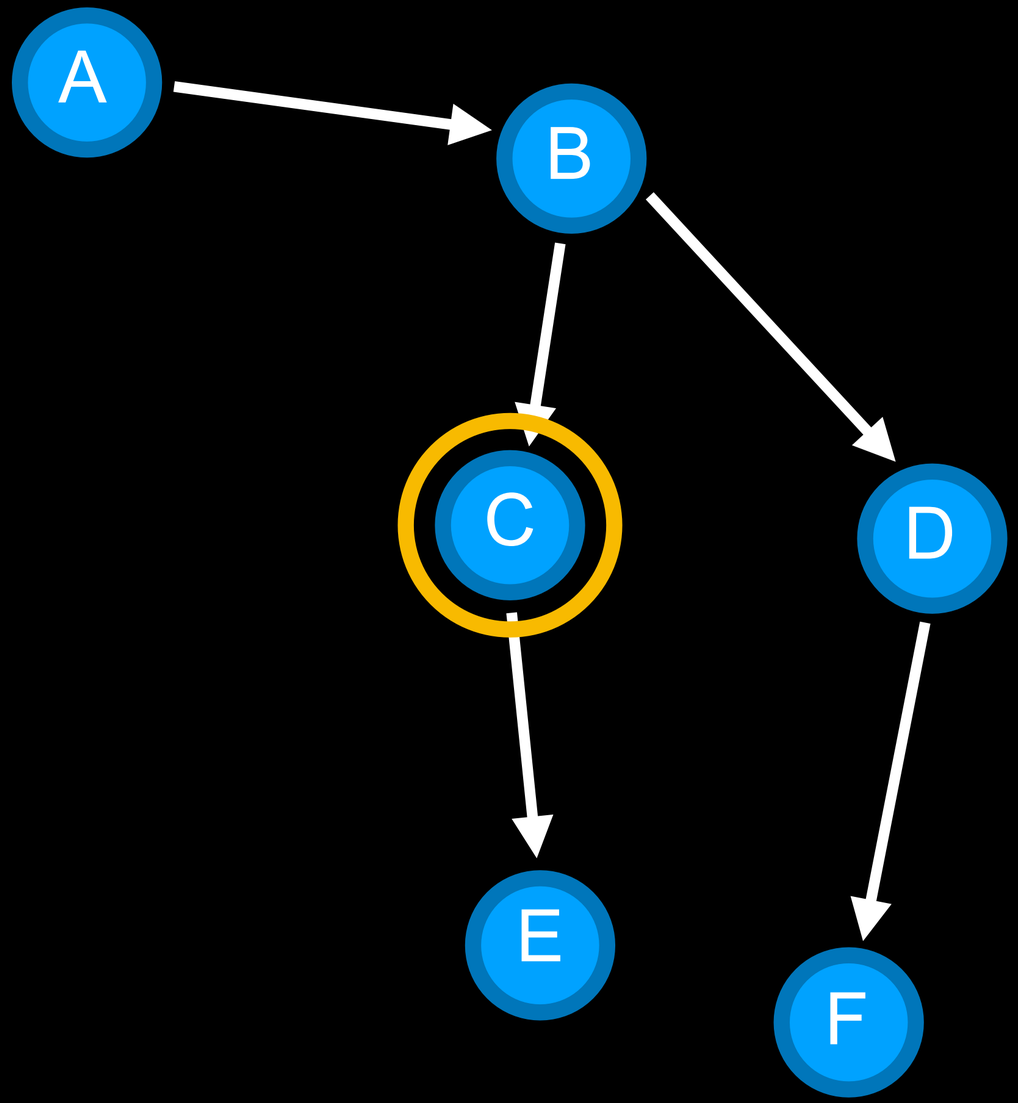
Find a path from A to E.

Frontier

D E

Explored Set

A B C



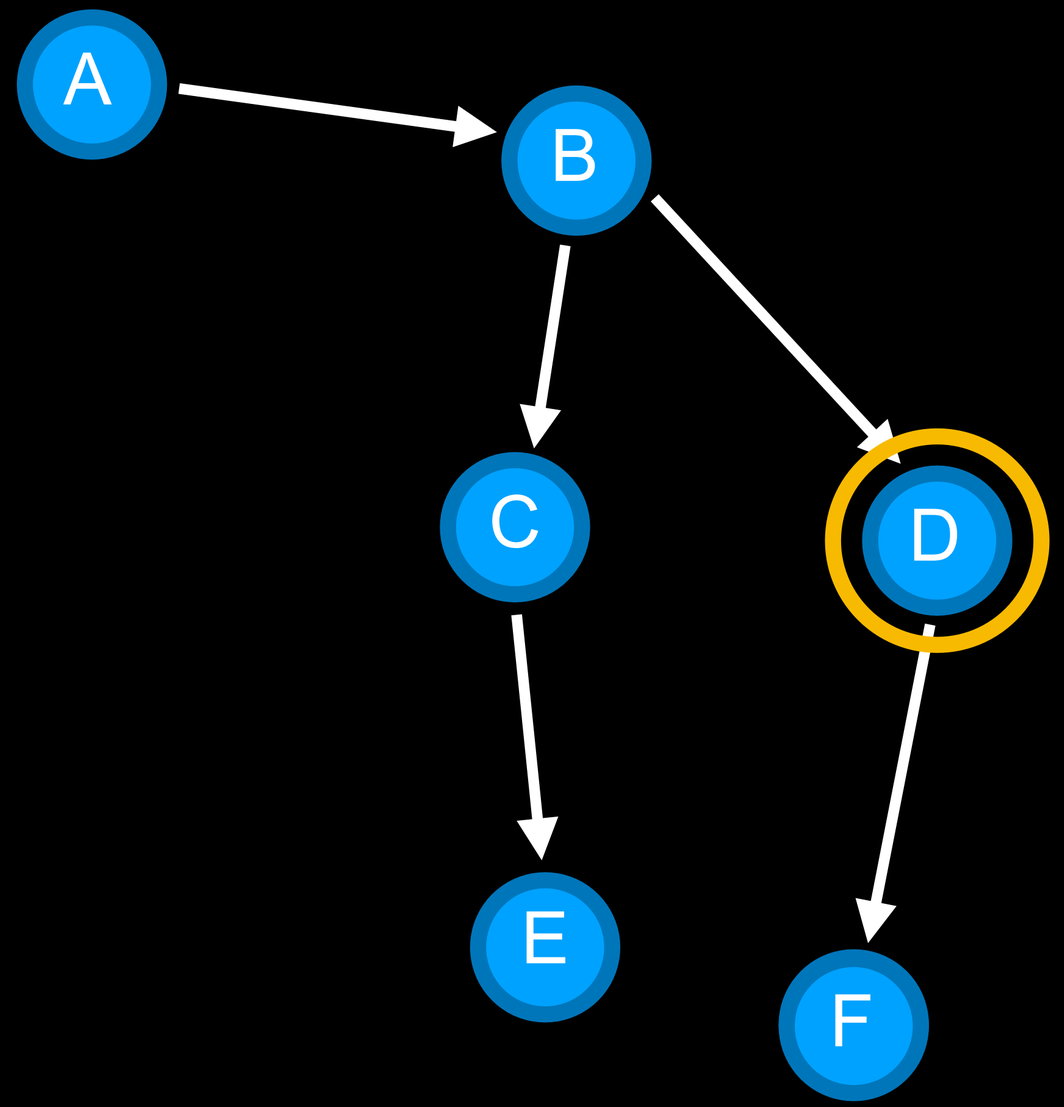
Find a path from A to E.

Frontier

E

Explored Set

A B C D



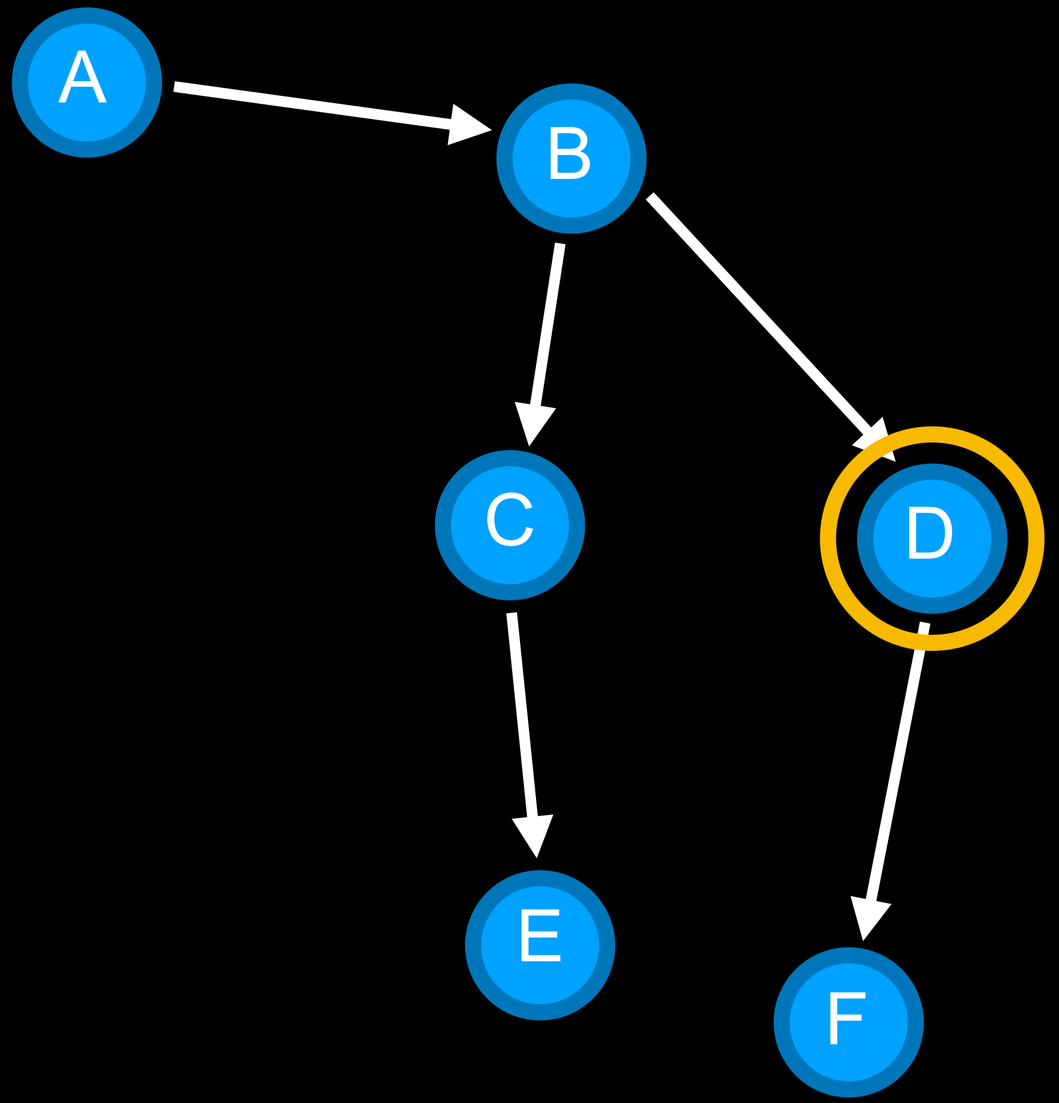
Find a path from A to E.

Frontier

E F

Explored Set

A B C D



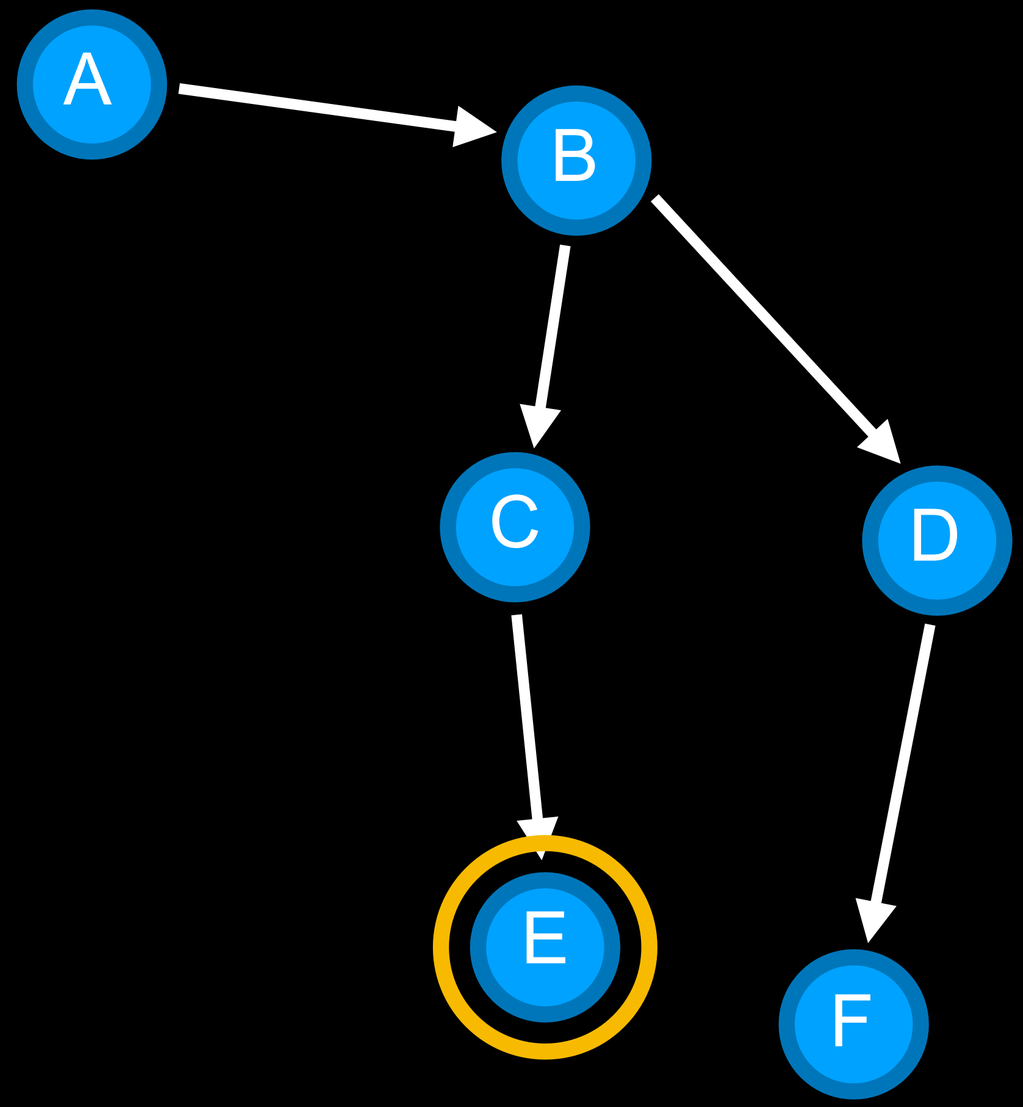
Find a path from A to E.

Frontier

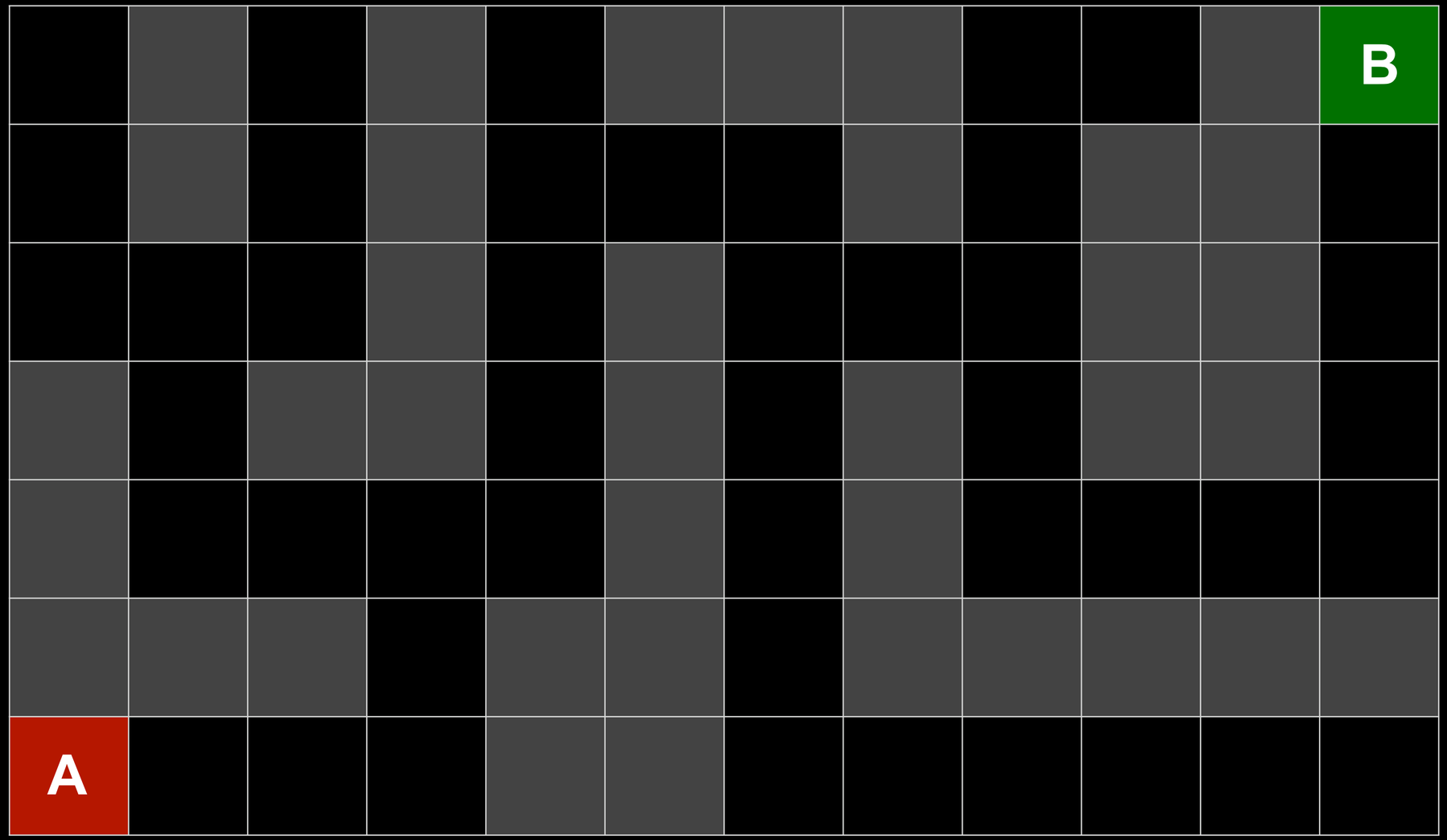
F

Explored Set

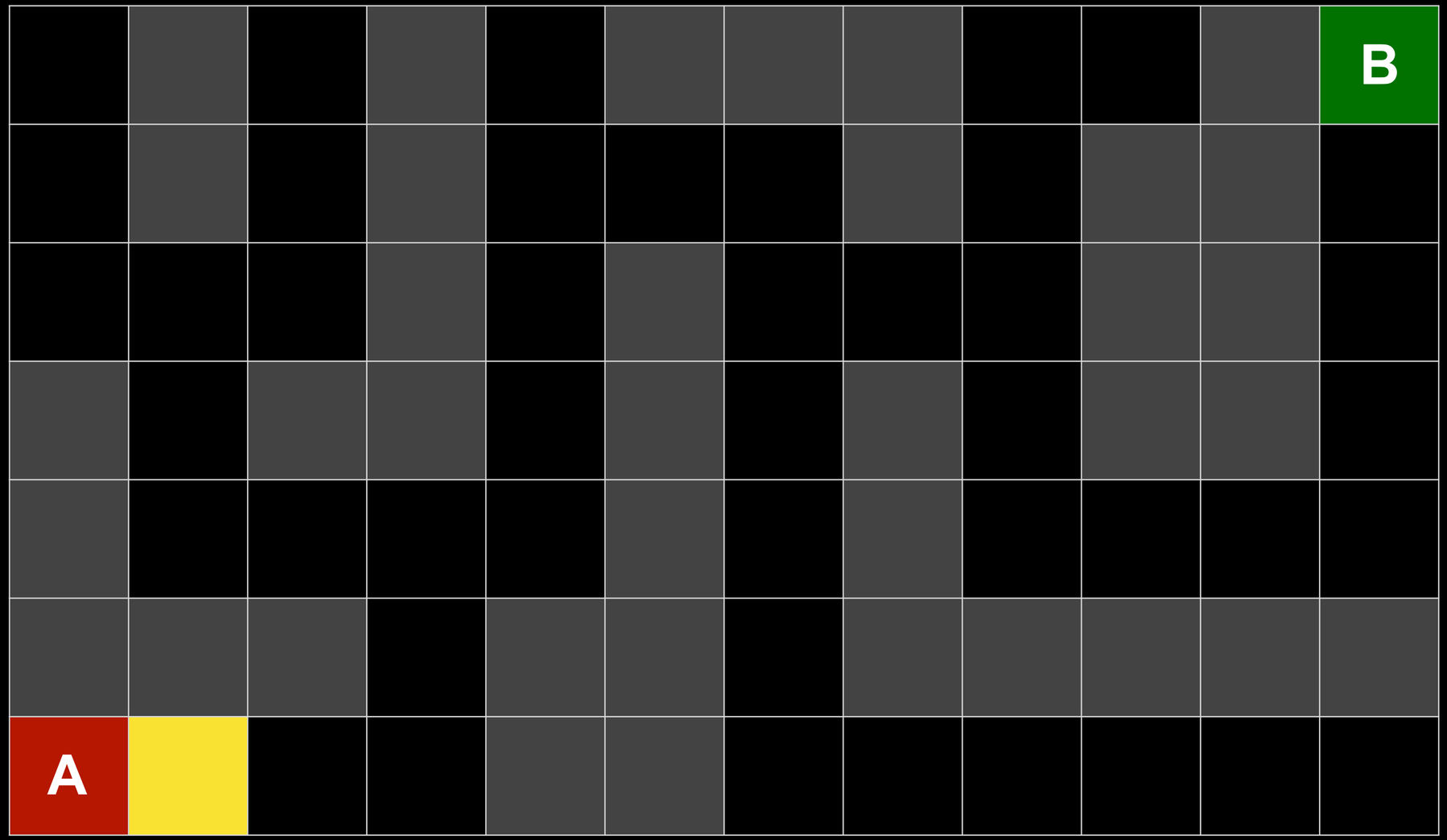
A B C D



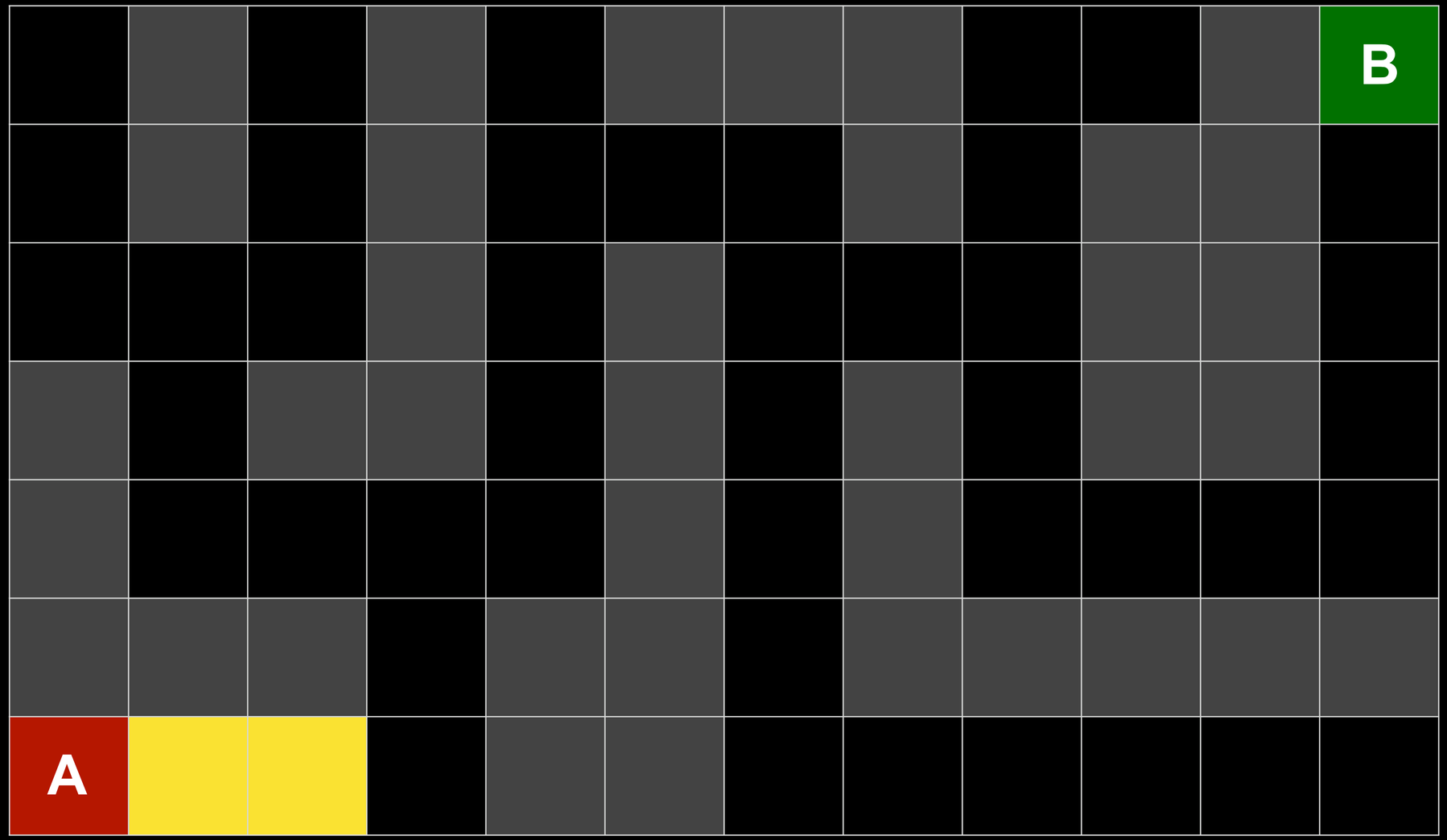
Depth-First Search



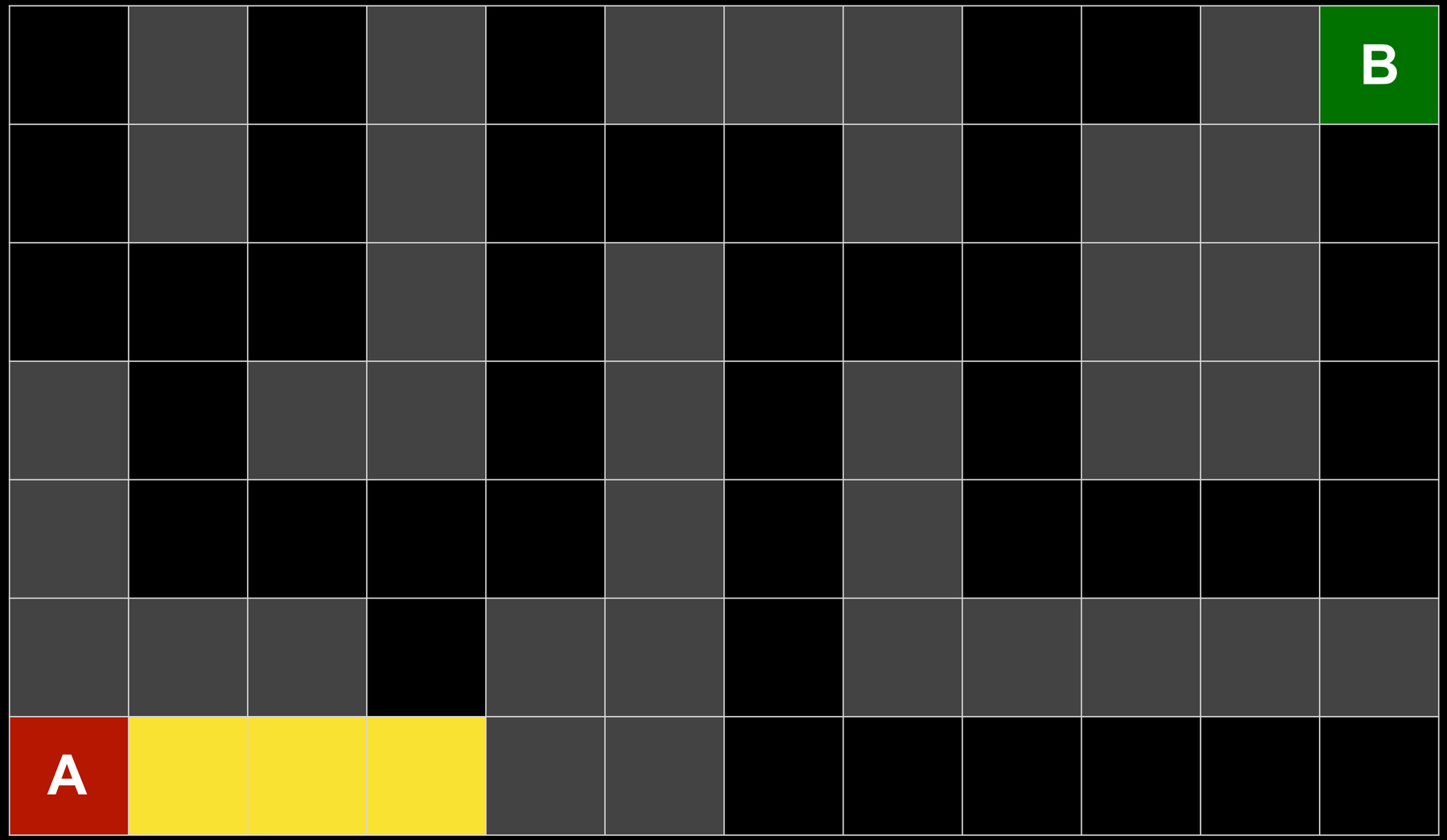
Depth-First Search



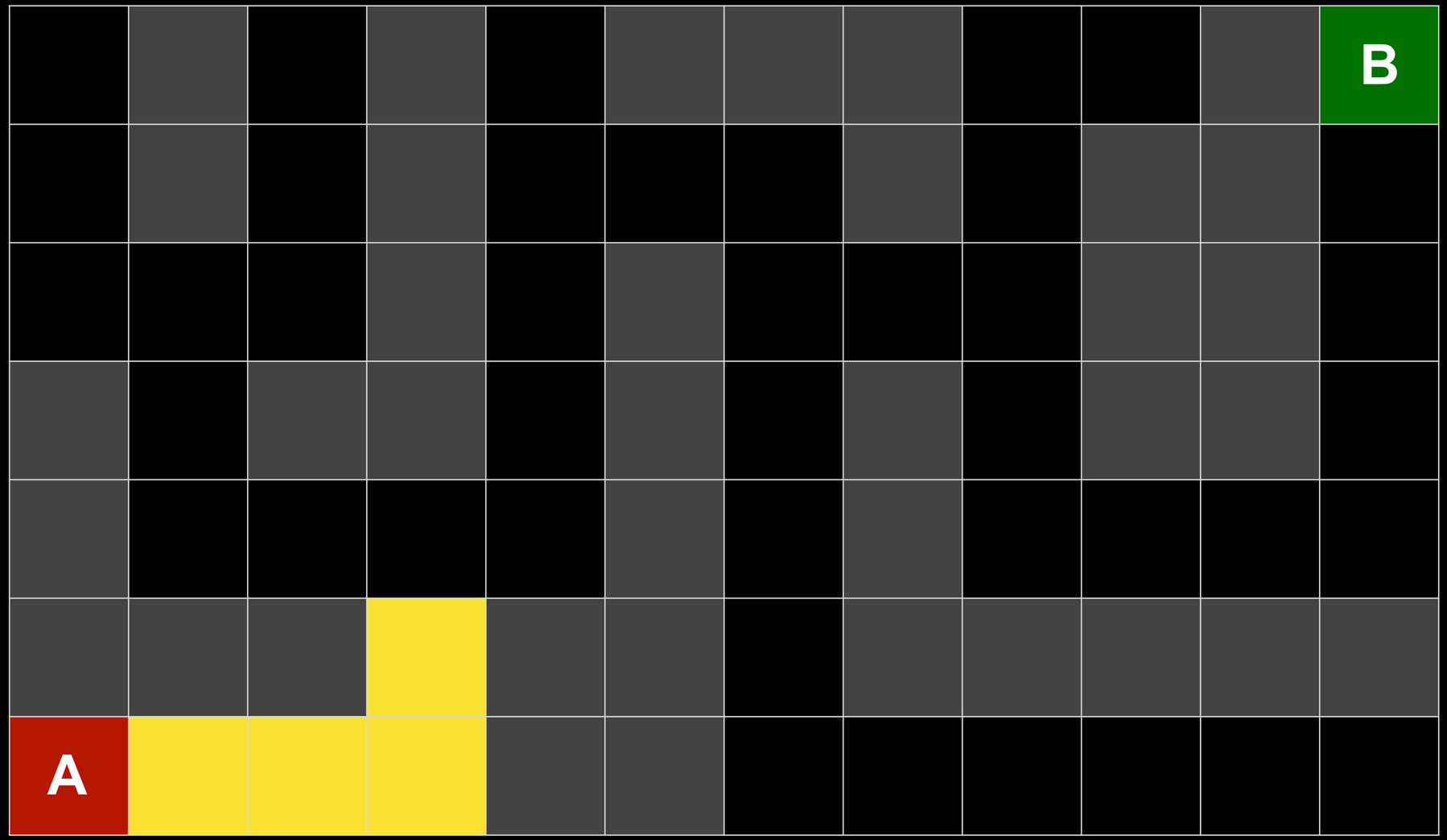
Depth-First Search



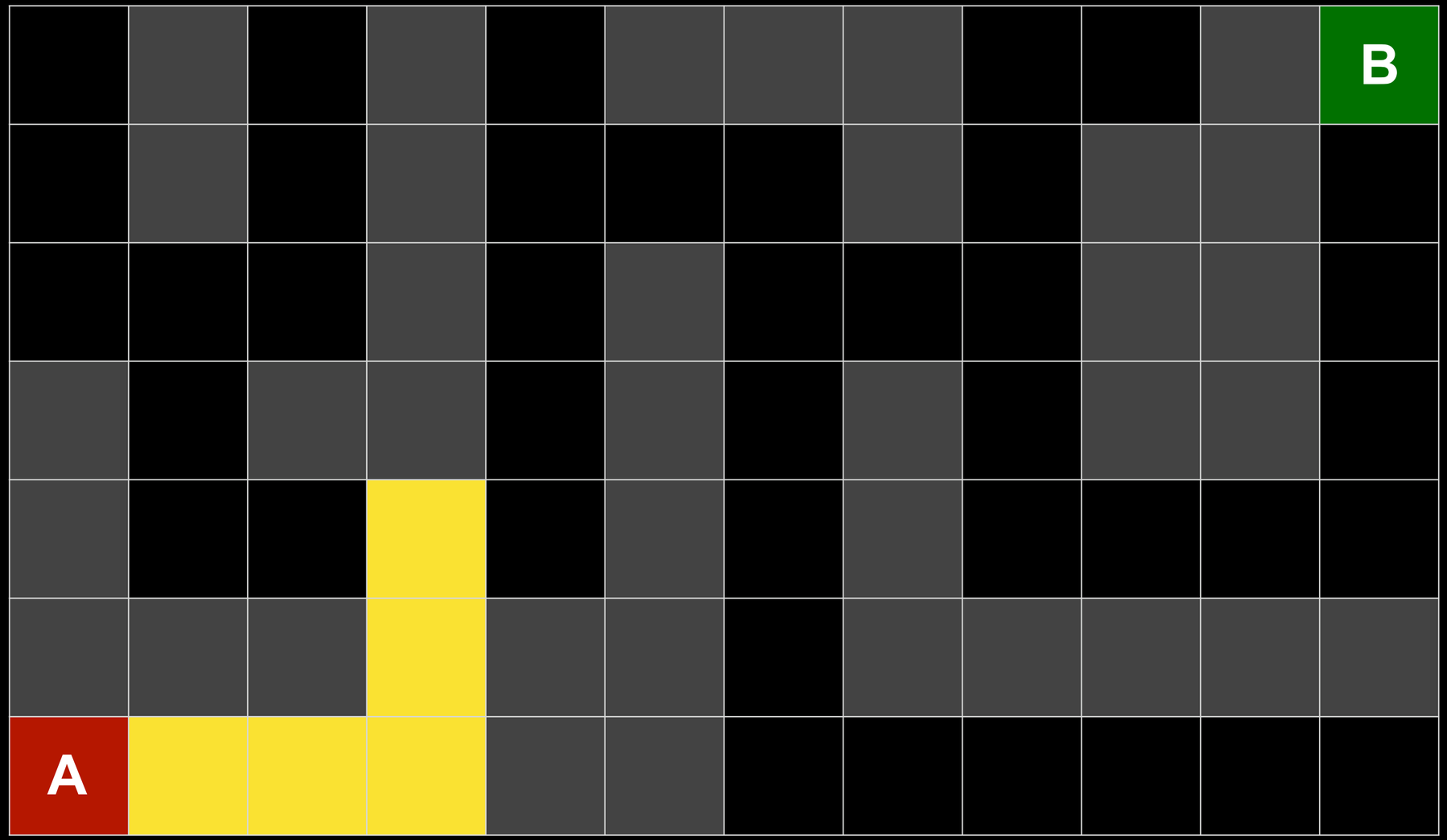
Depth-First Search



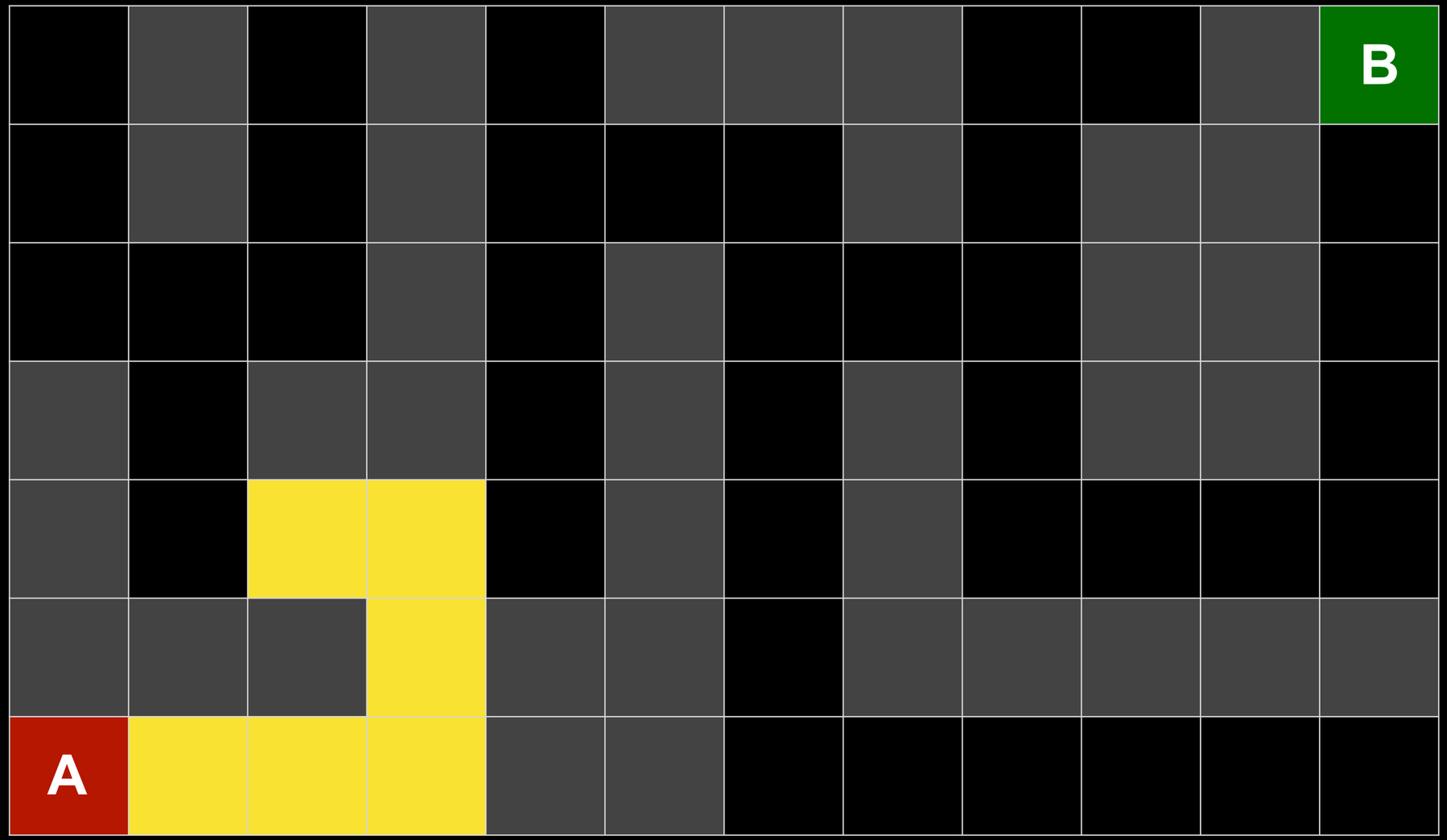
Depth-First Search



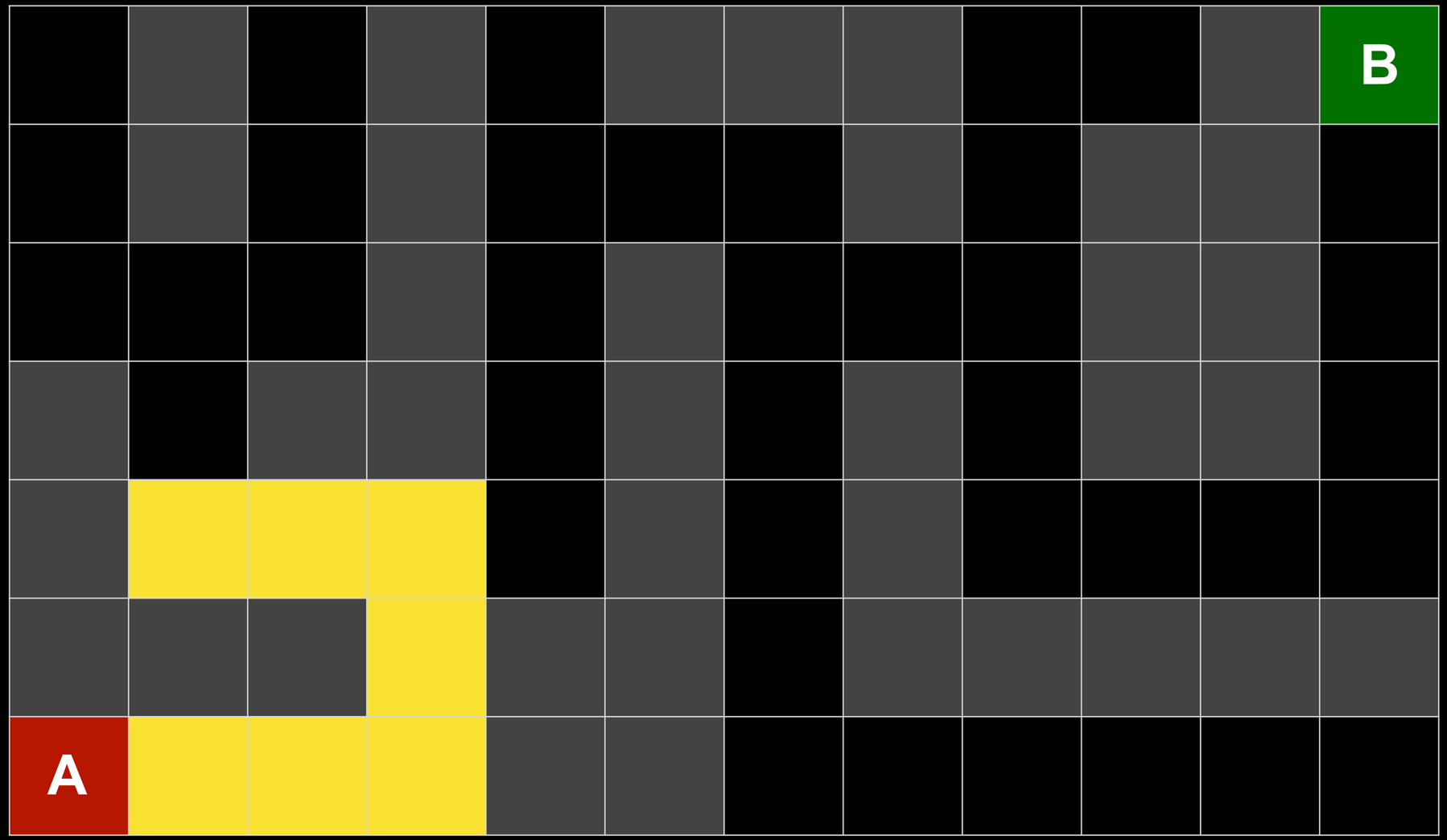
Depth-First Search



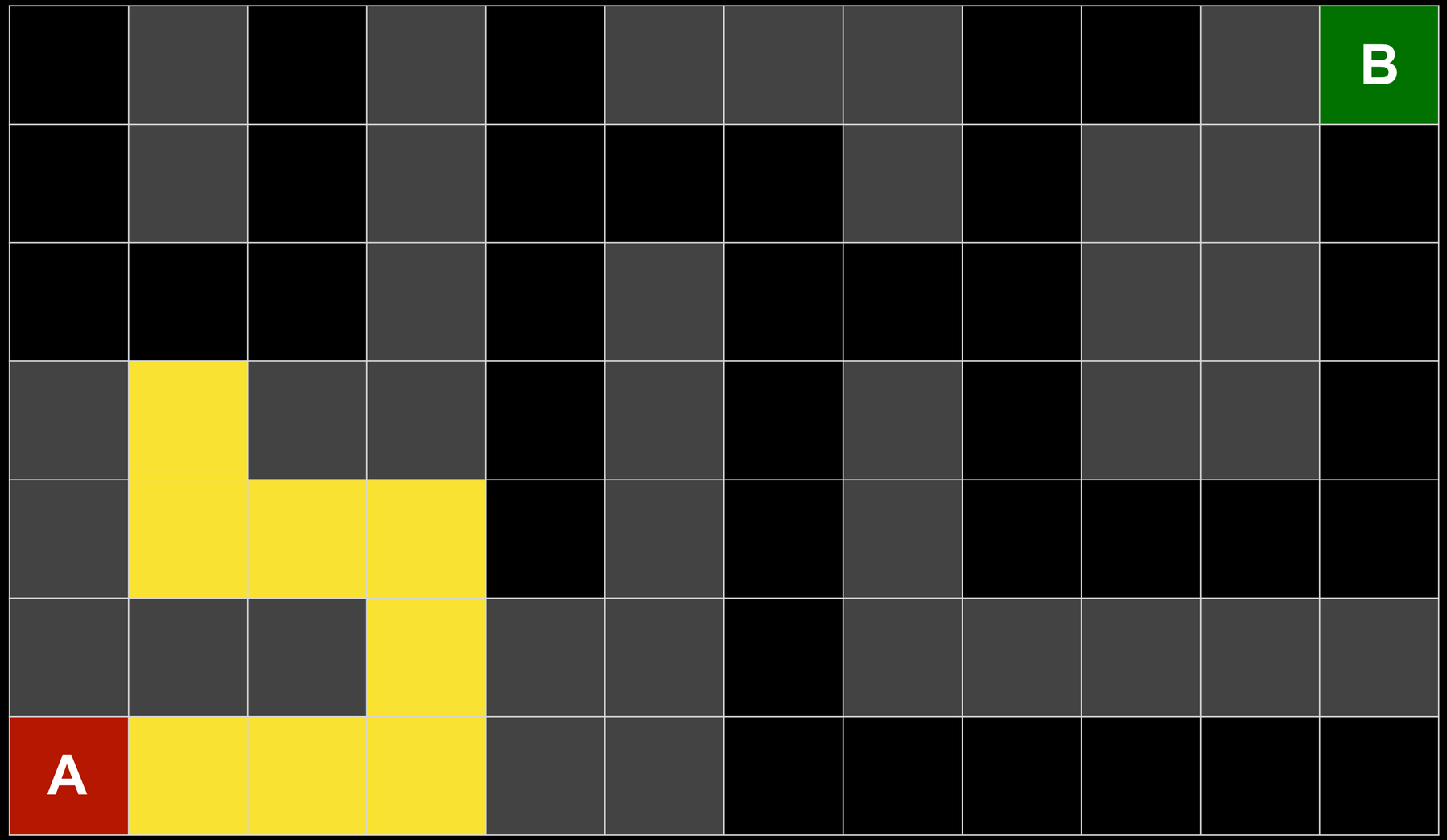
Depth-First Search



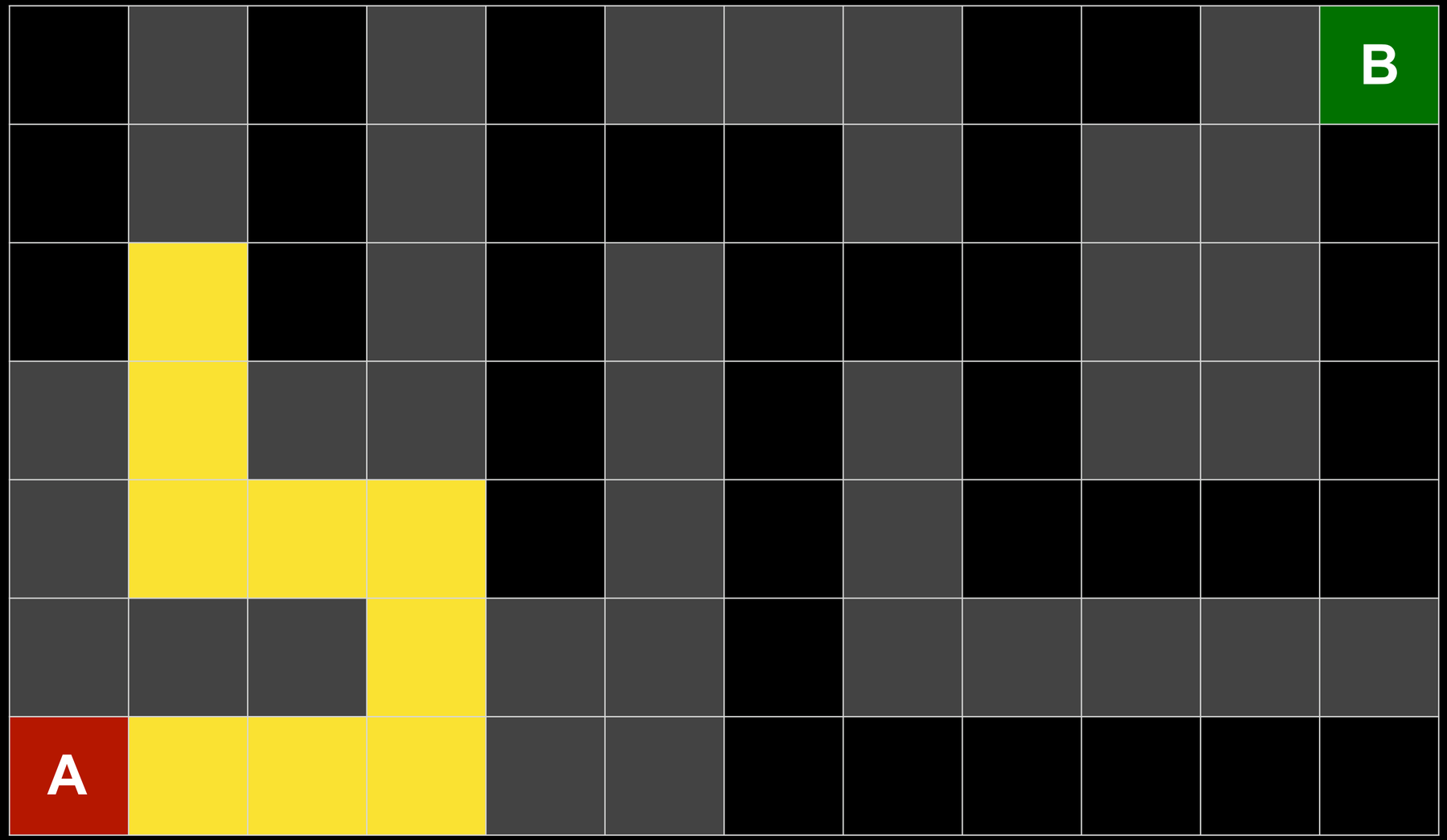
Depth-First Search



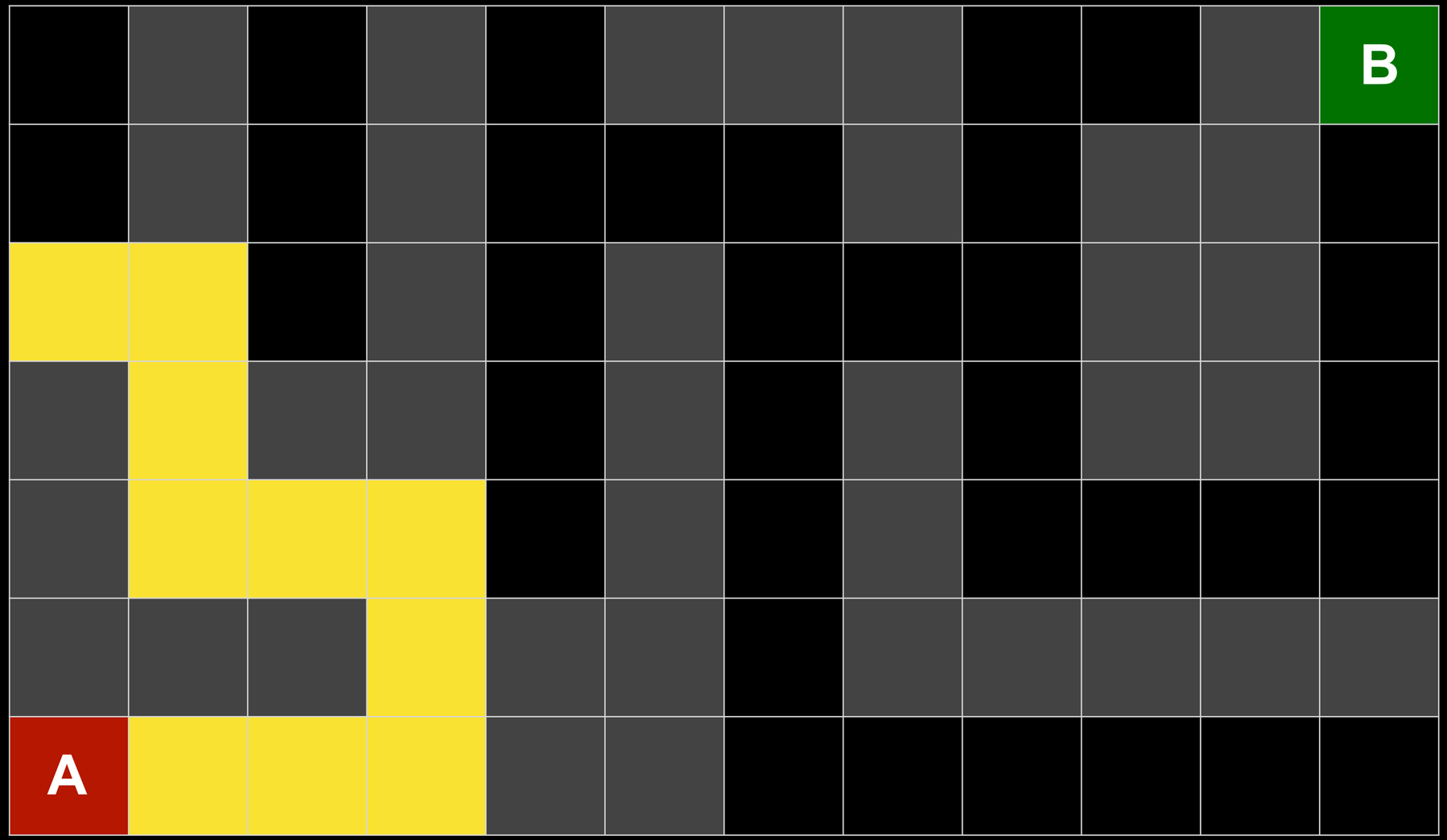
Depth-First Search



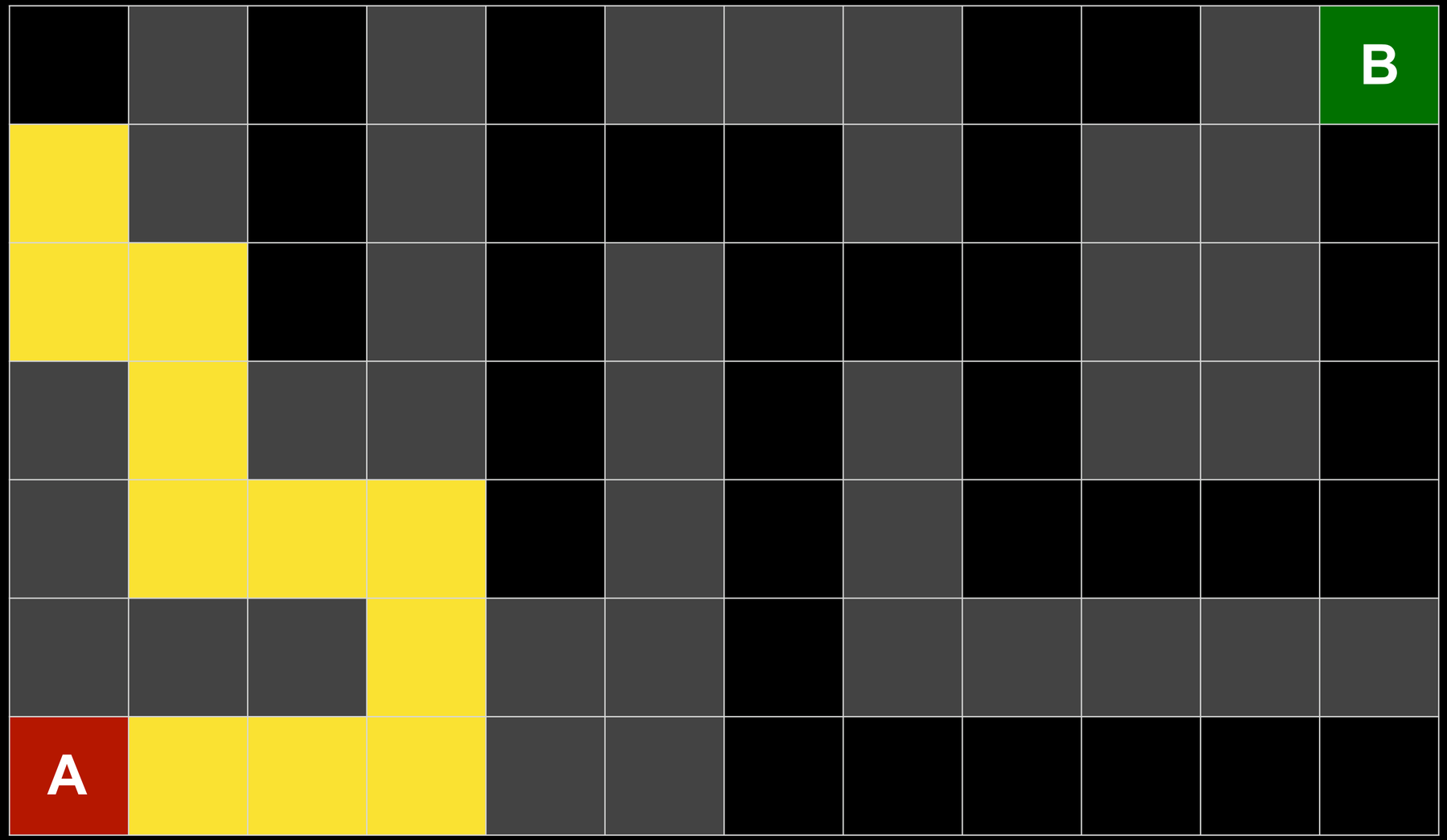
Depth-First Search



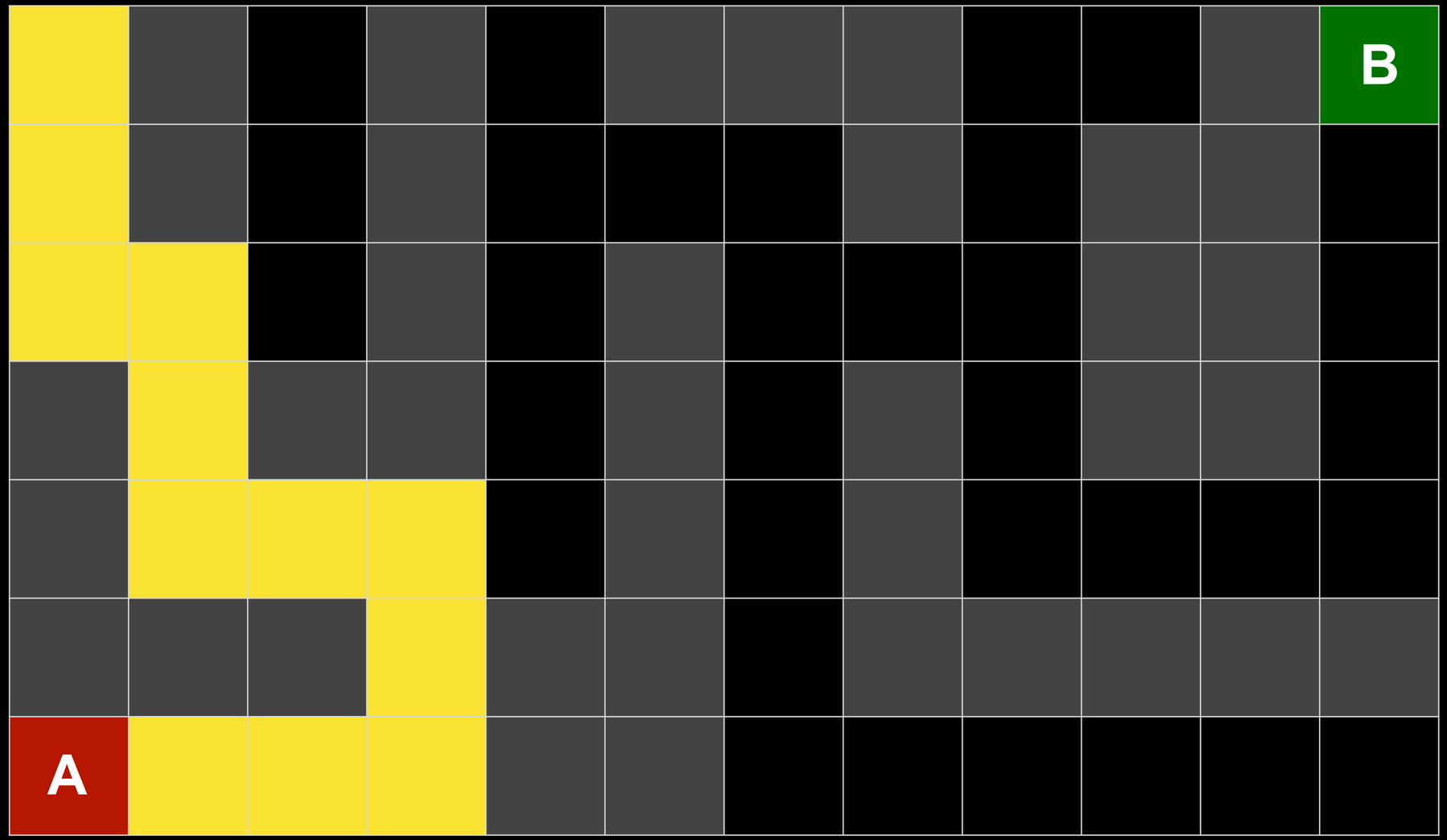
Depth-First Search



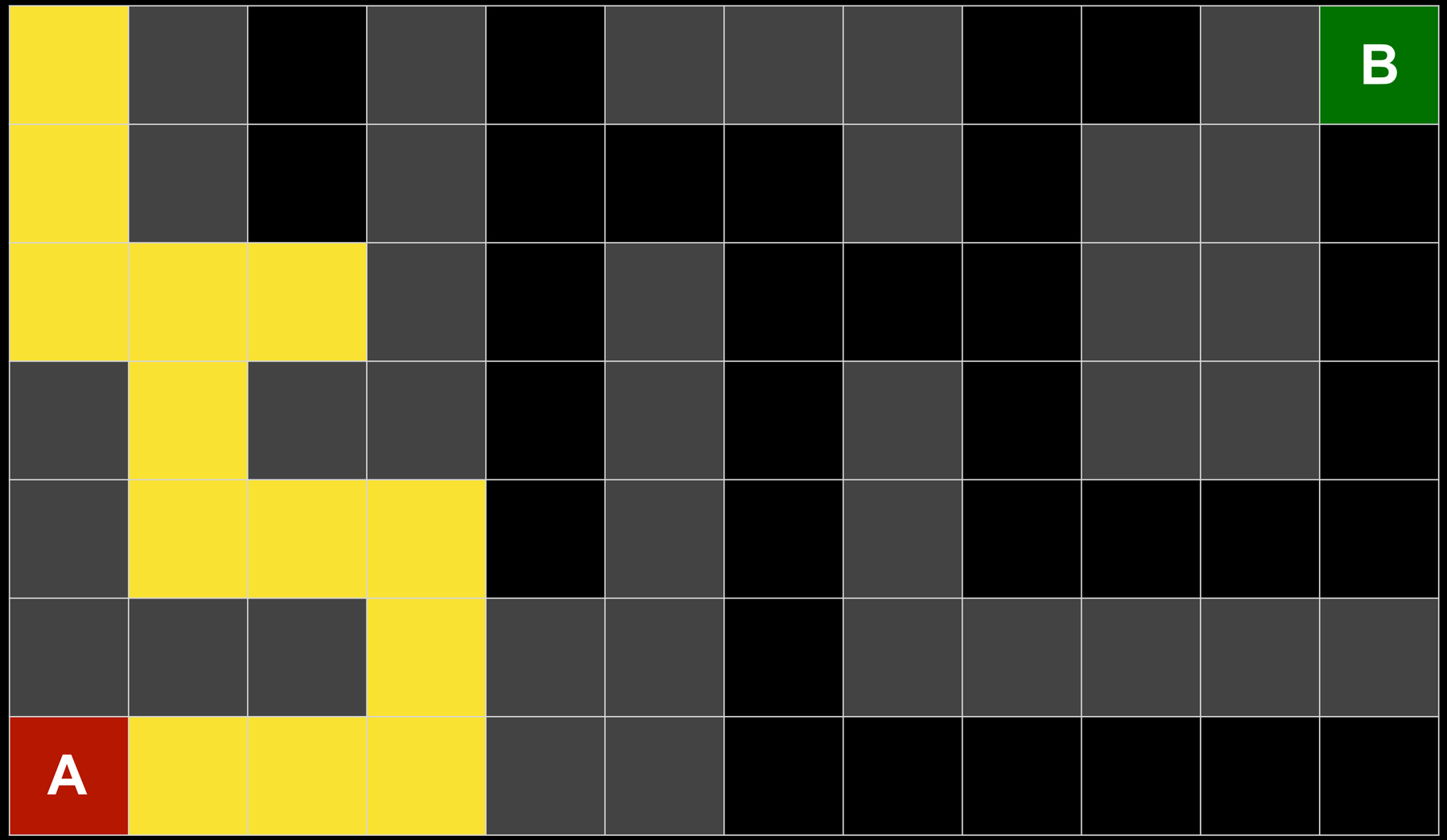
Depth-First Search



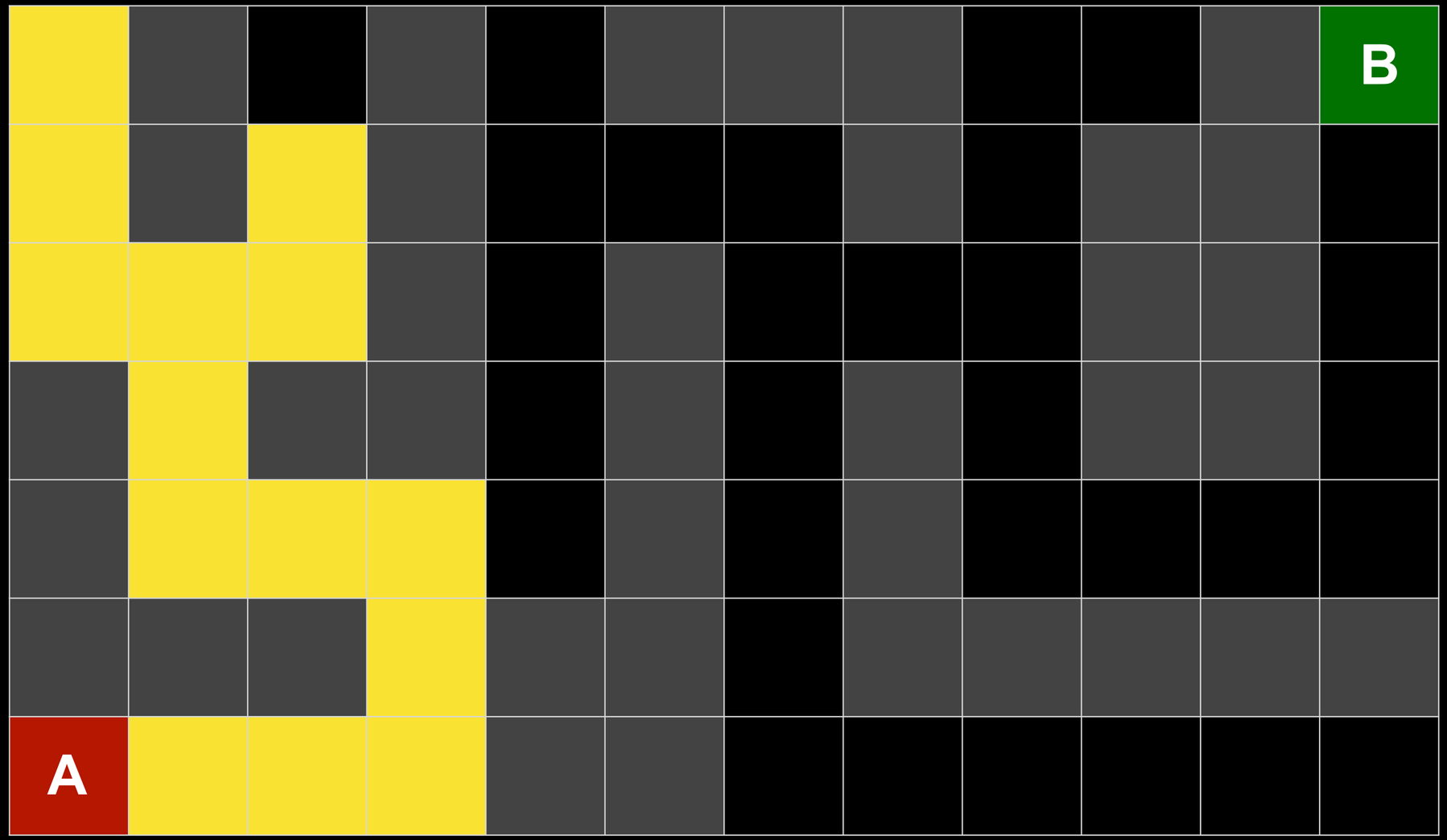
Depth-First Search



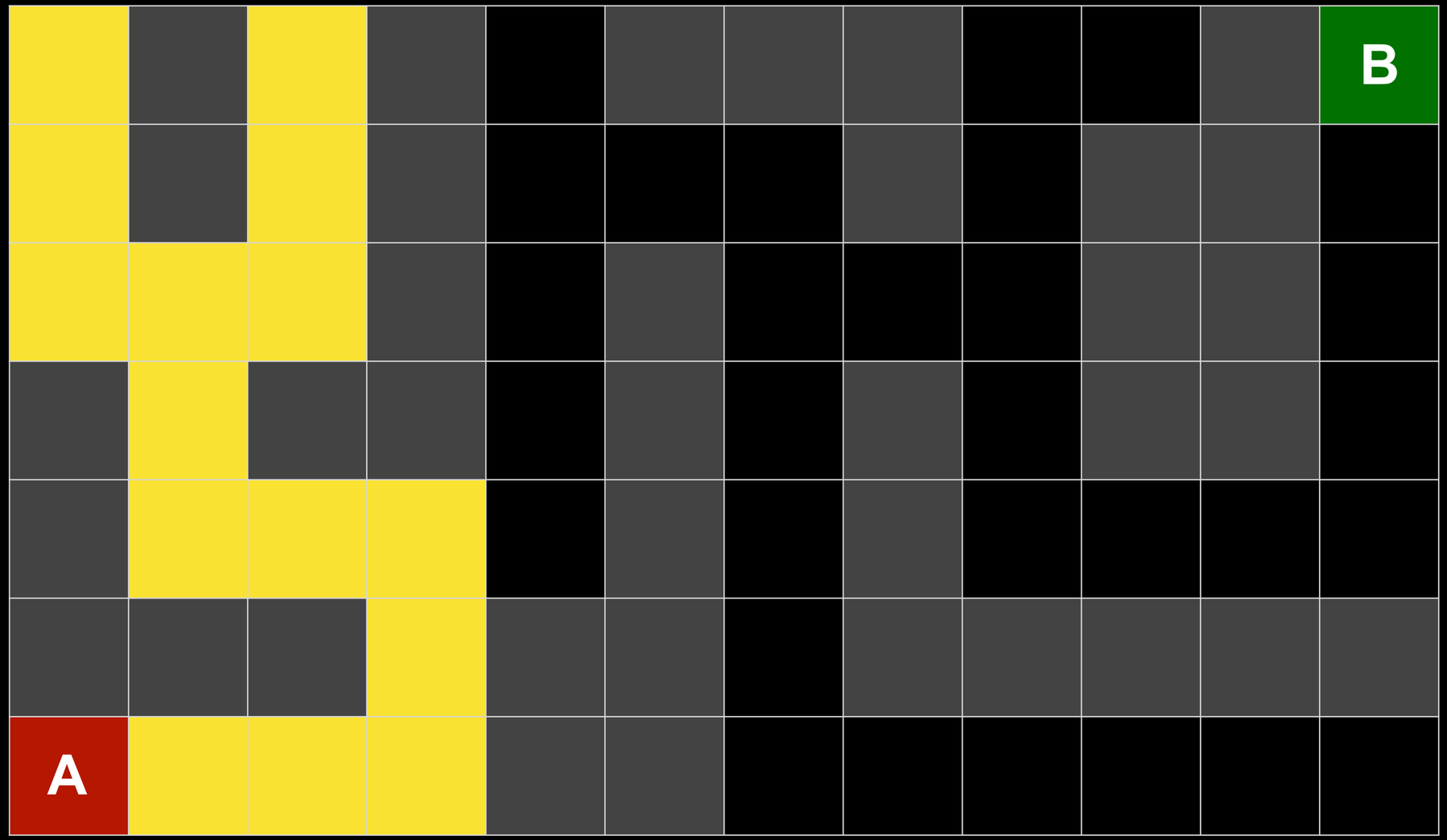
Depth-First Search



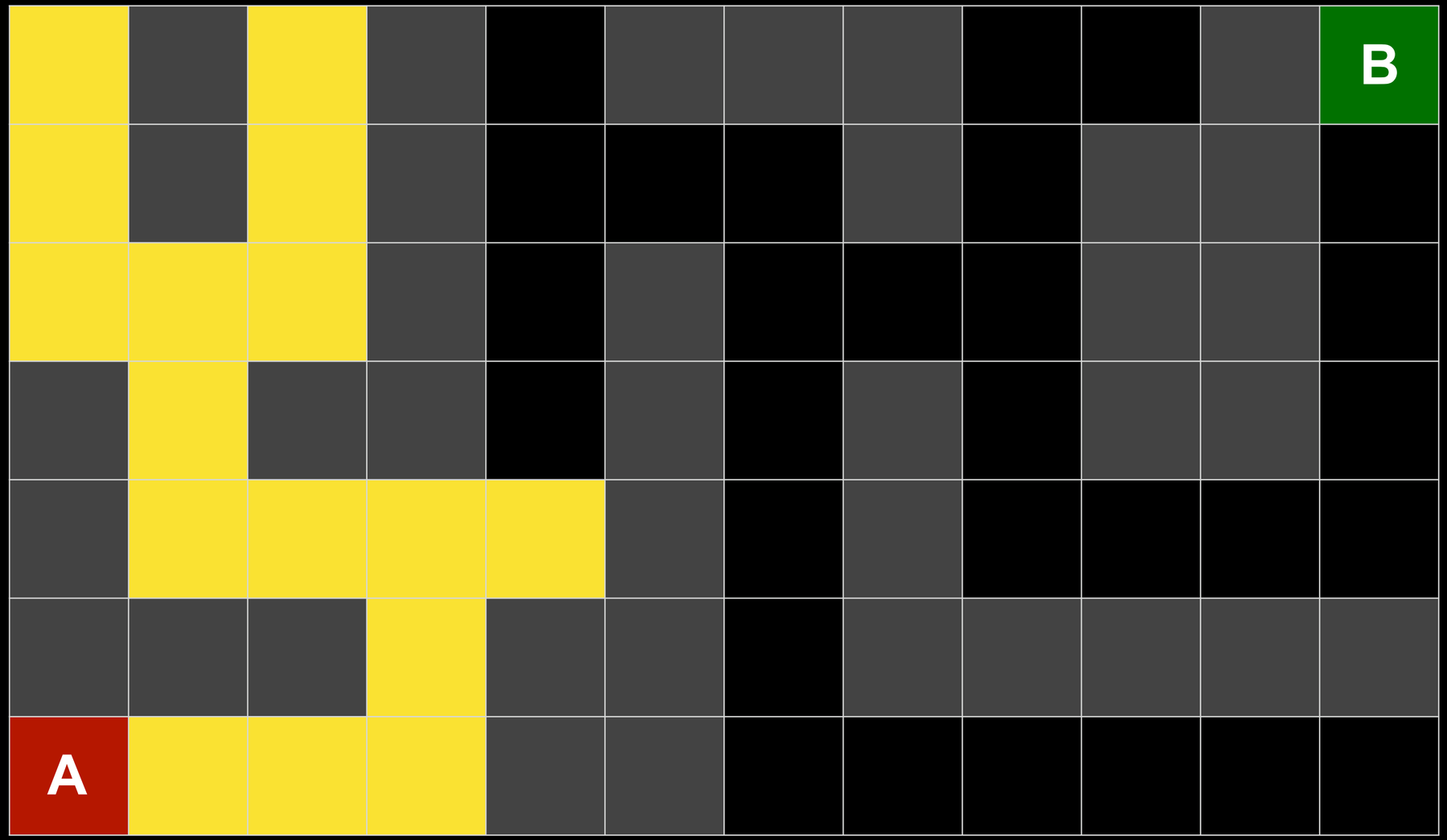
Depth-First Search



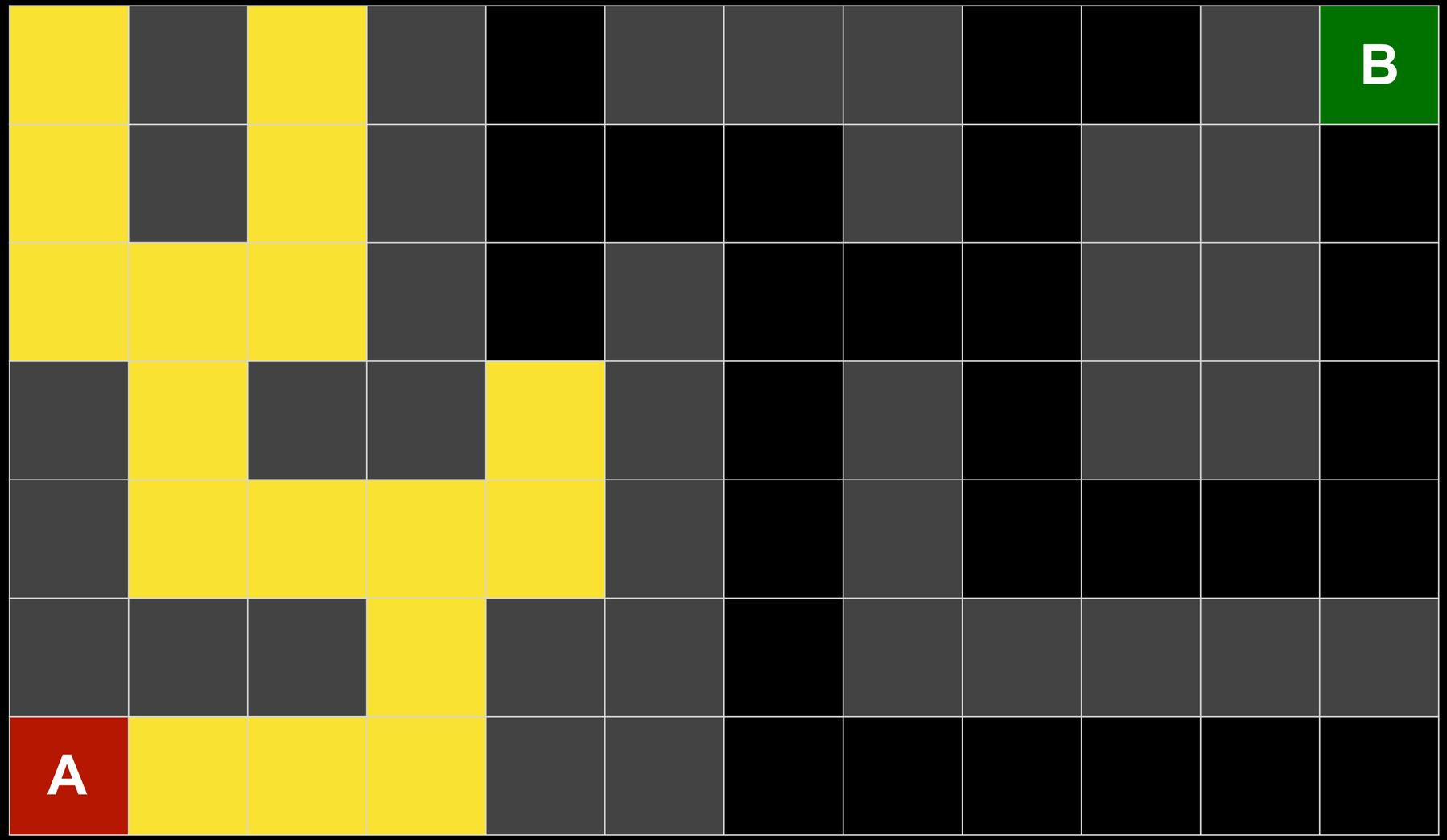
Depth-First Search



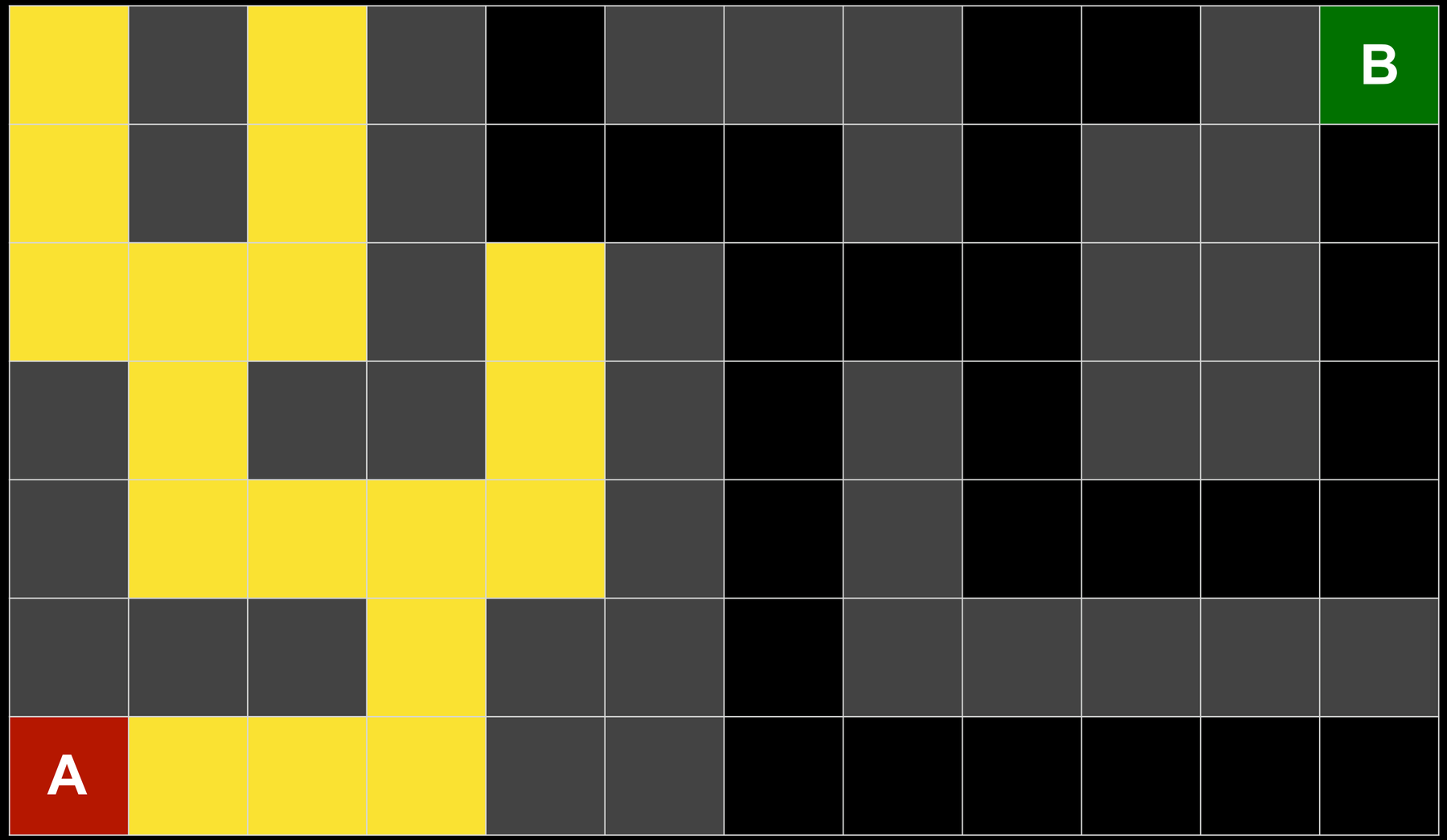
Depth-First Search



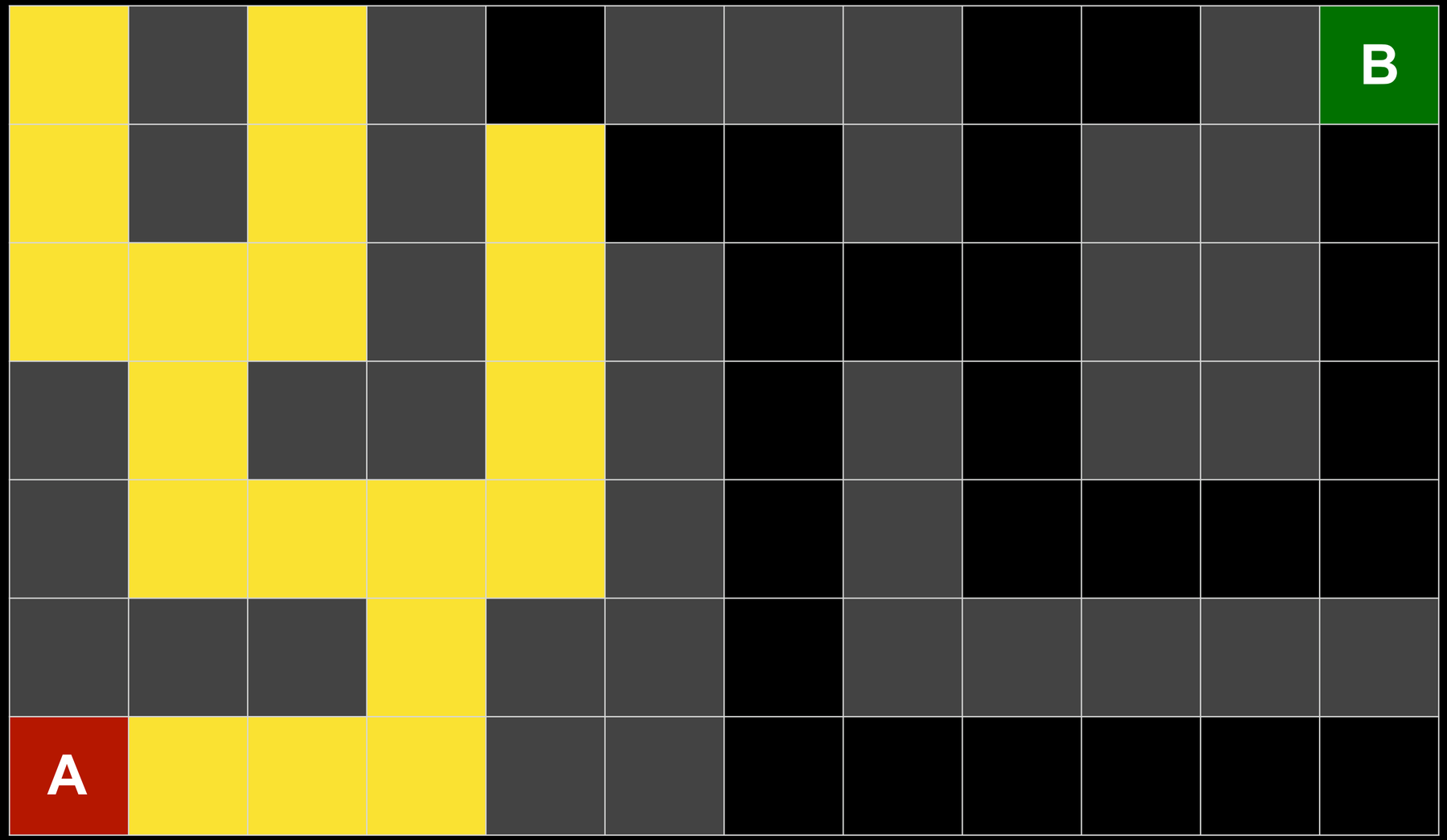
Depth-First Search



Depth-First Search



Depth-First Search



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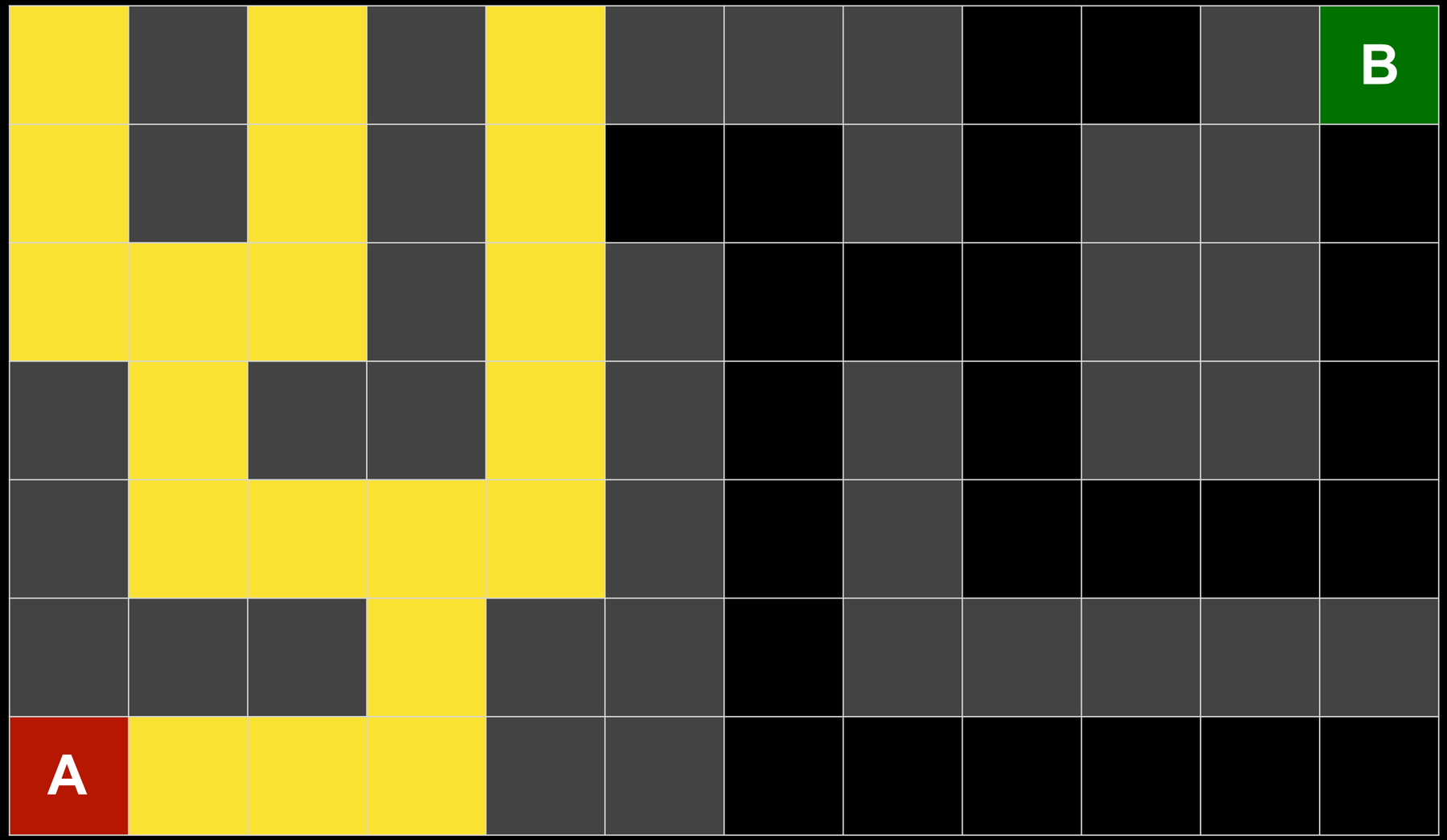

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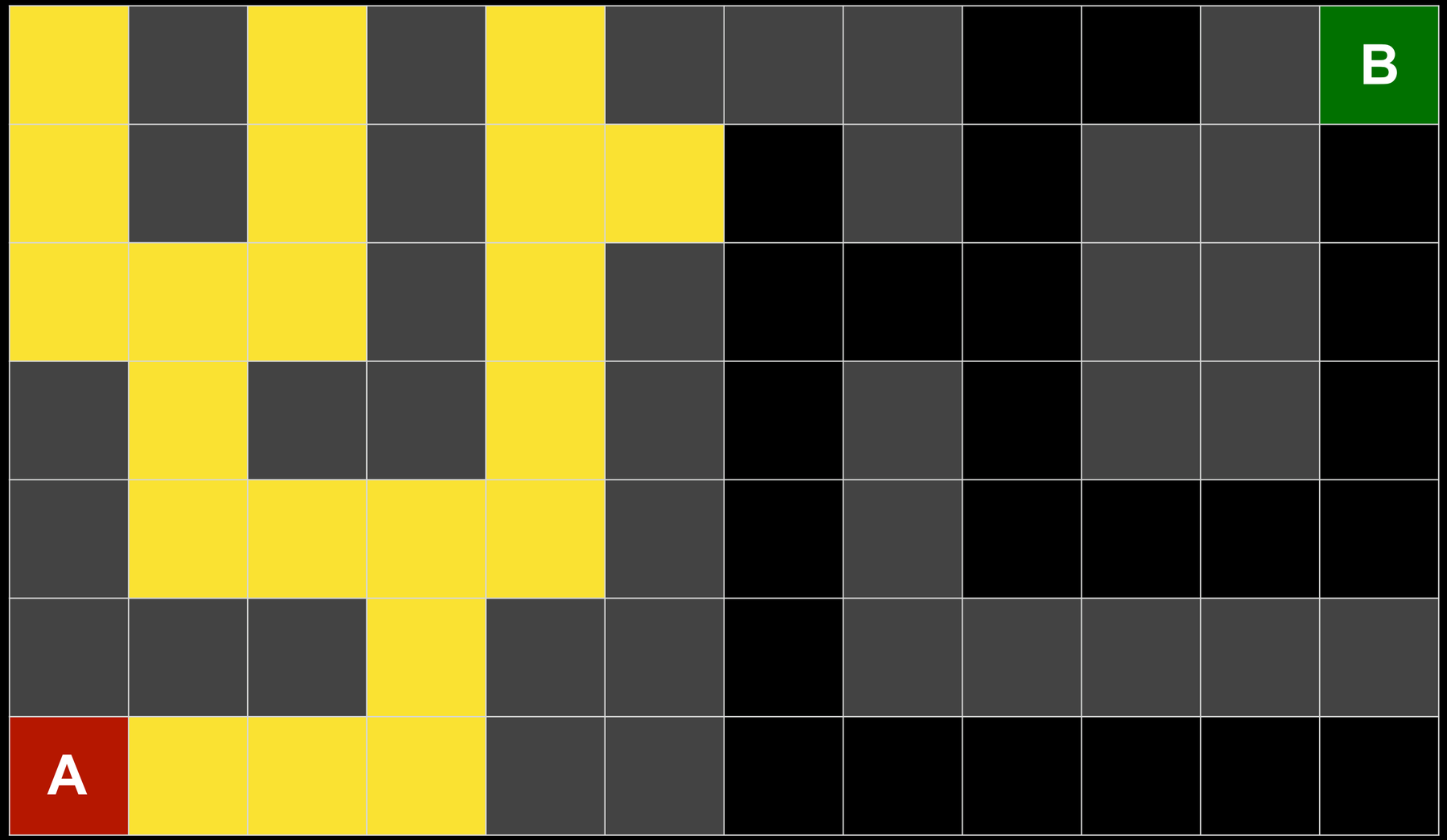

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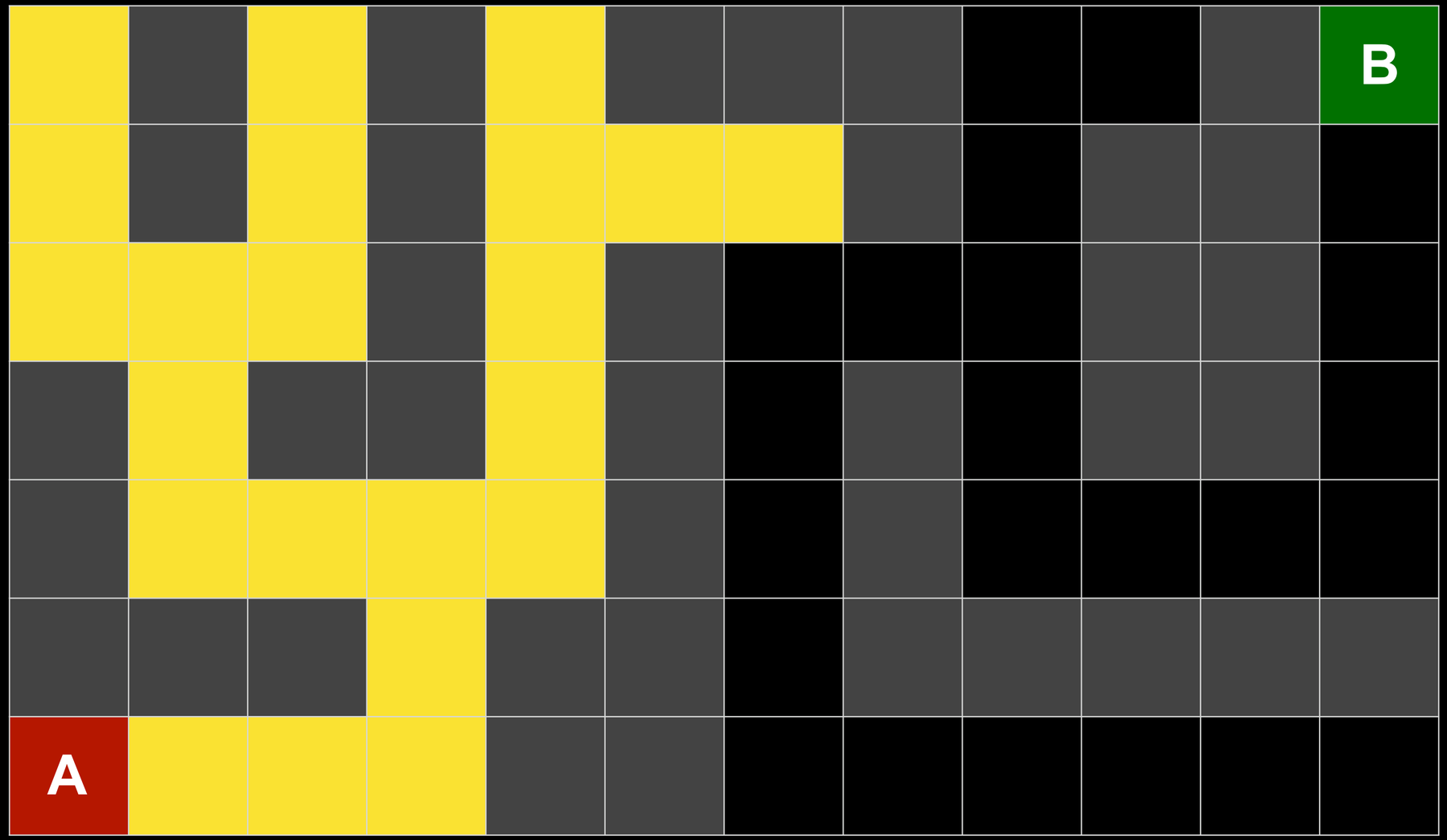

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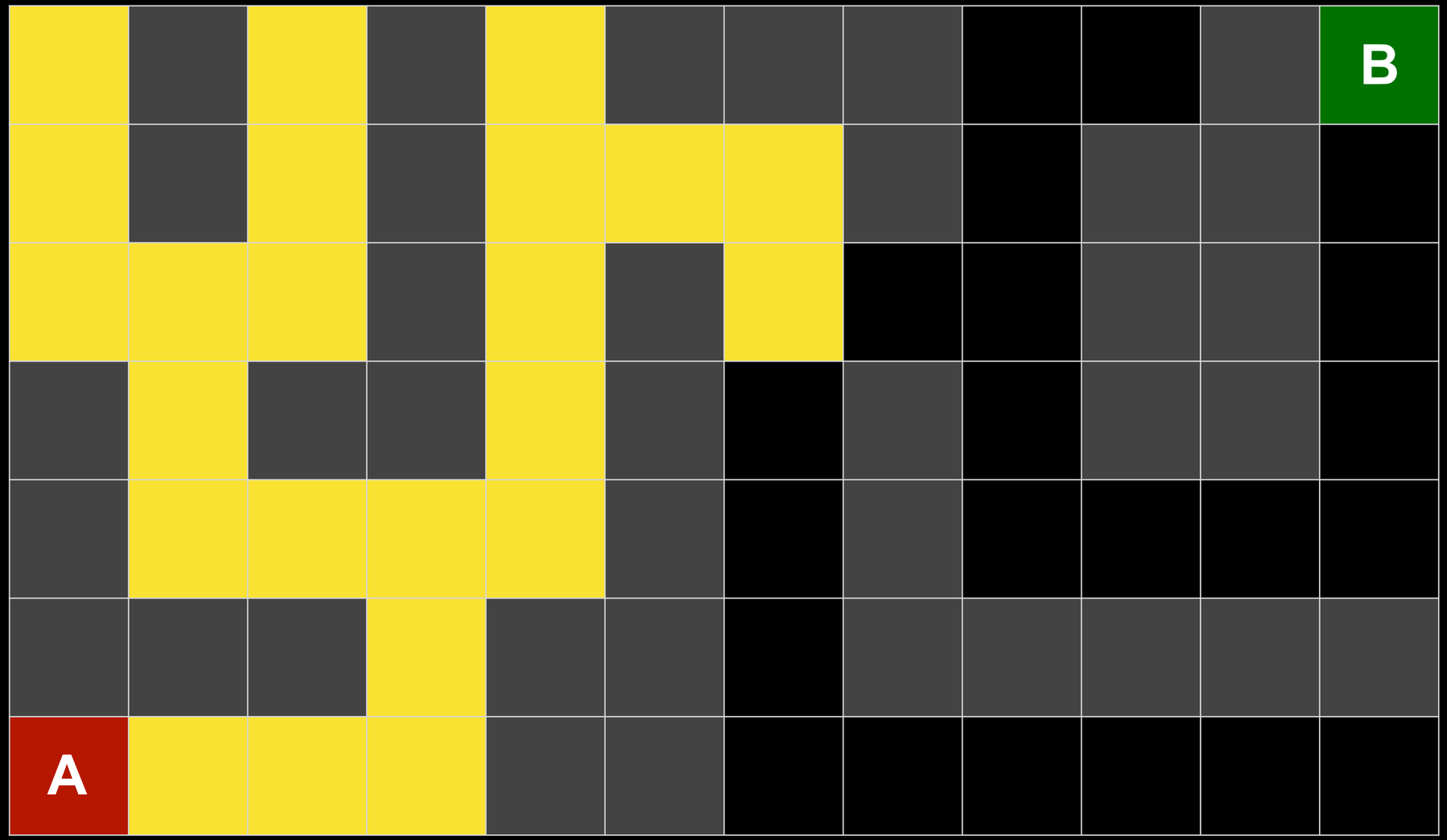

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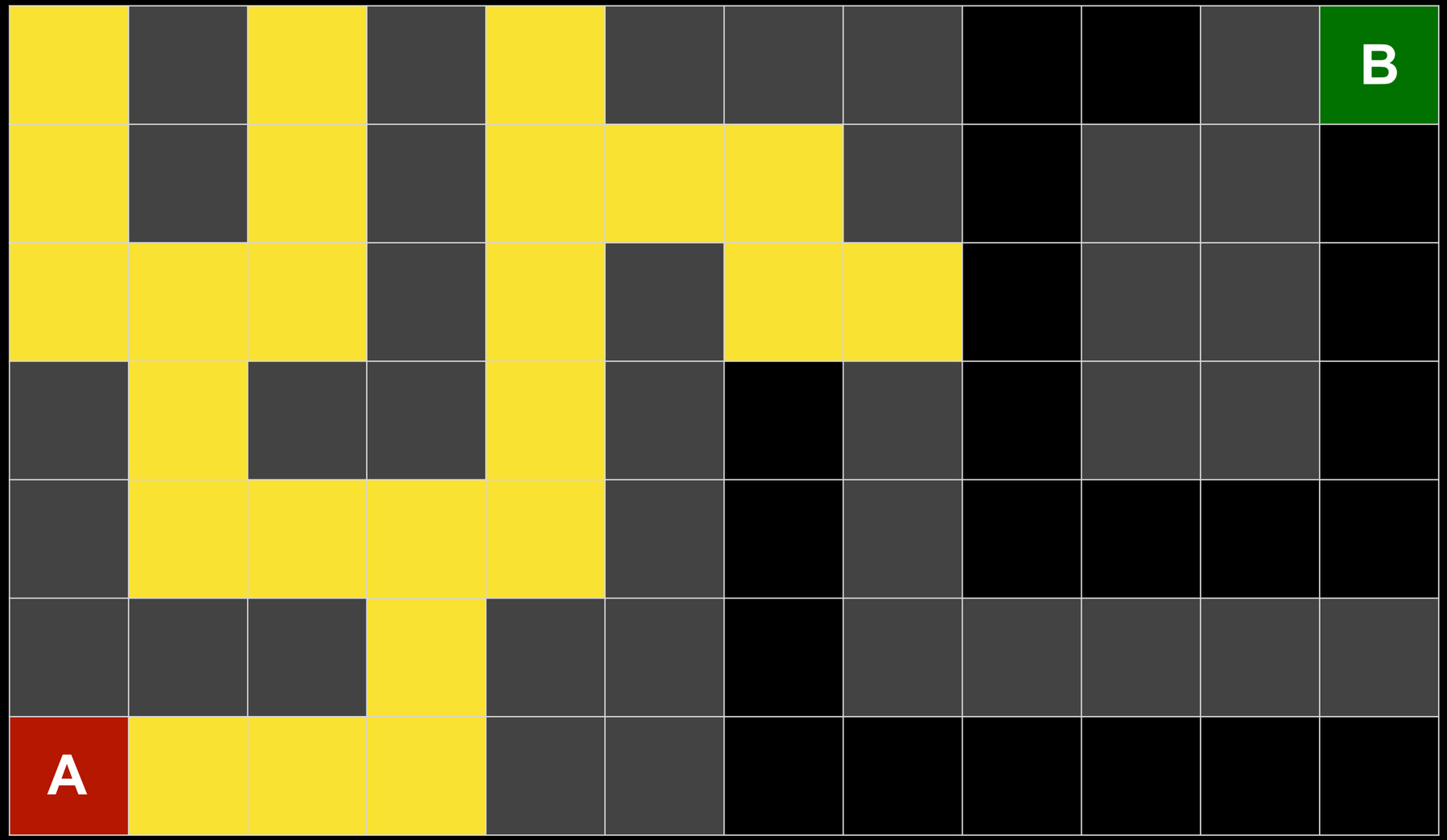

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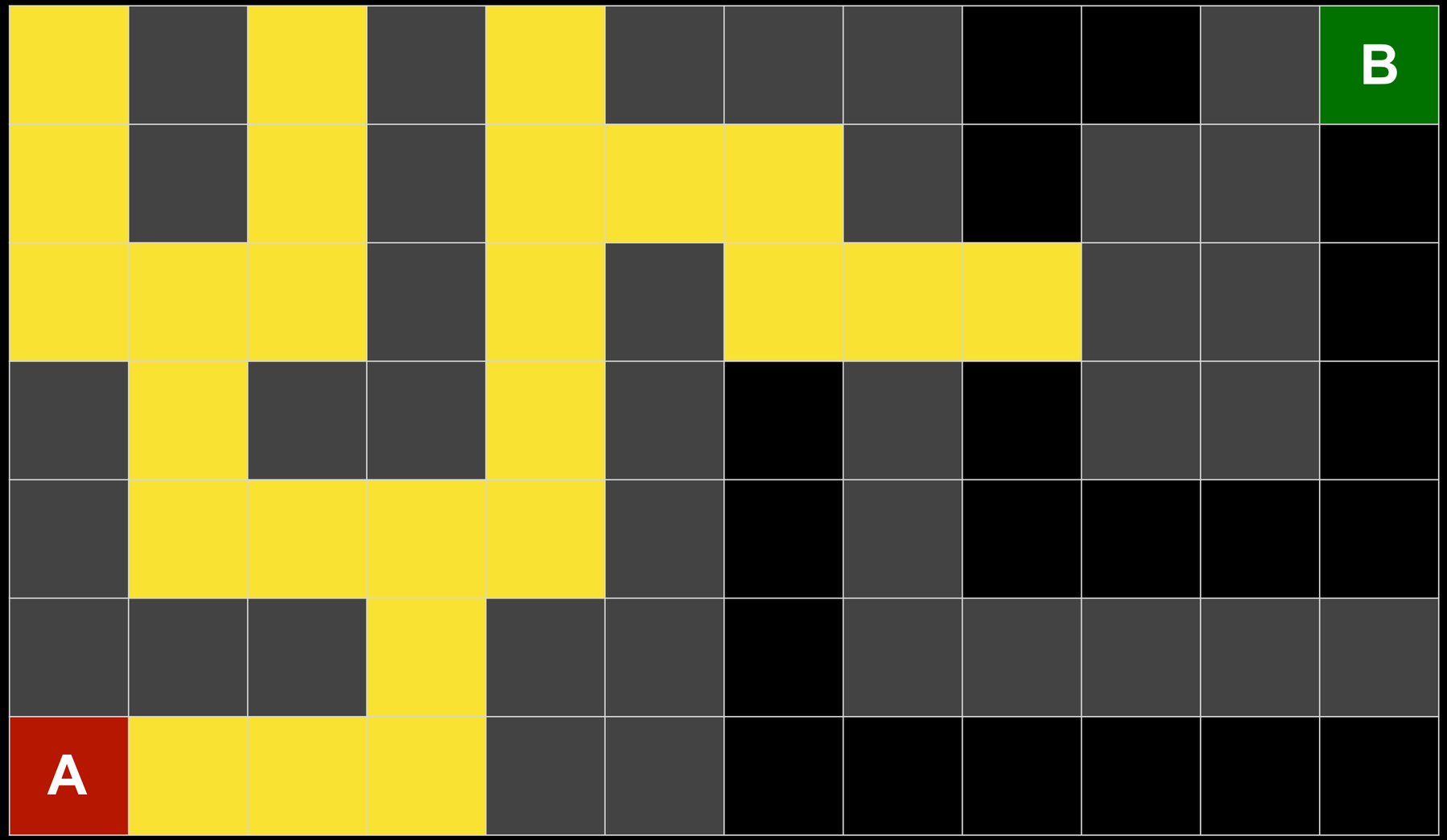

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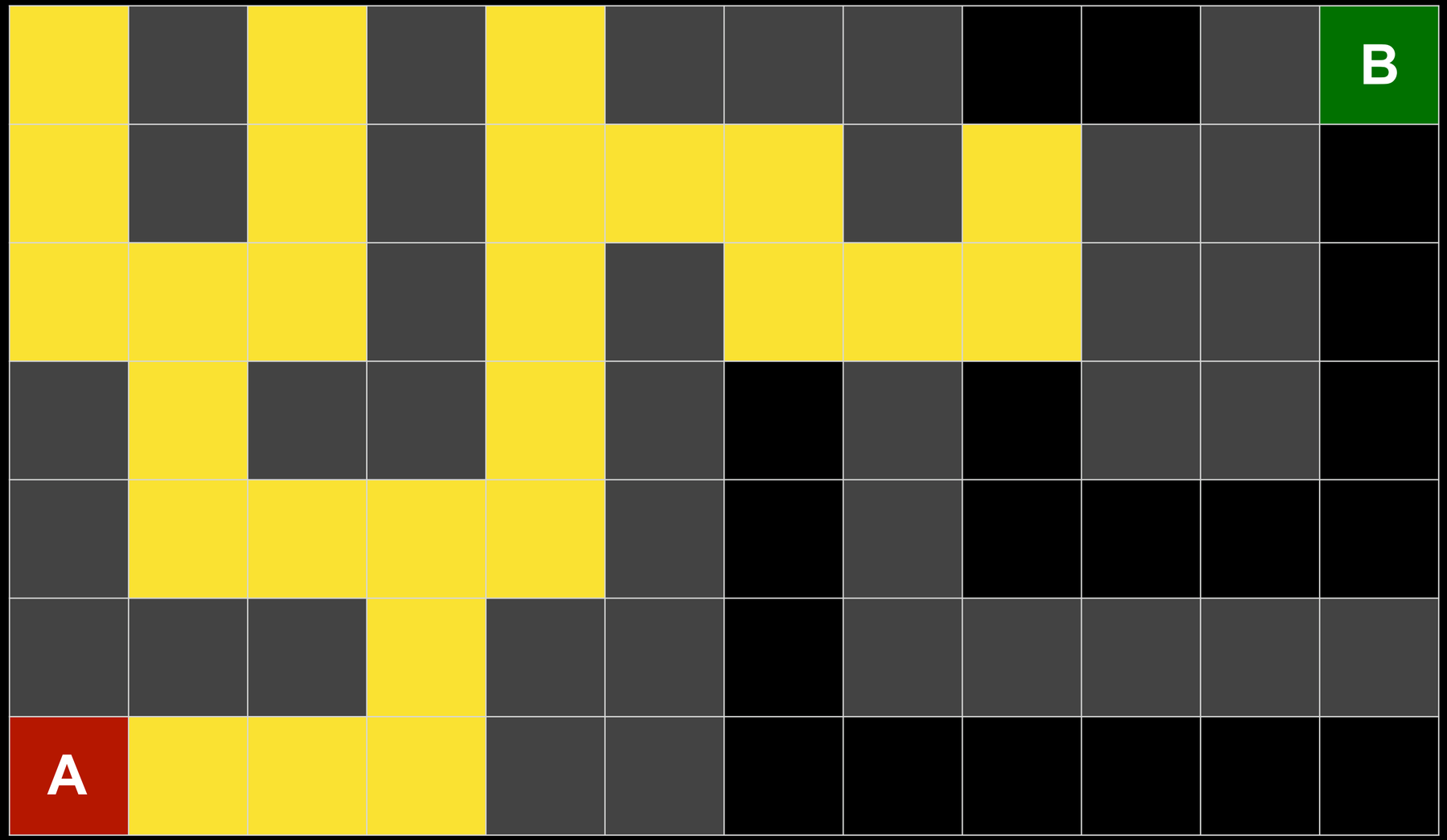

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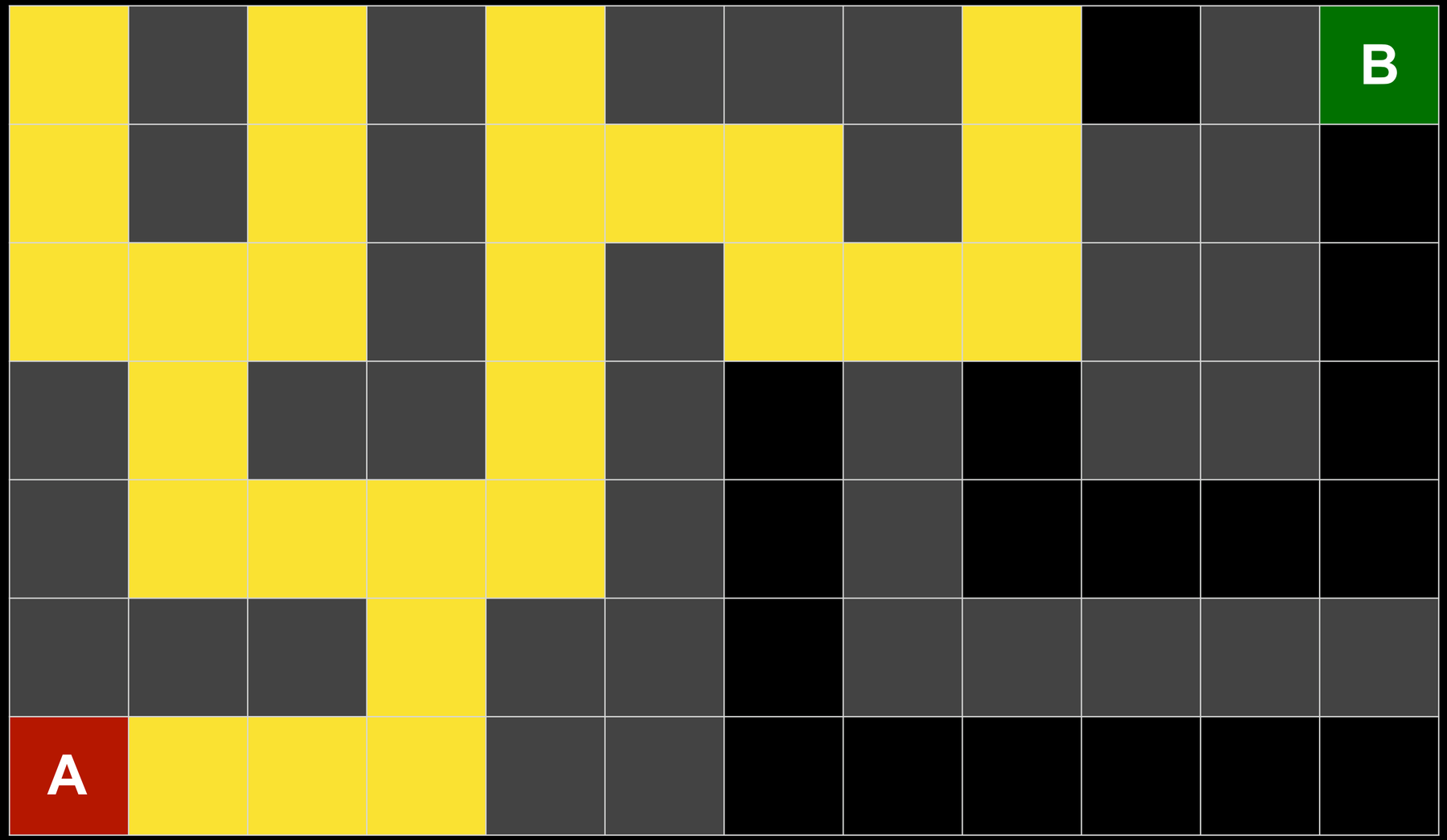

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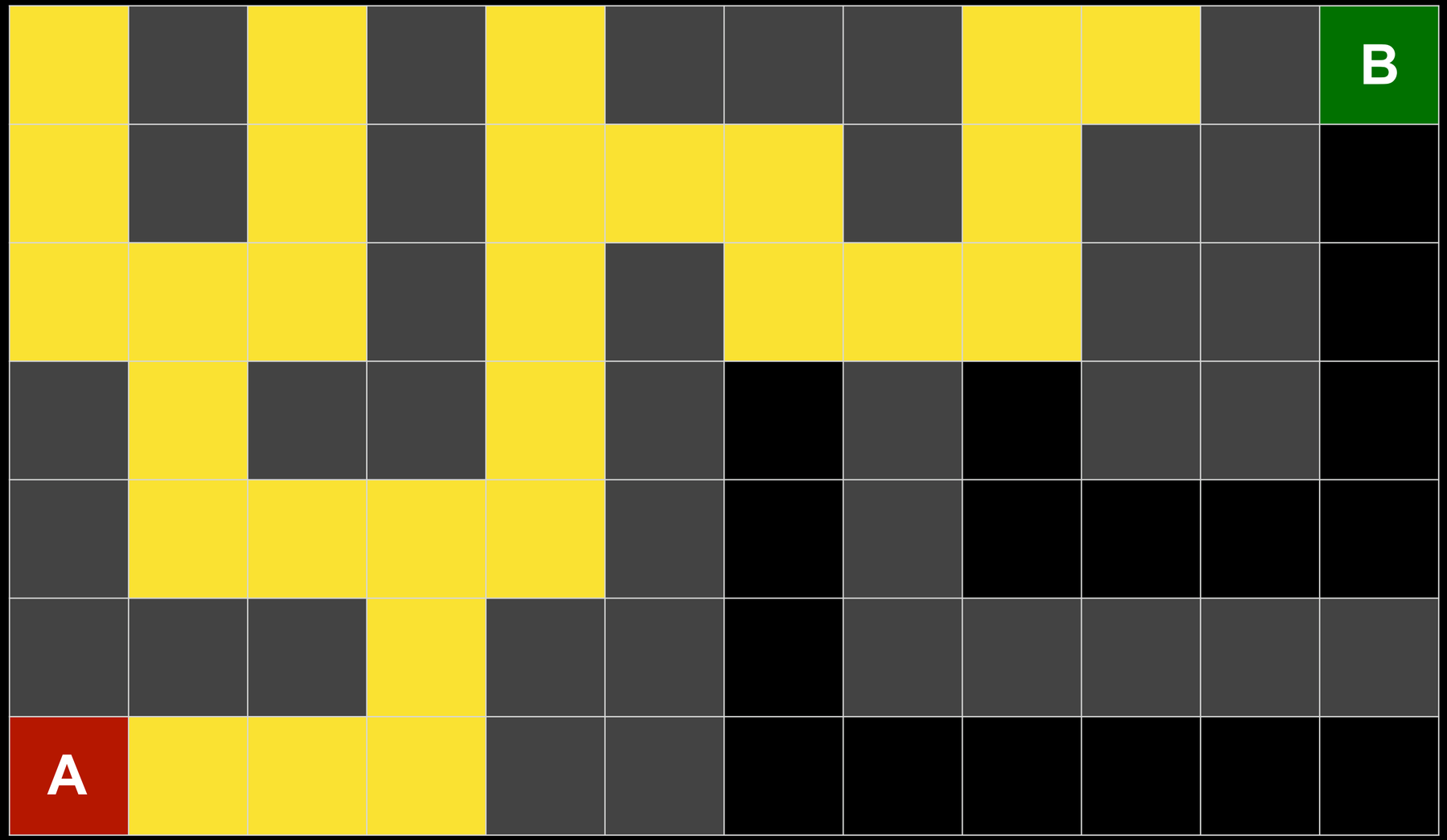

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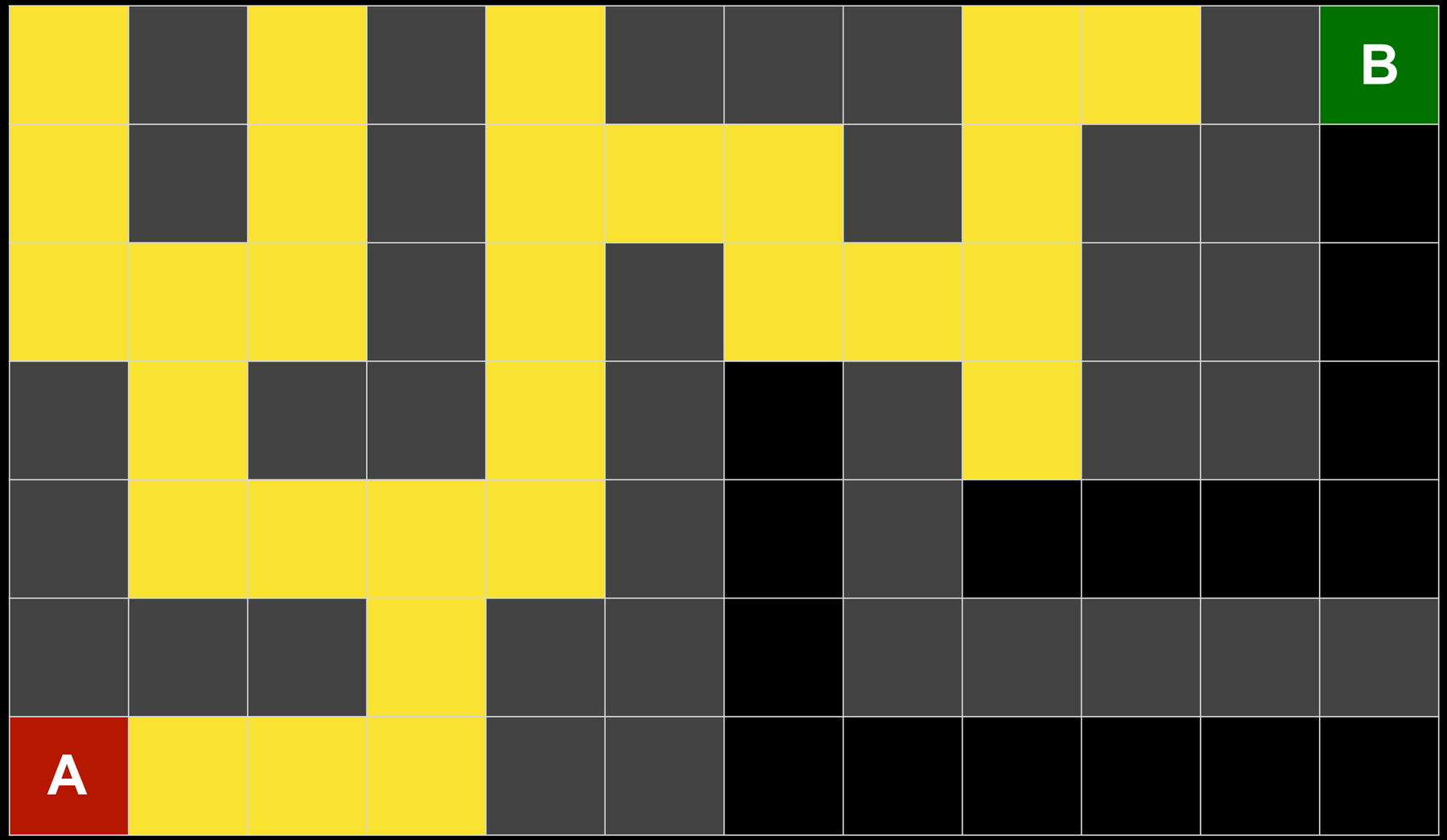

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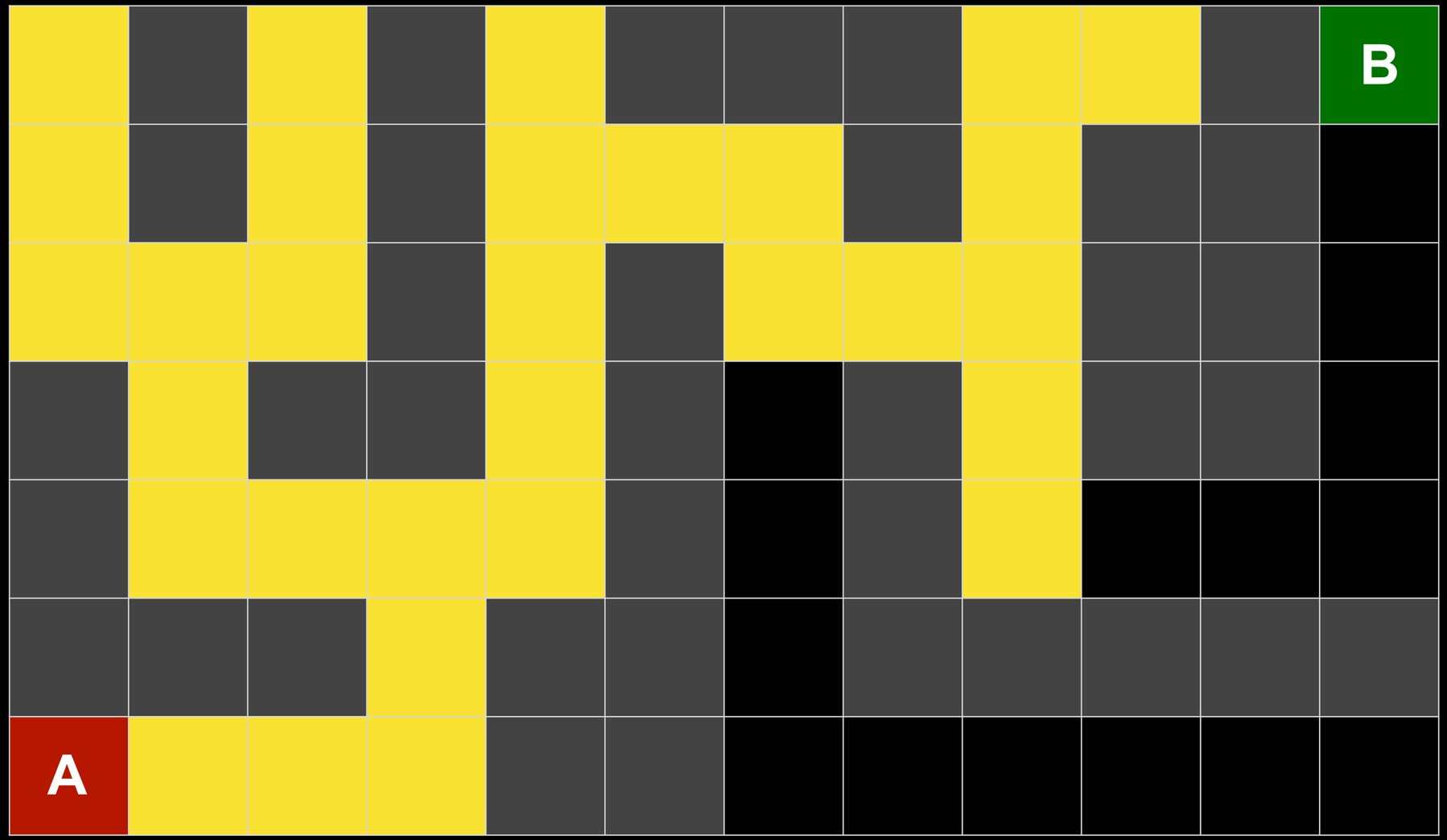

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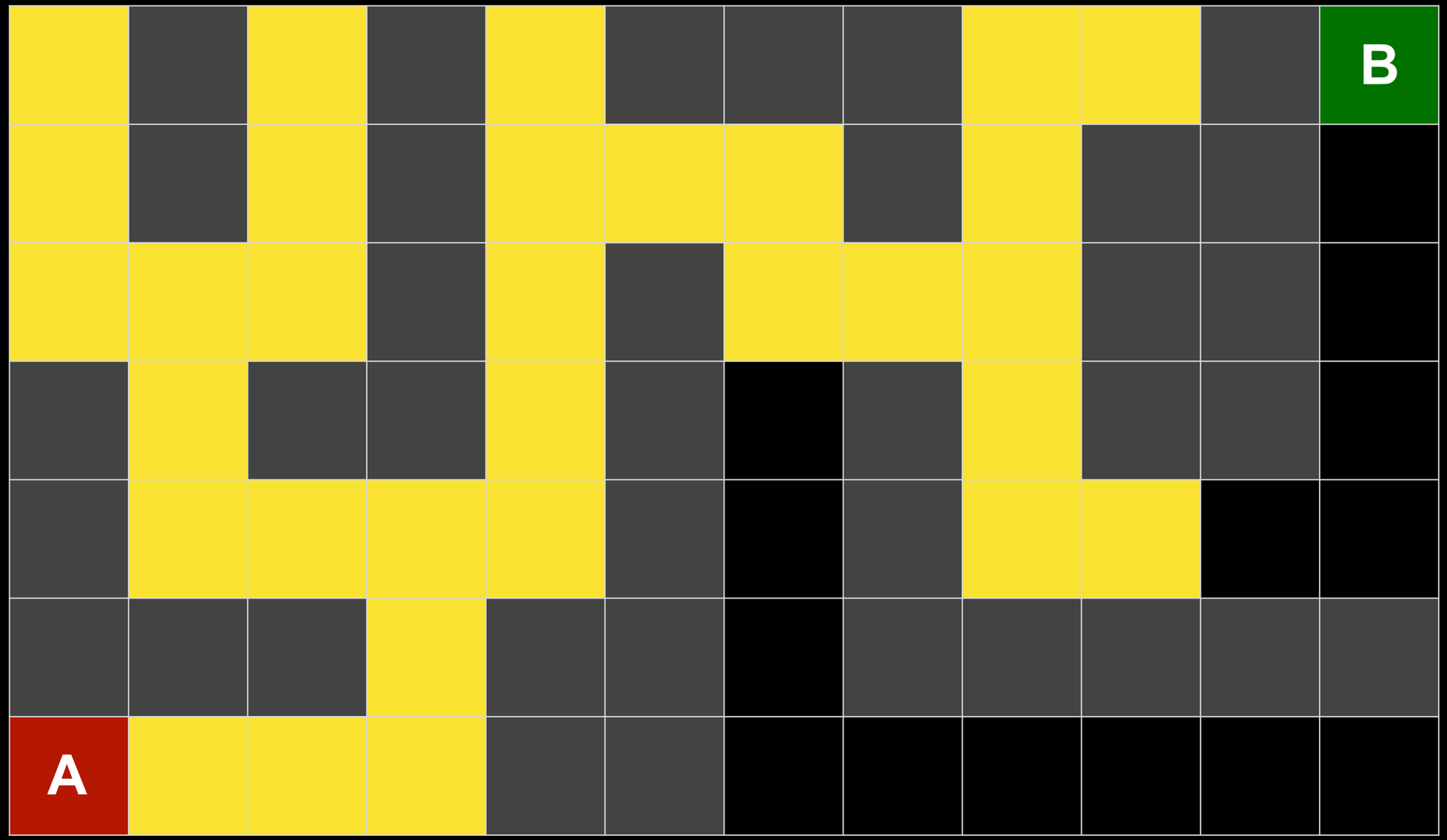

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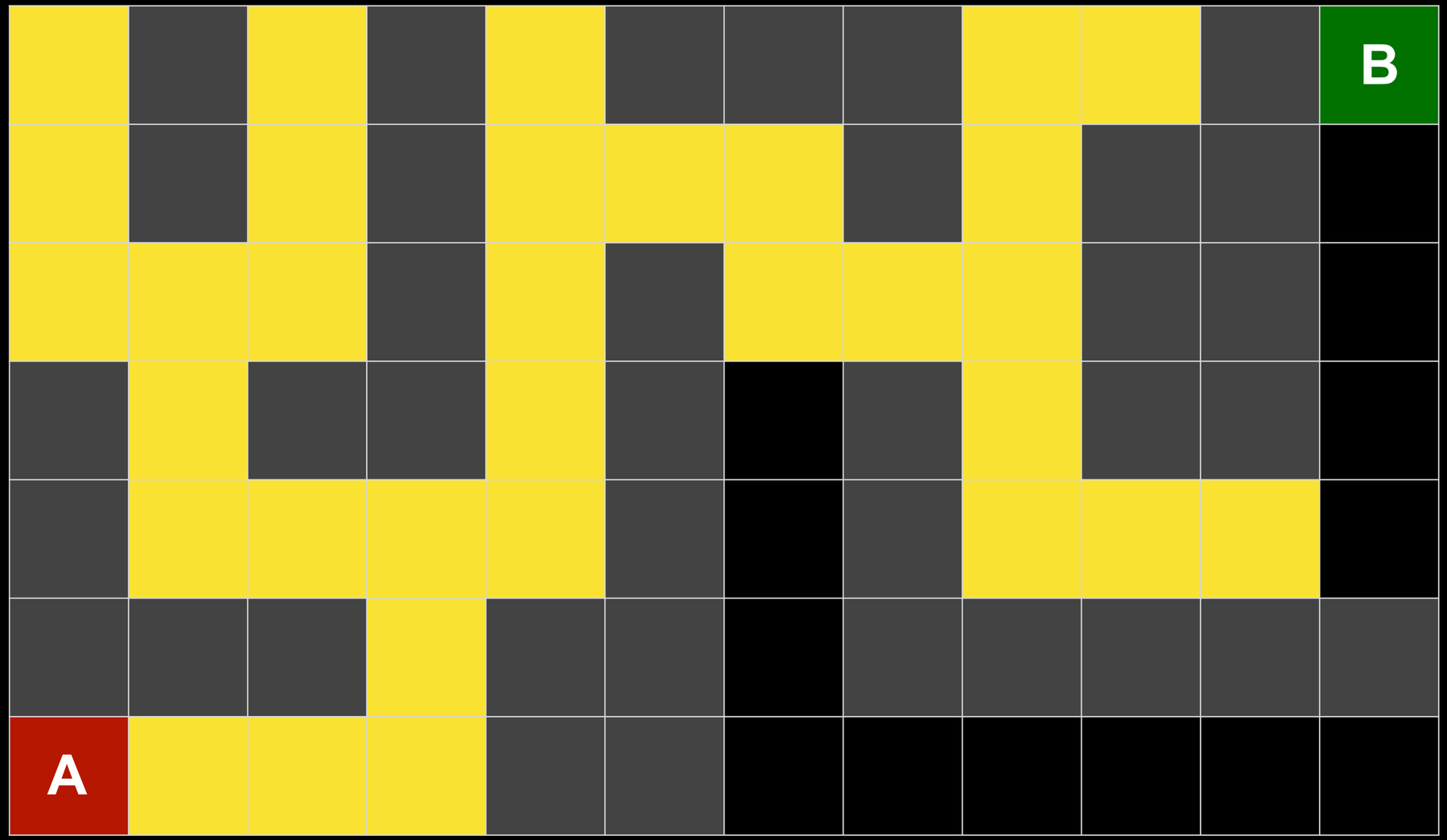

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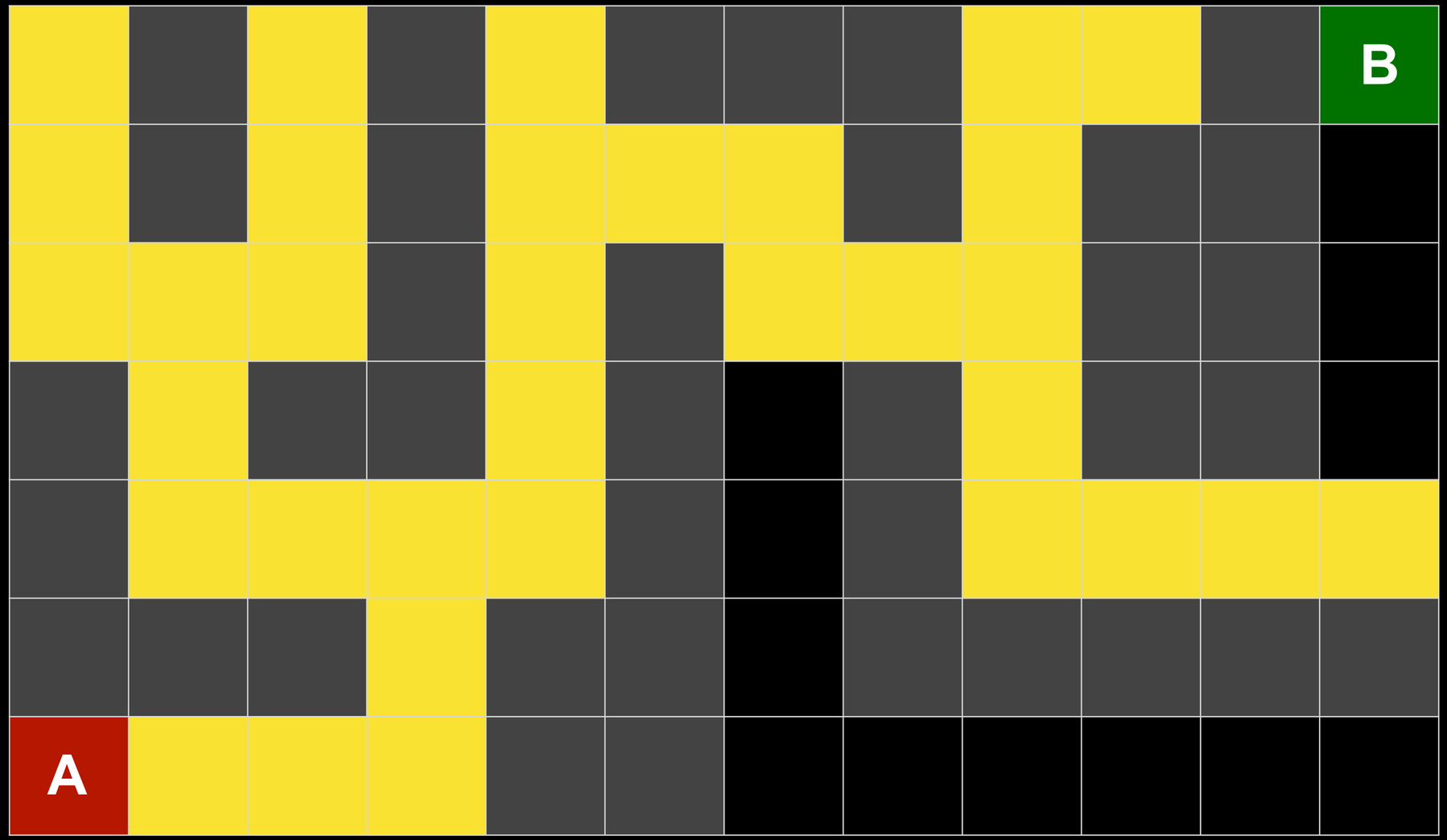

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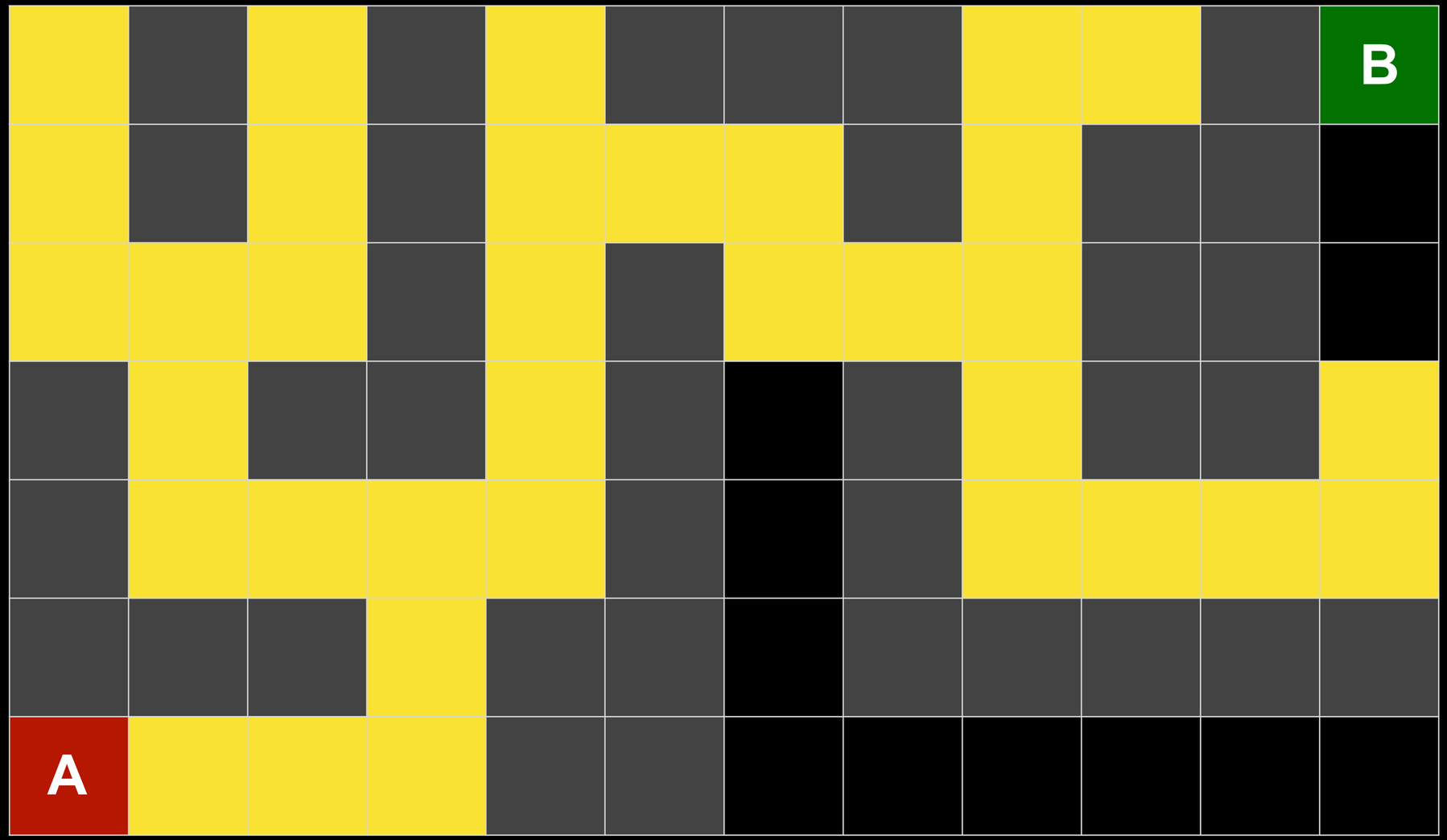

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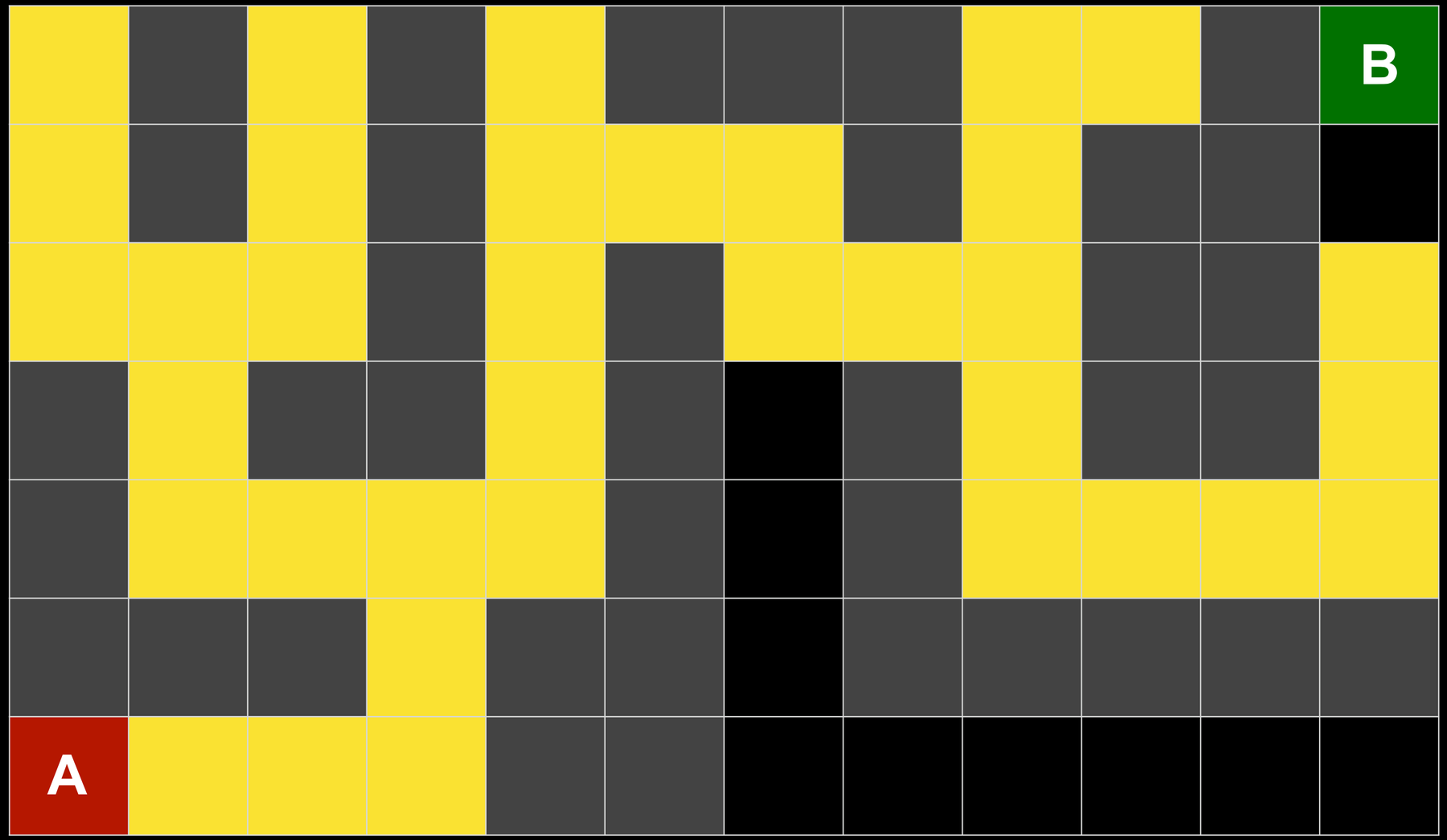

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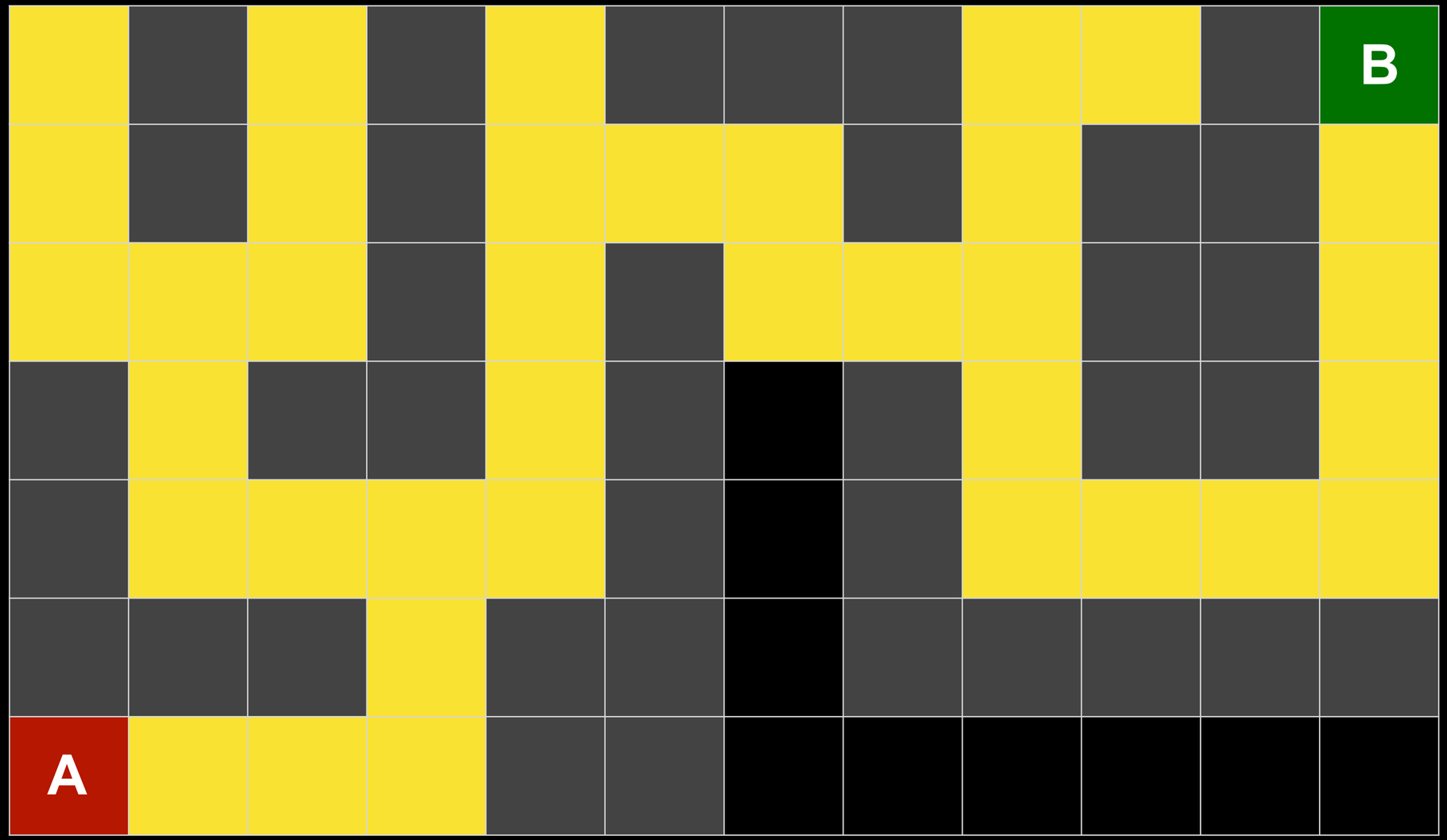

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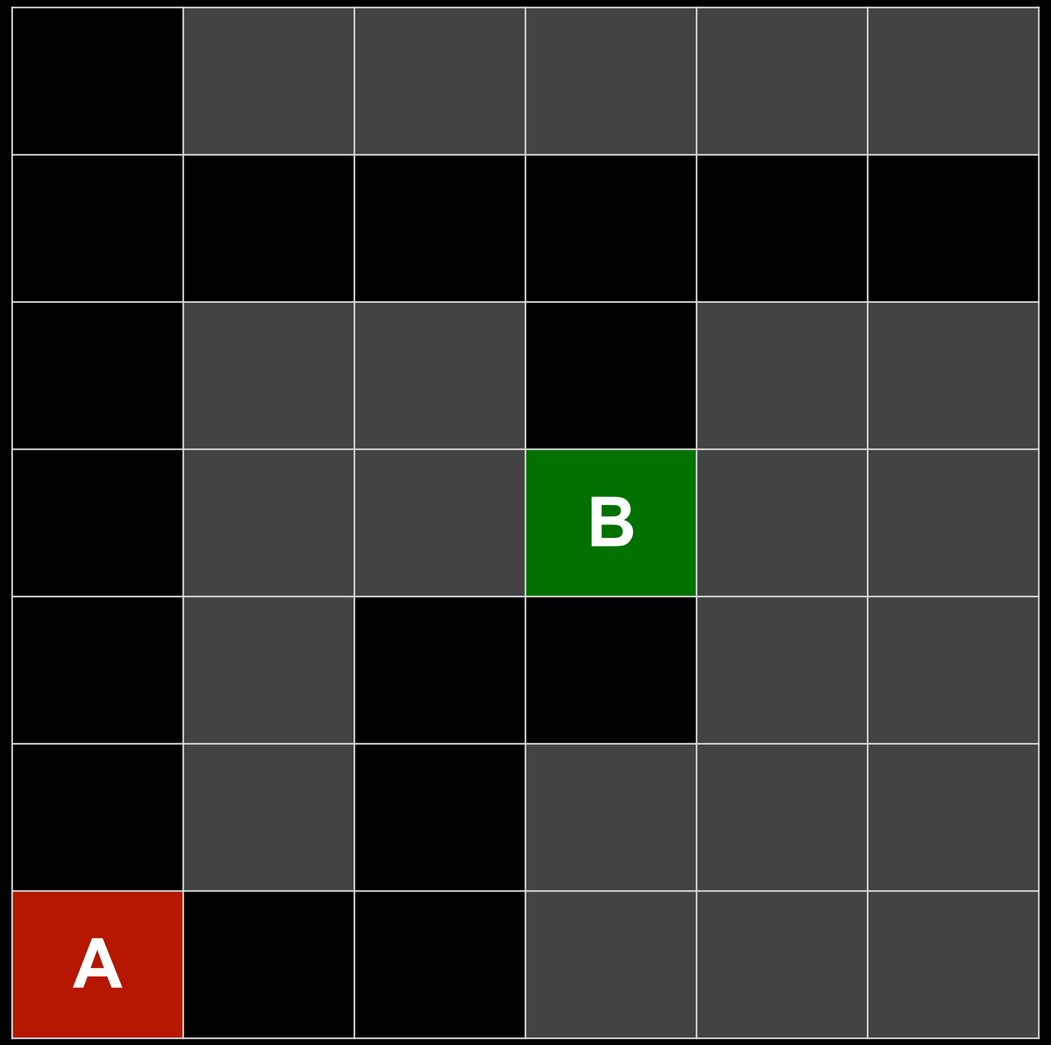

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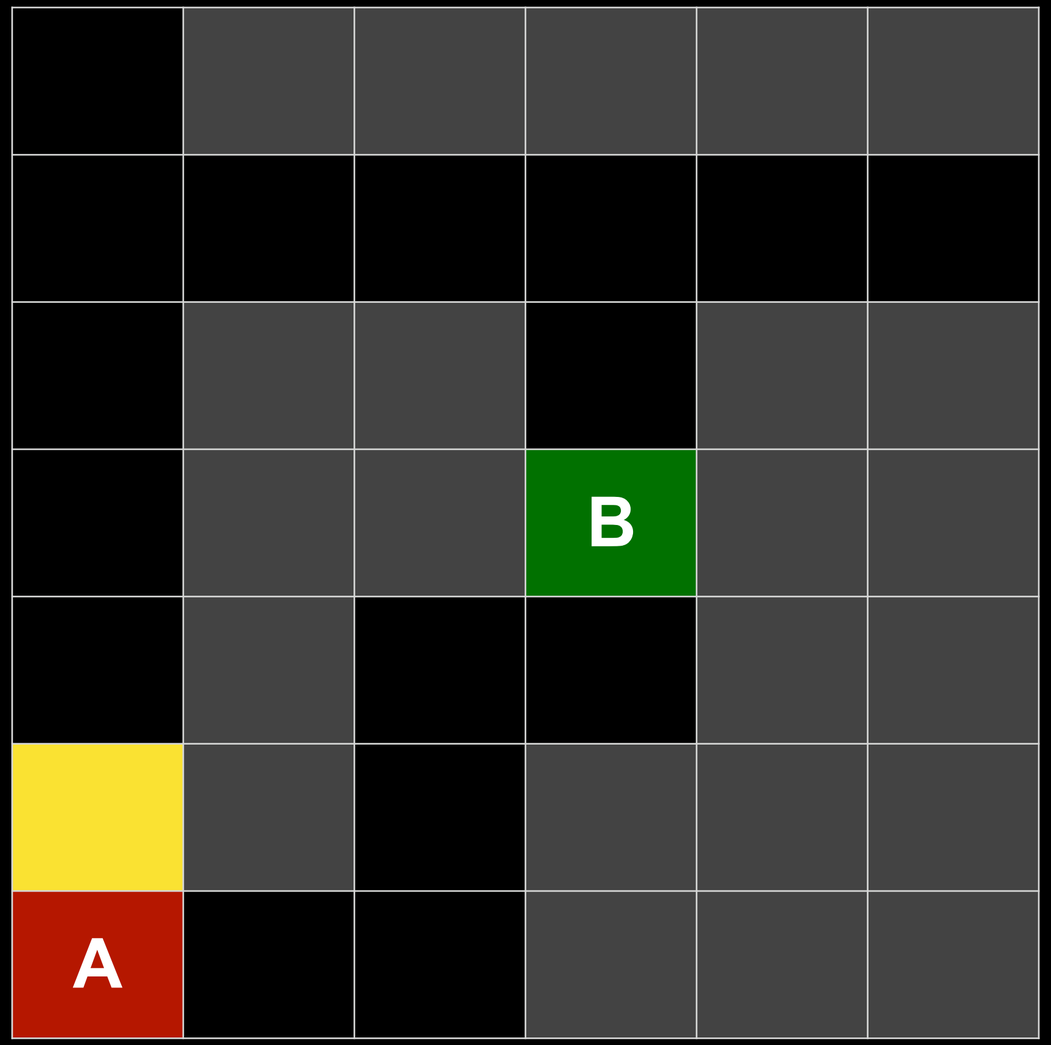

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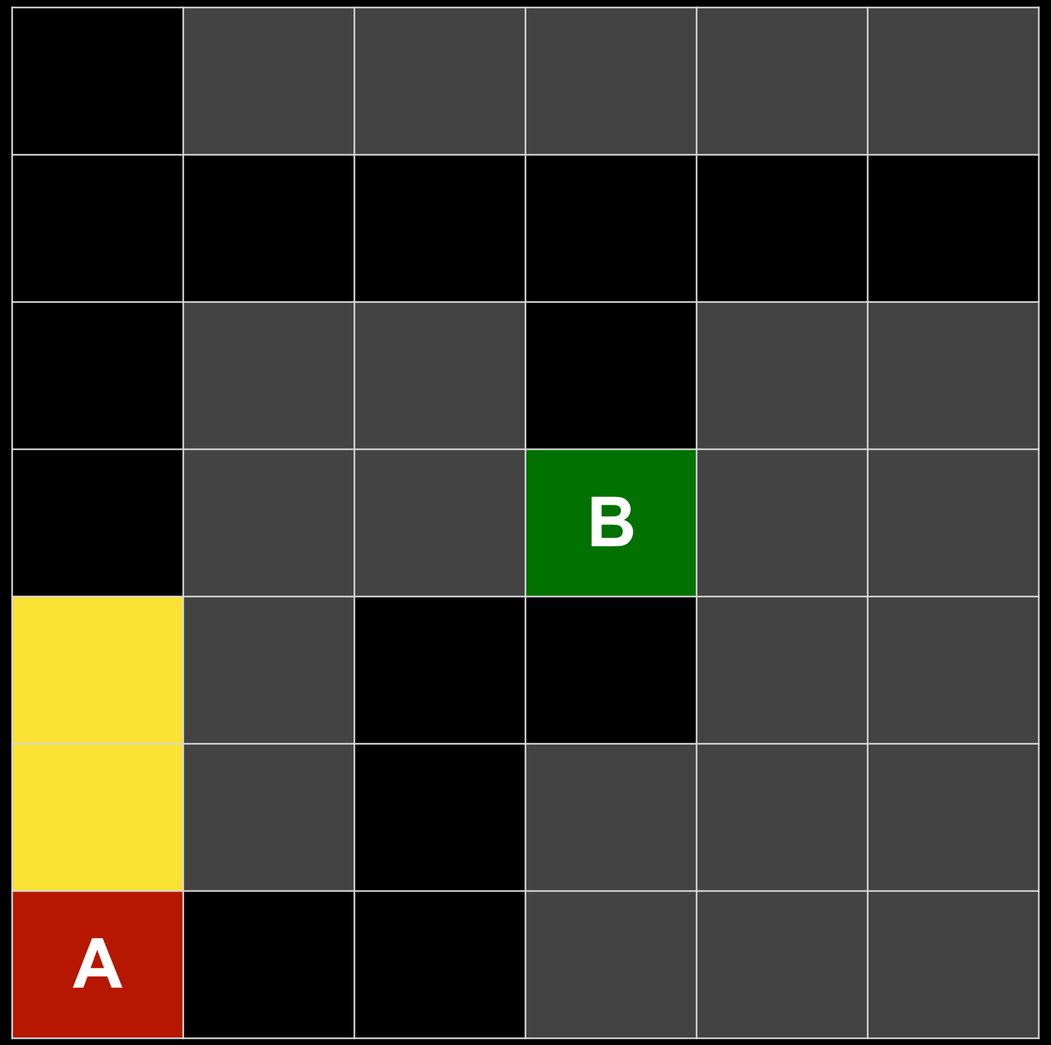

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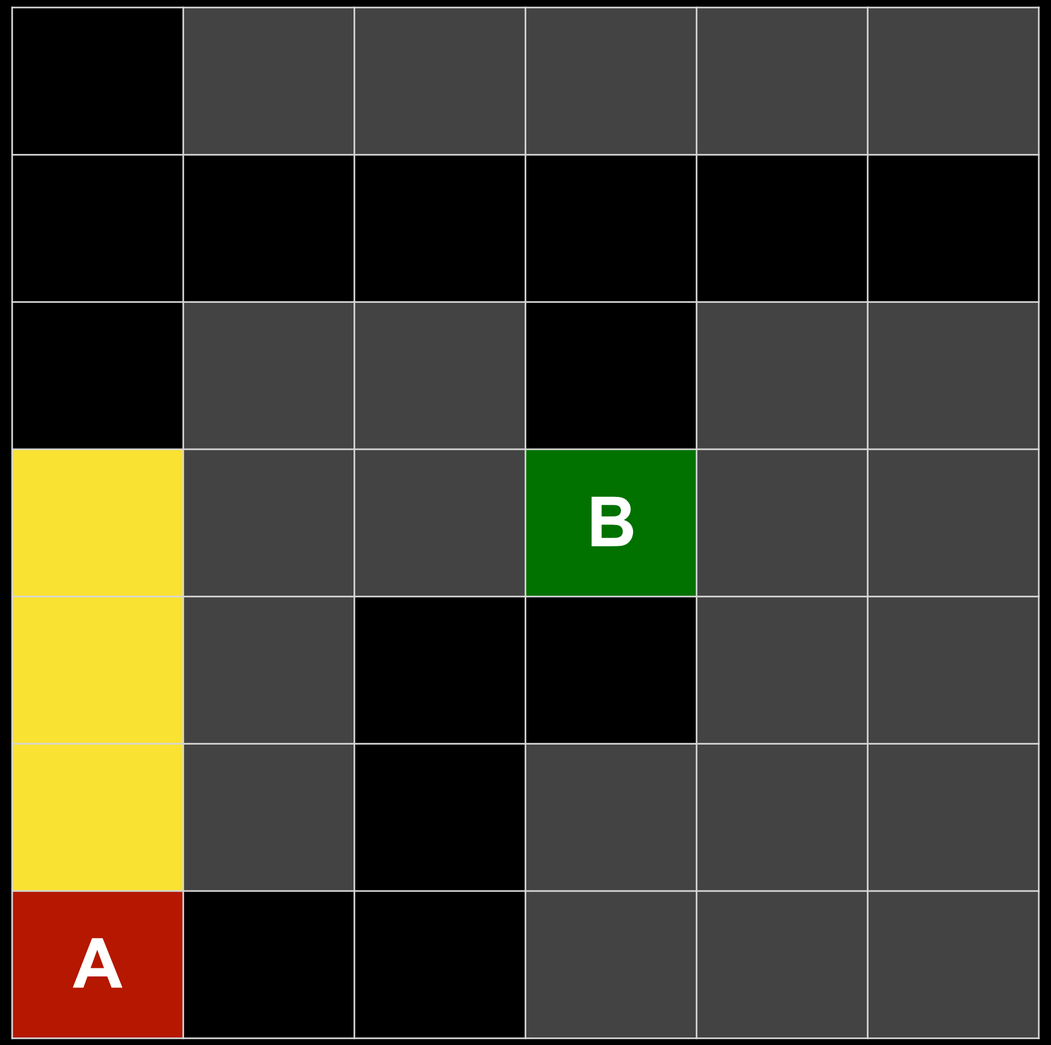

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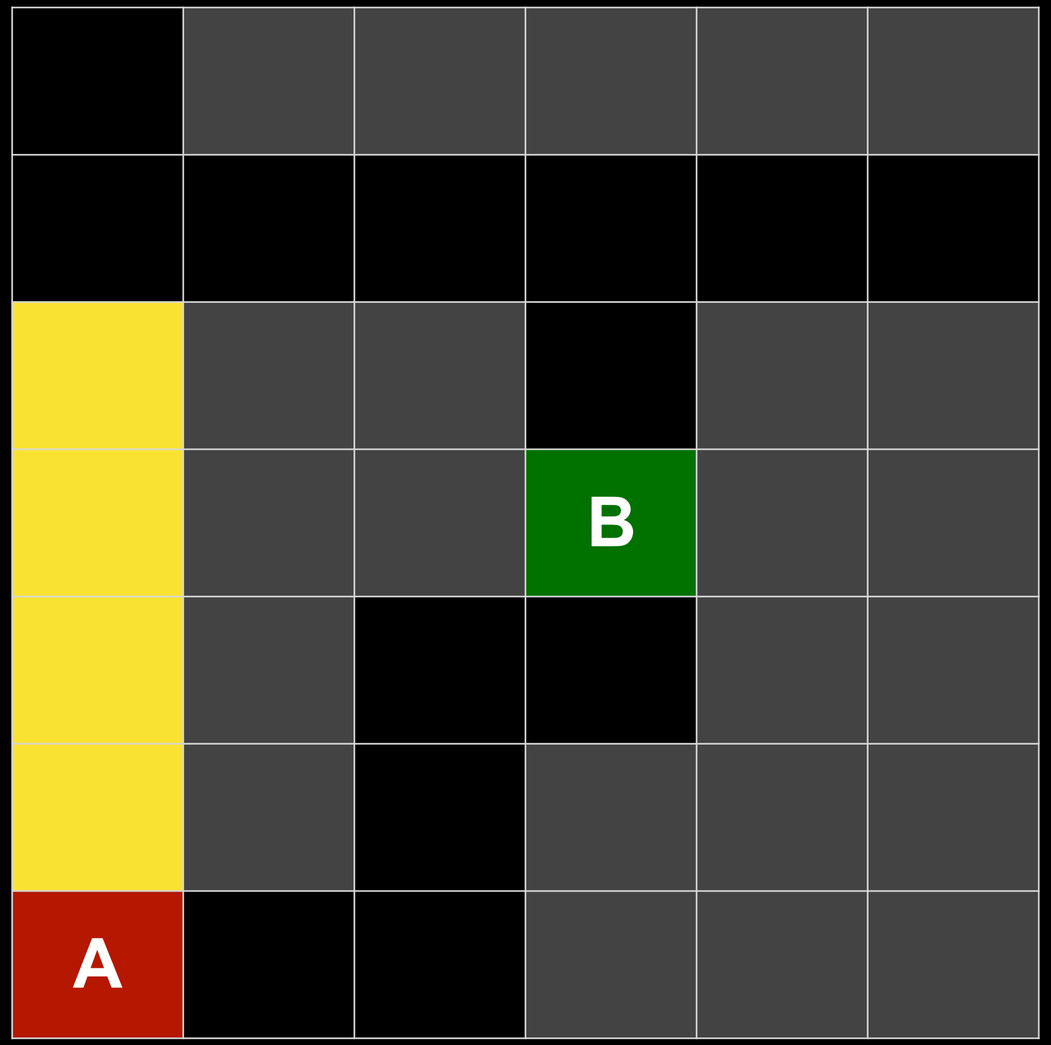

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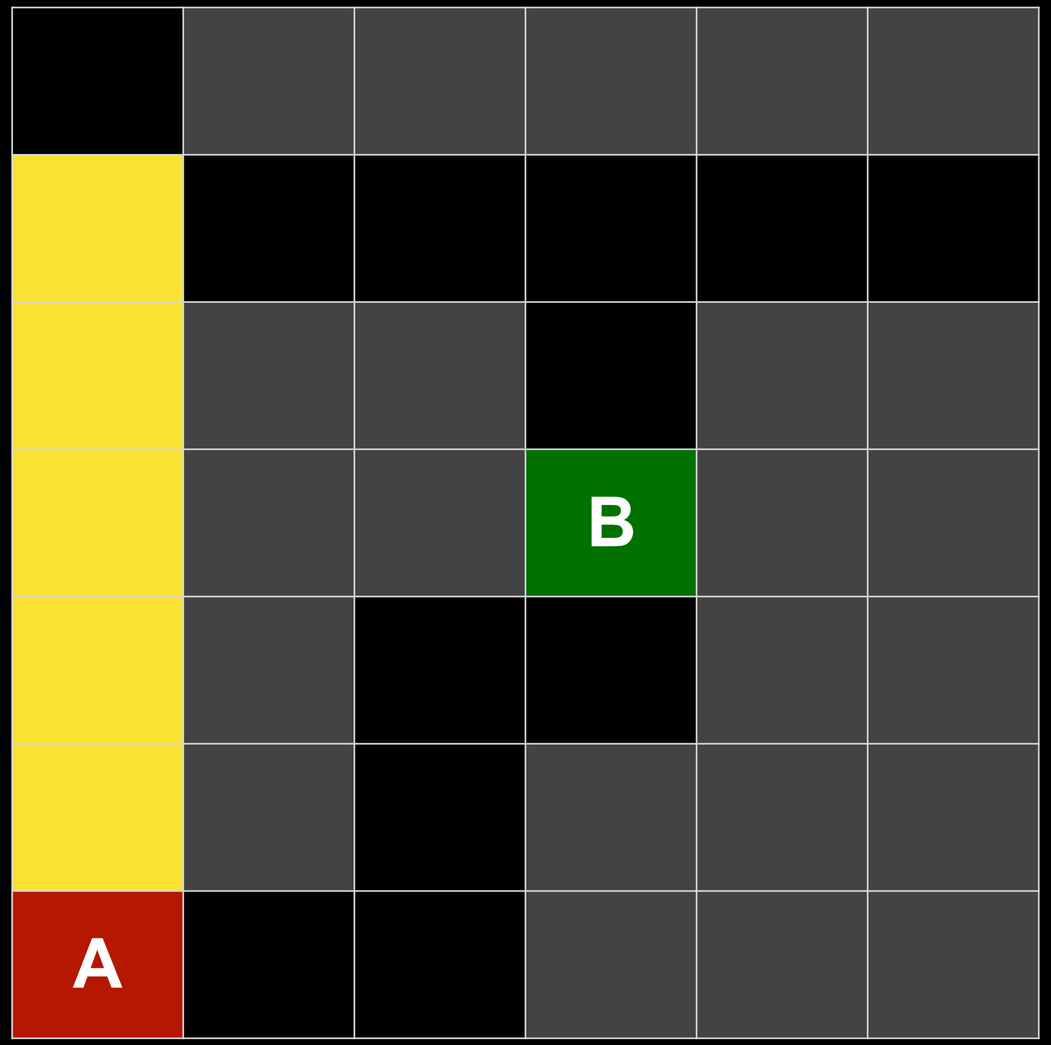

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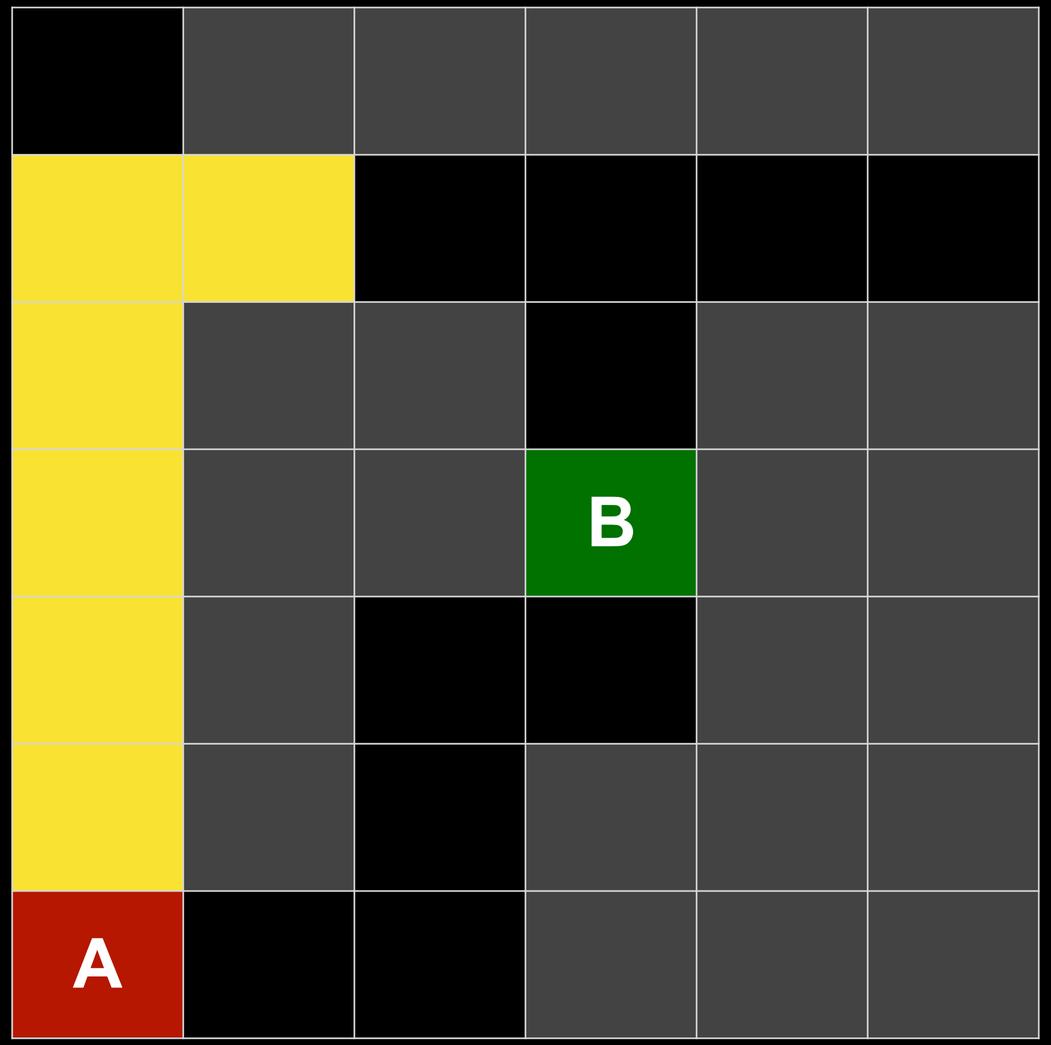

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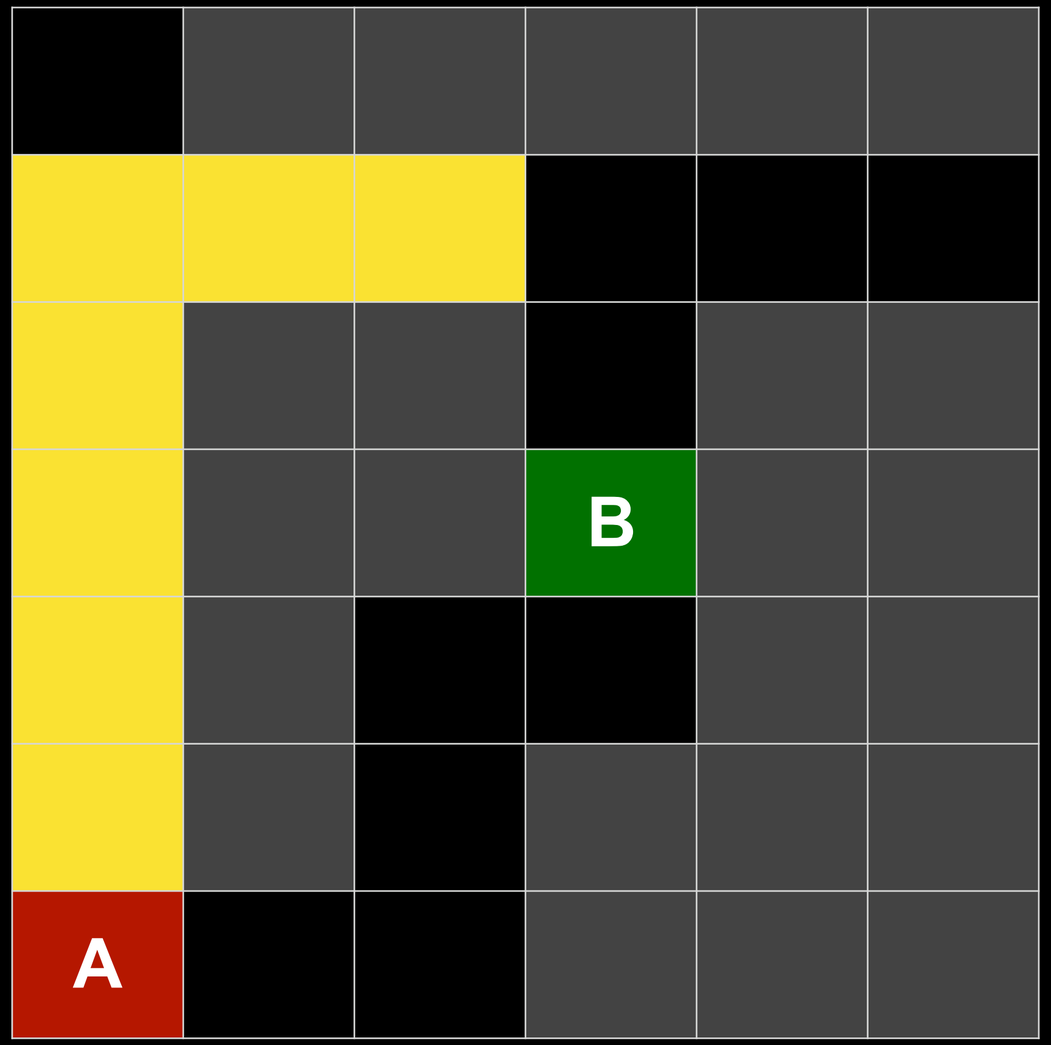

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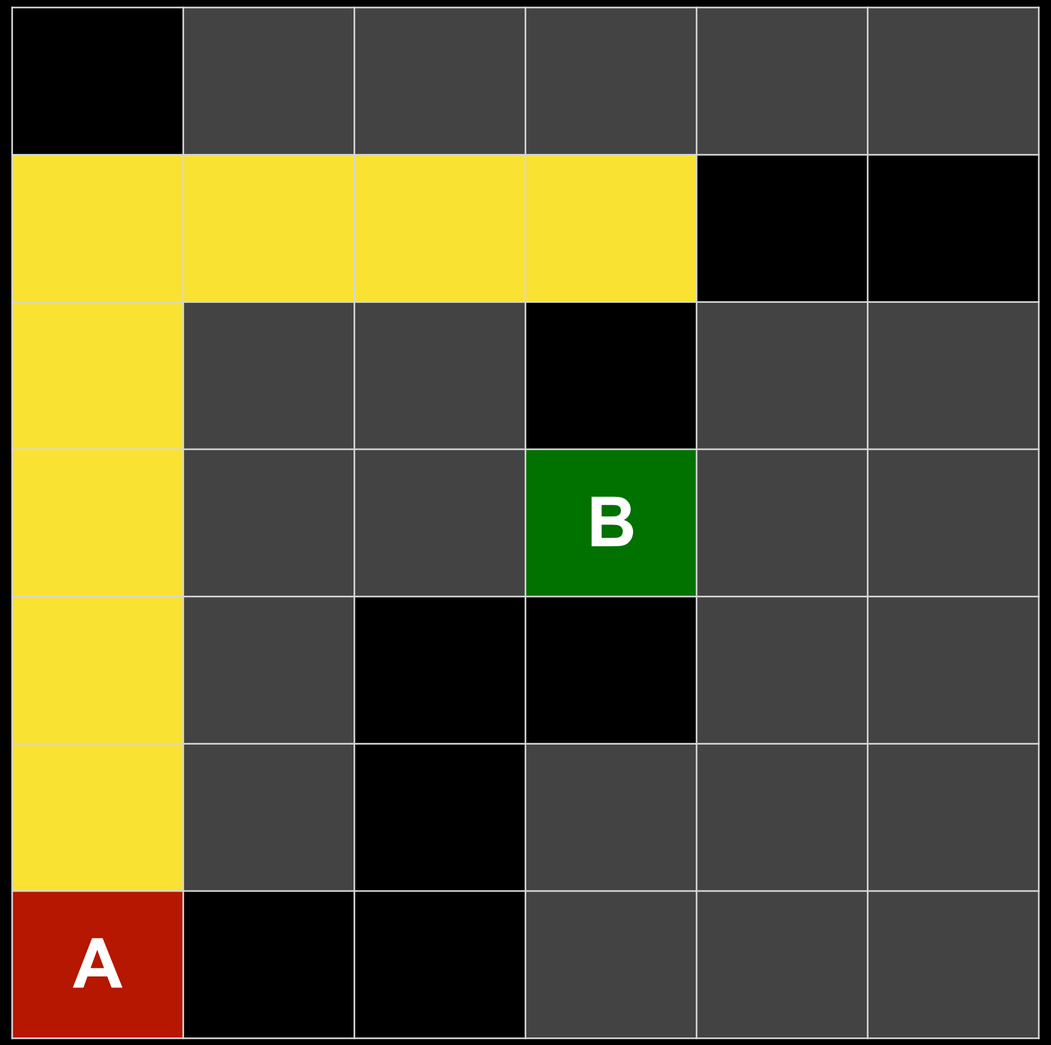

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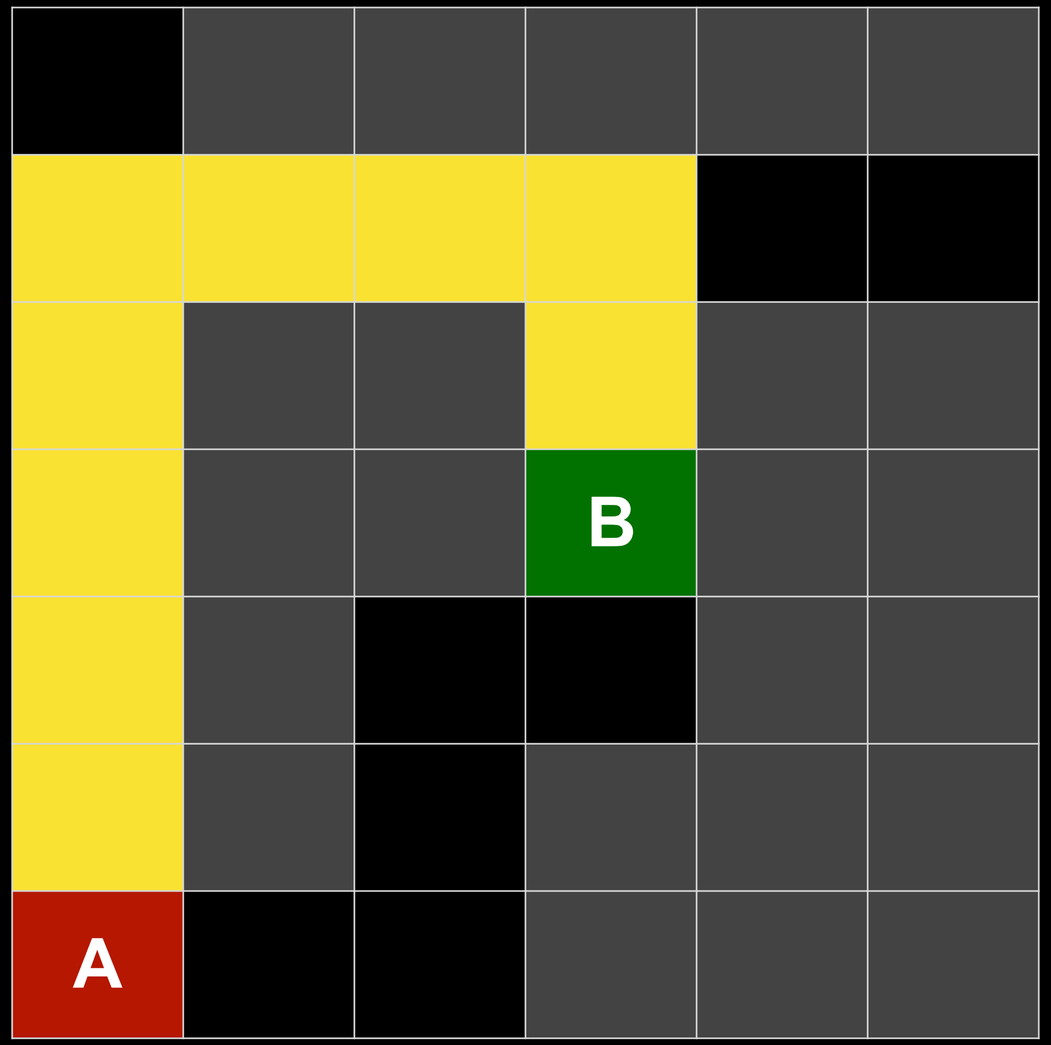

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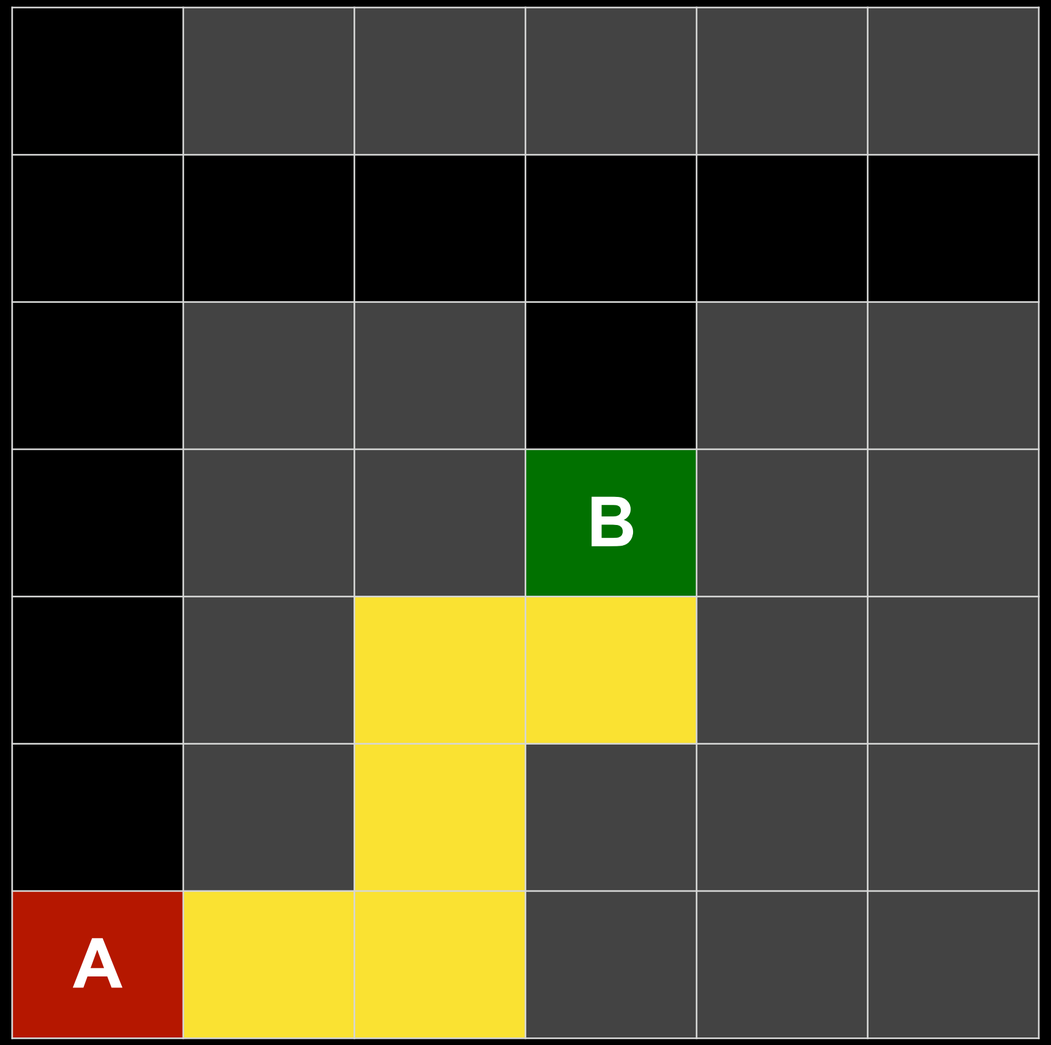

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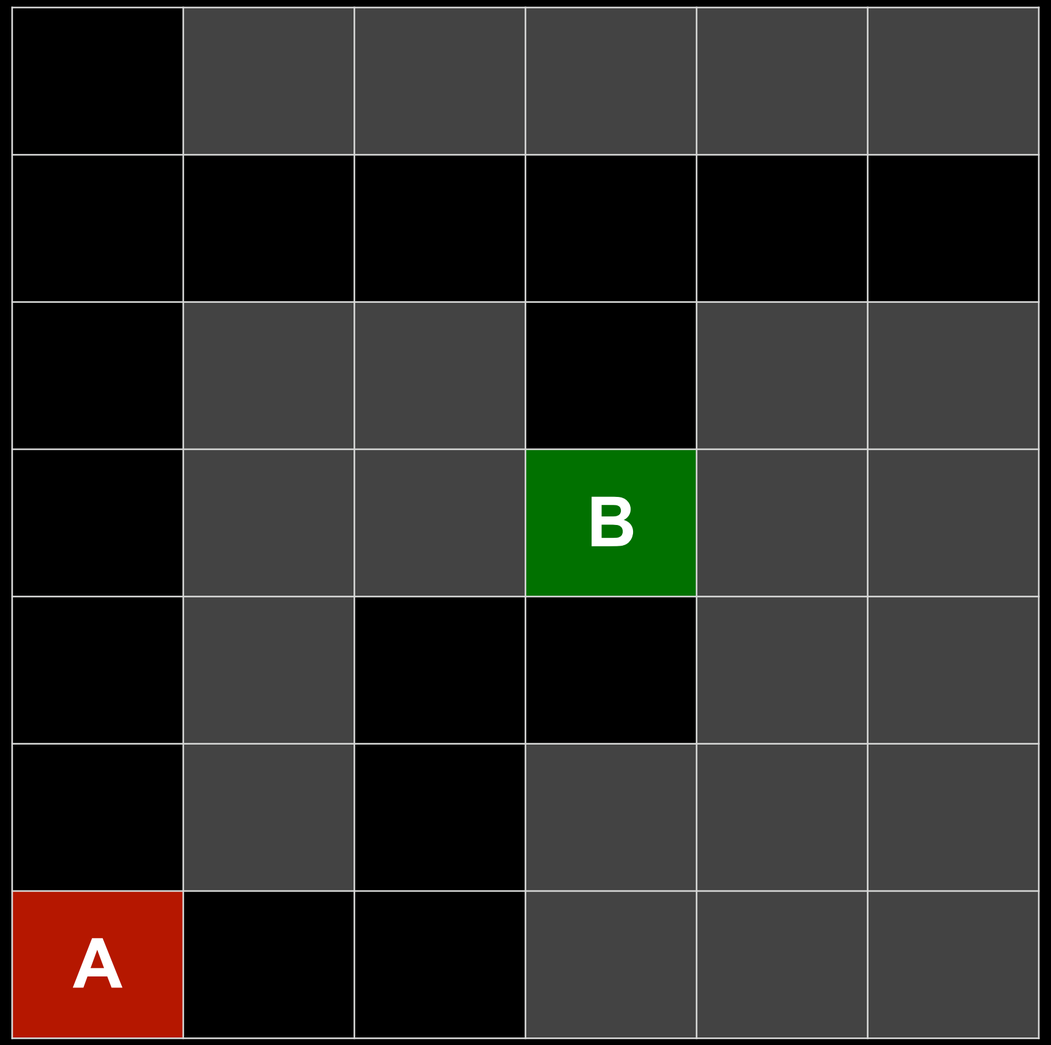

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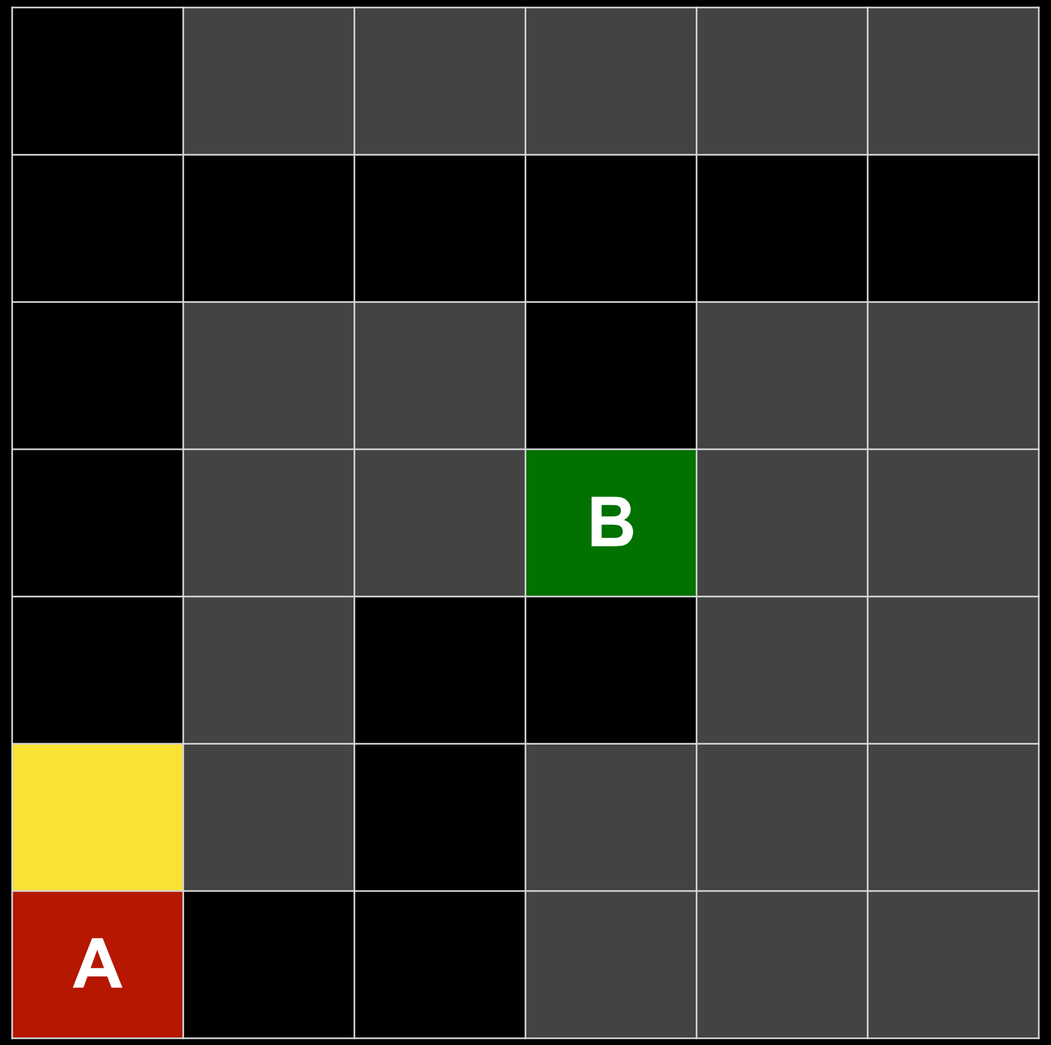

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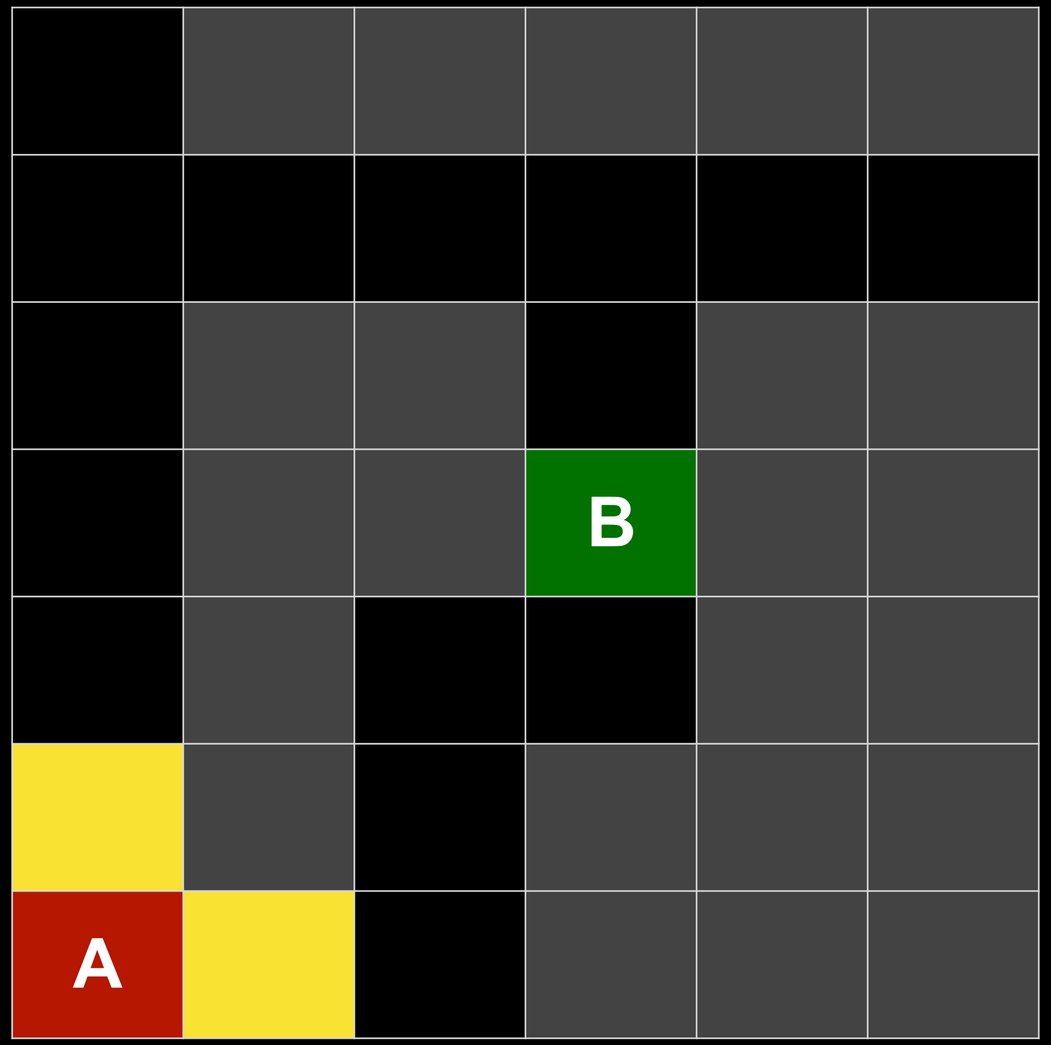

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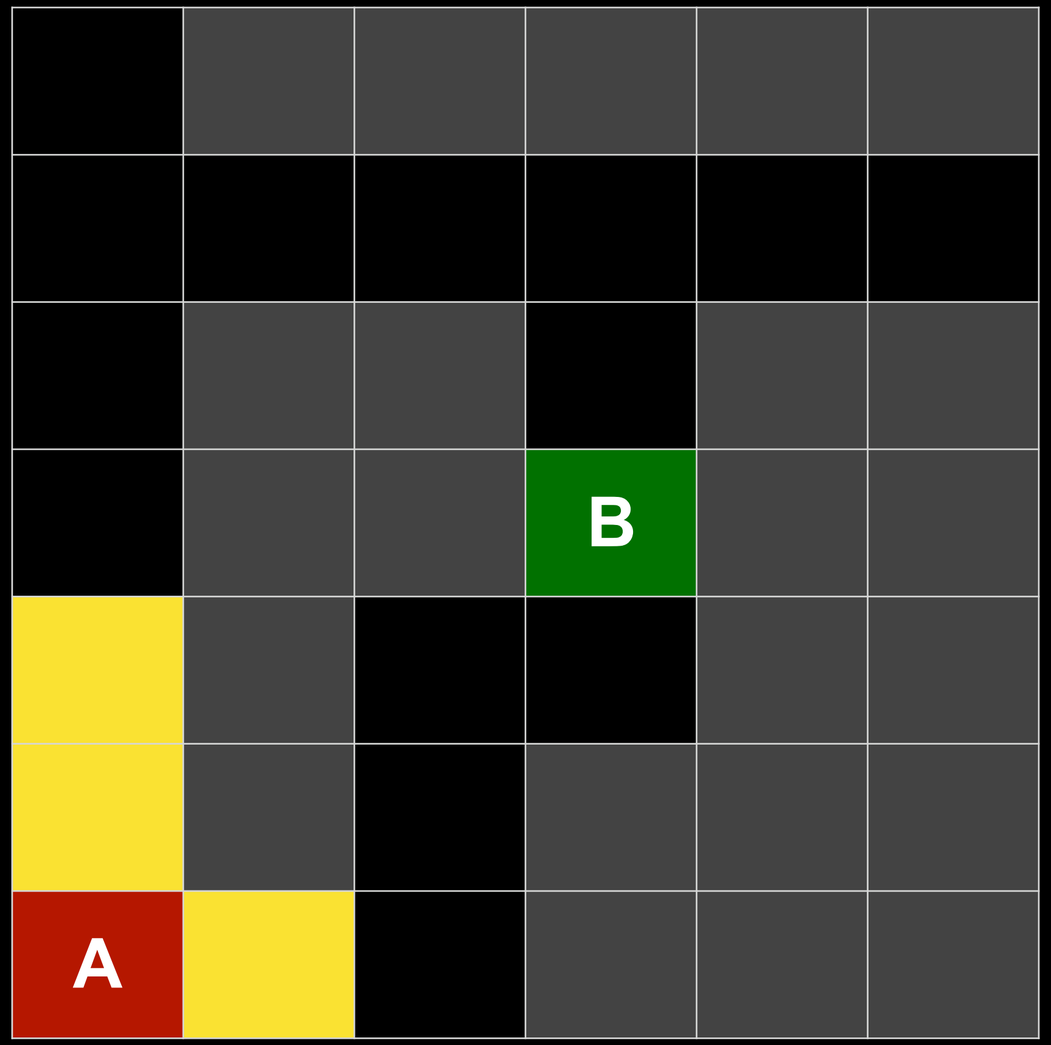

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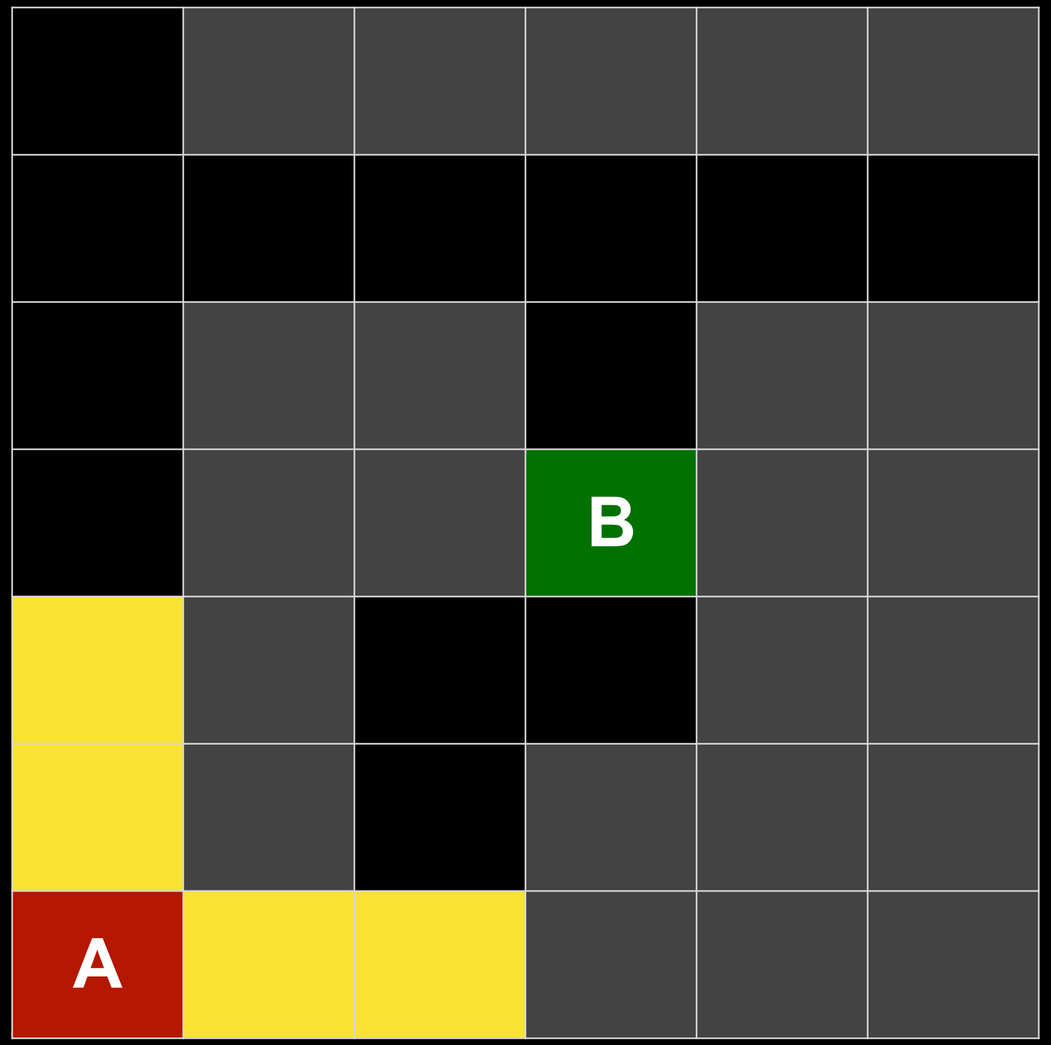

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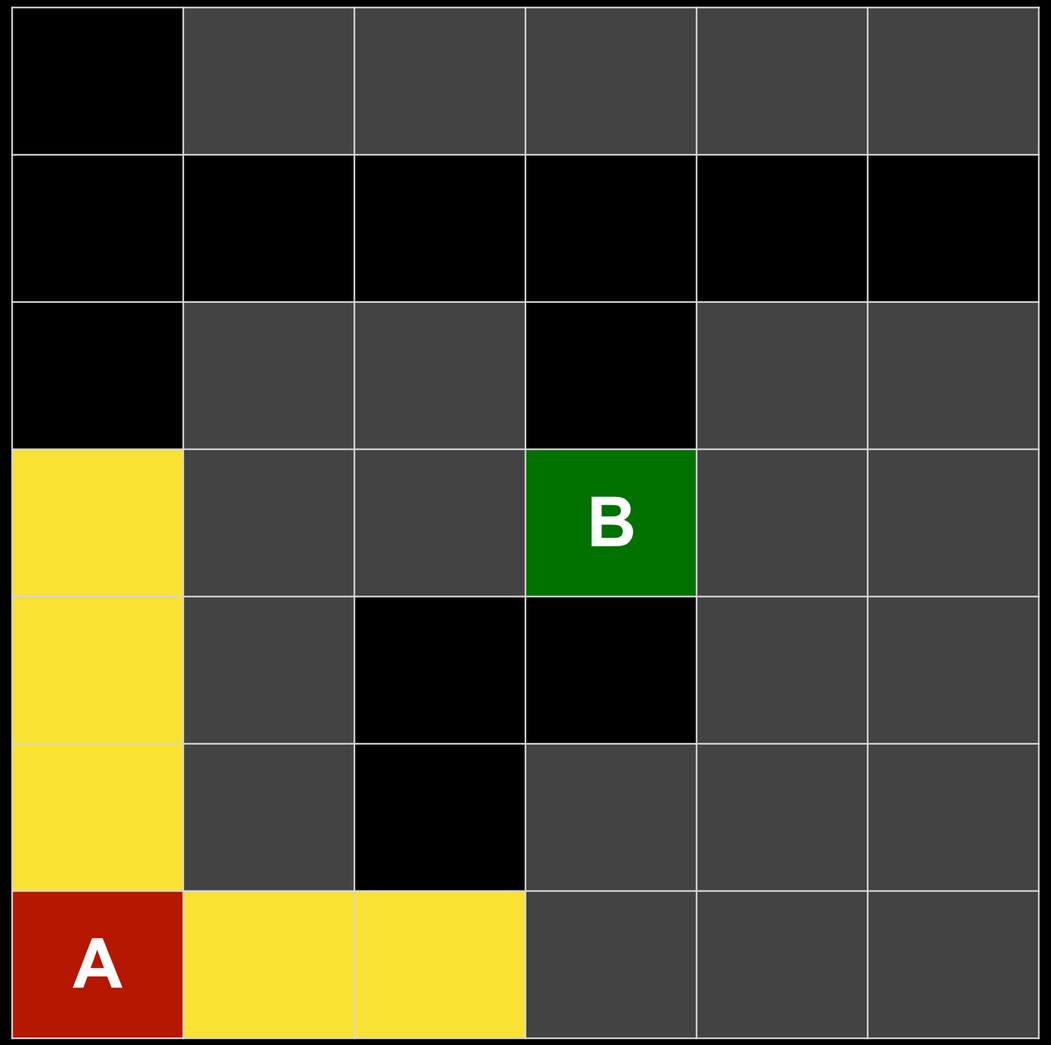

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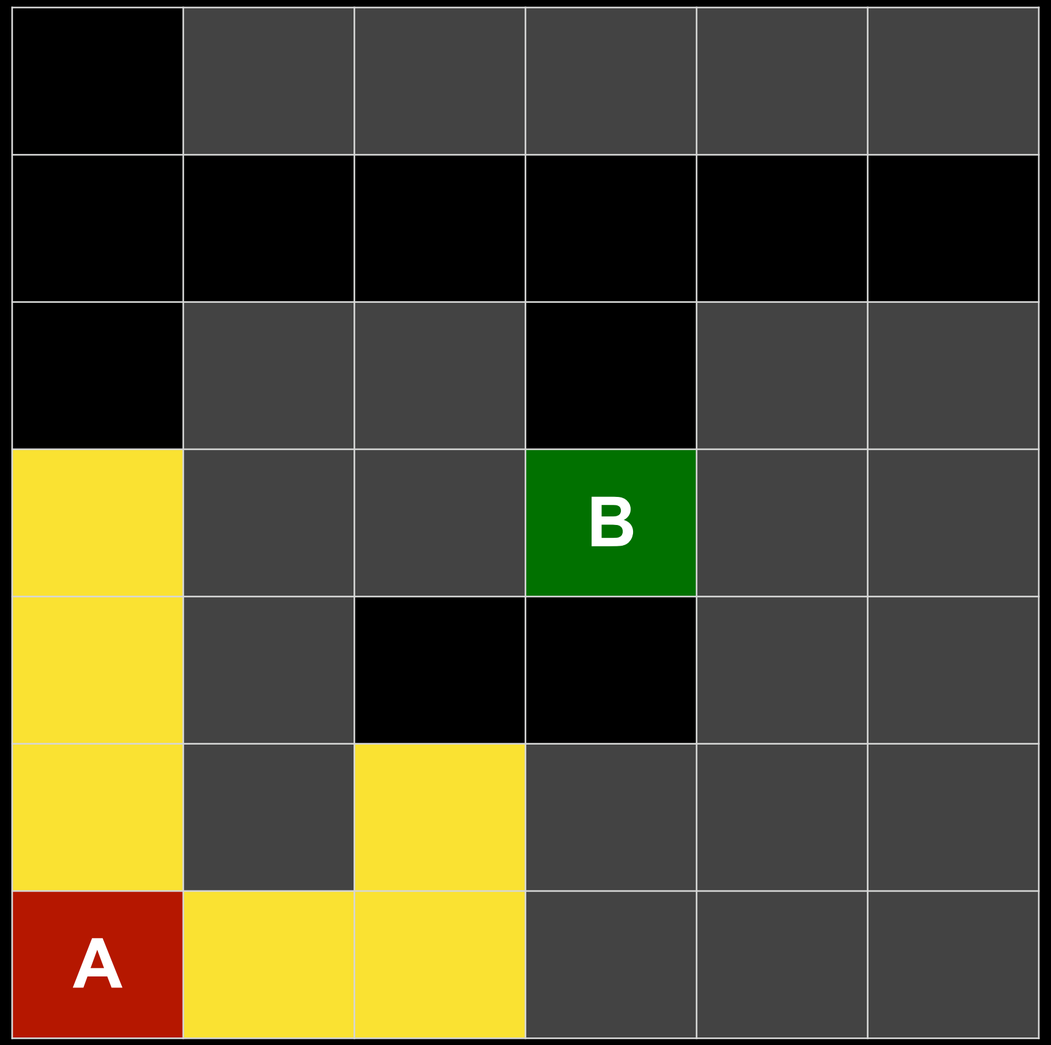

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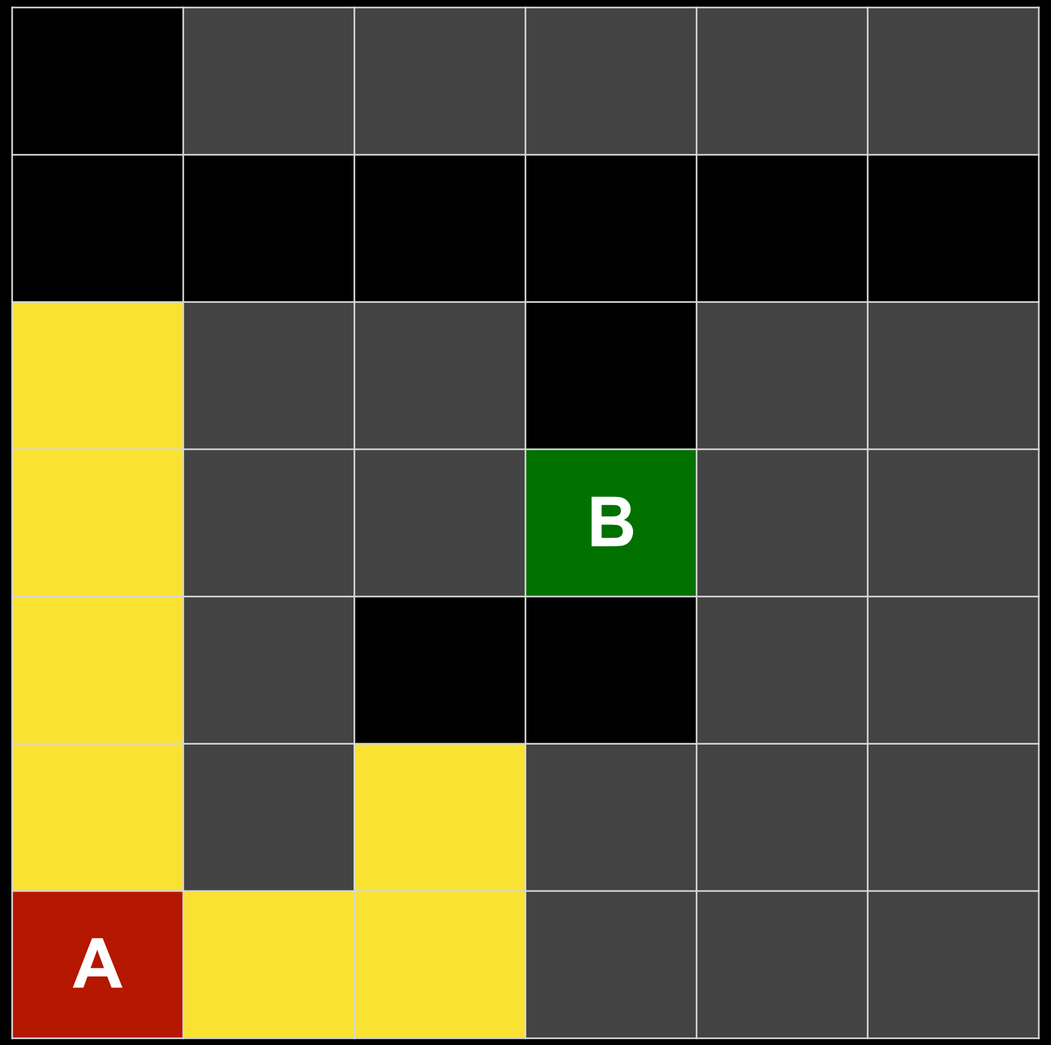

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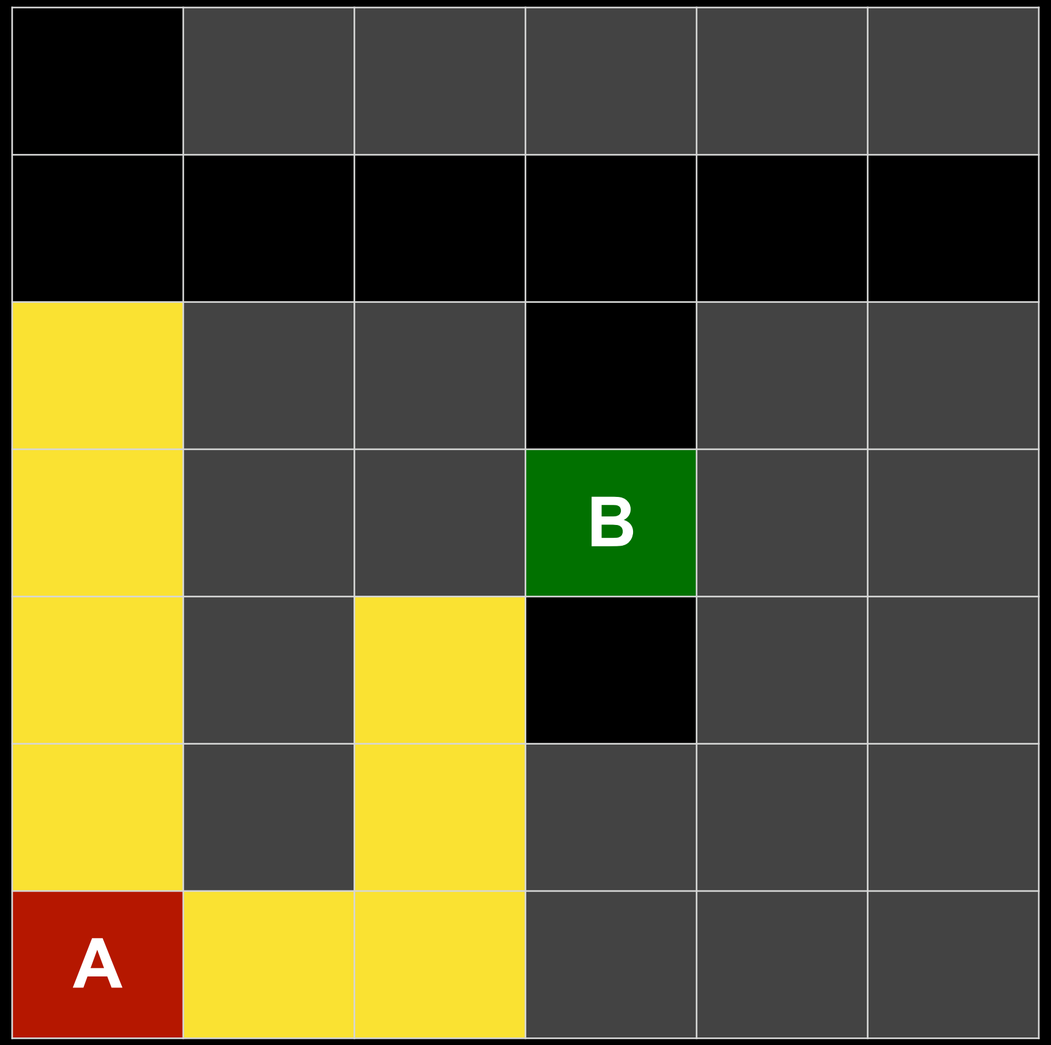

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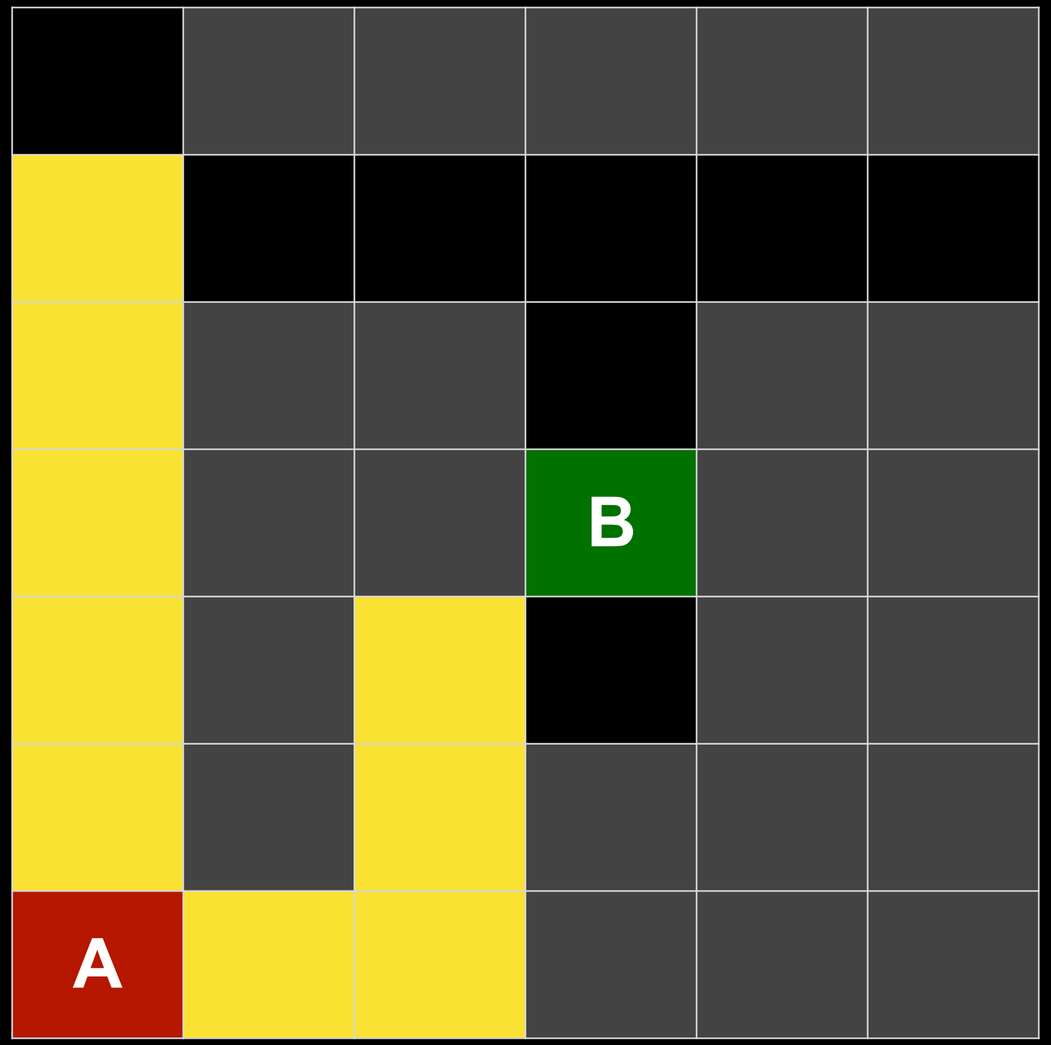

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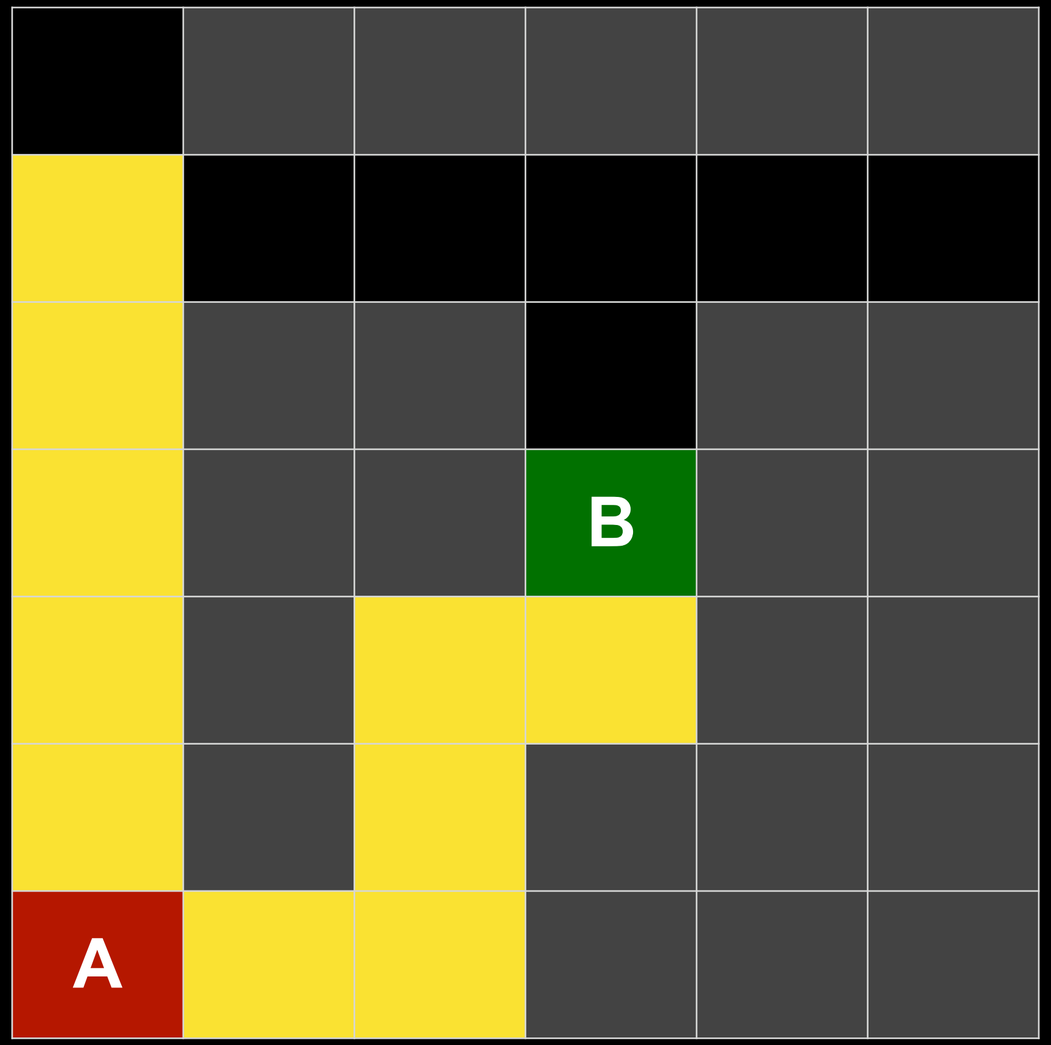

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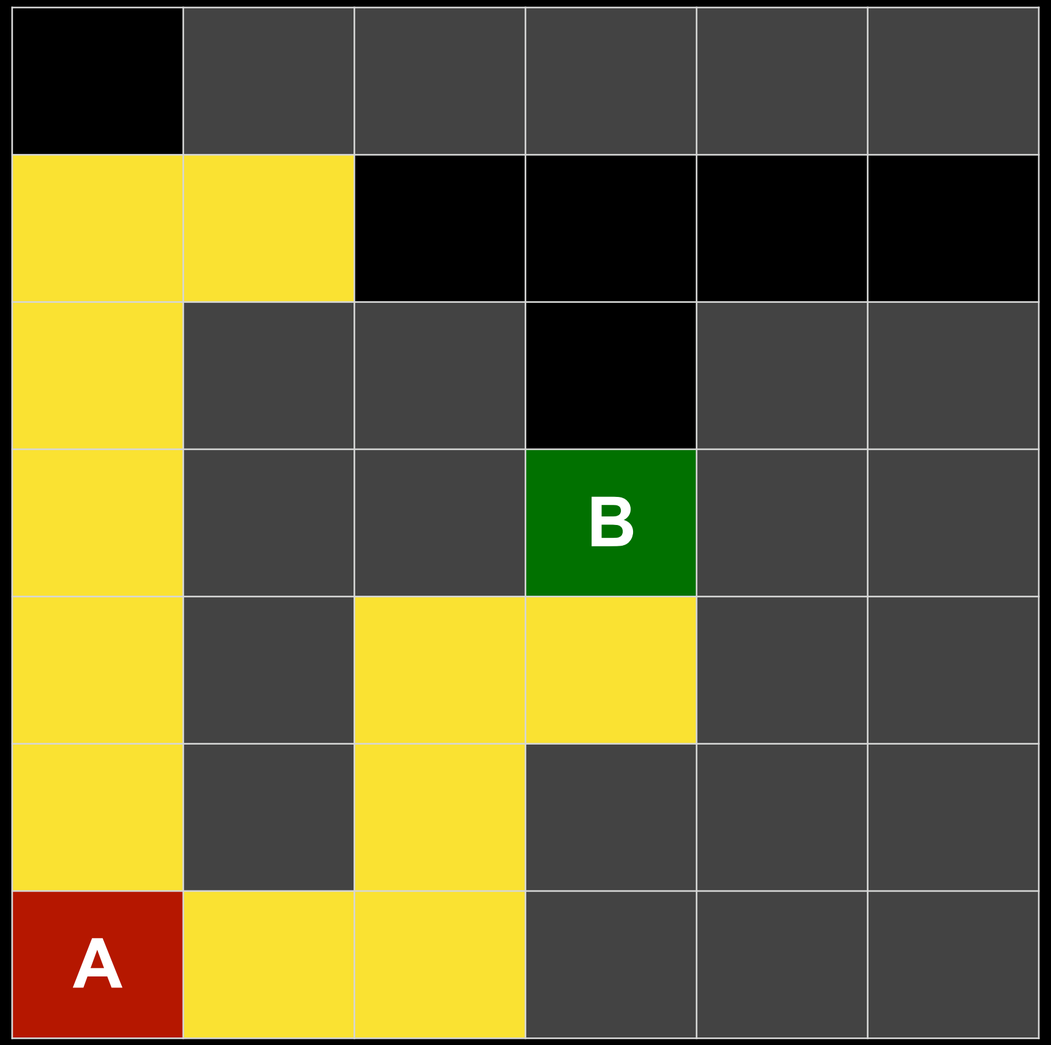

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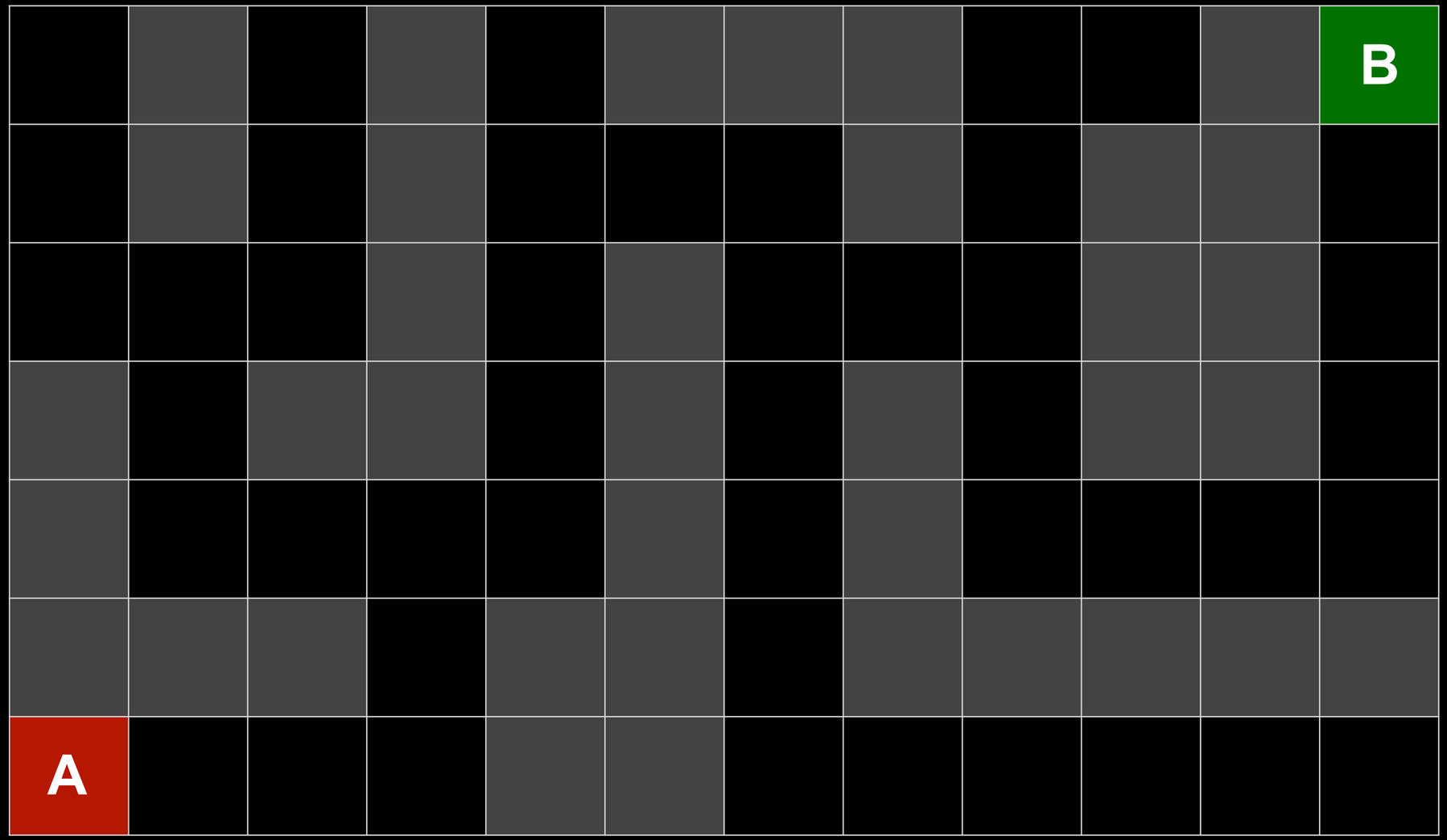

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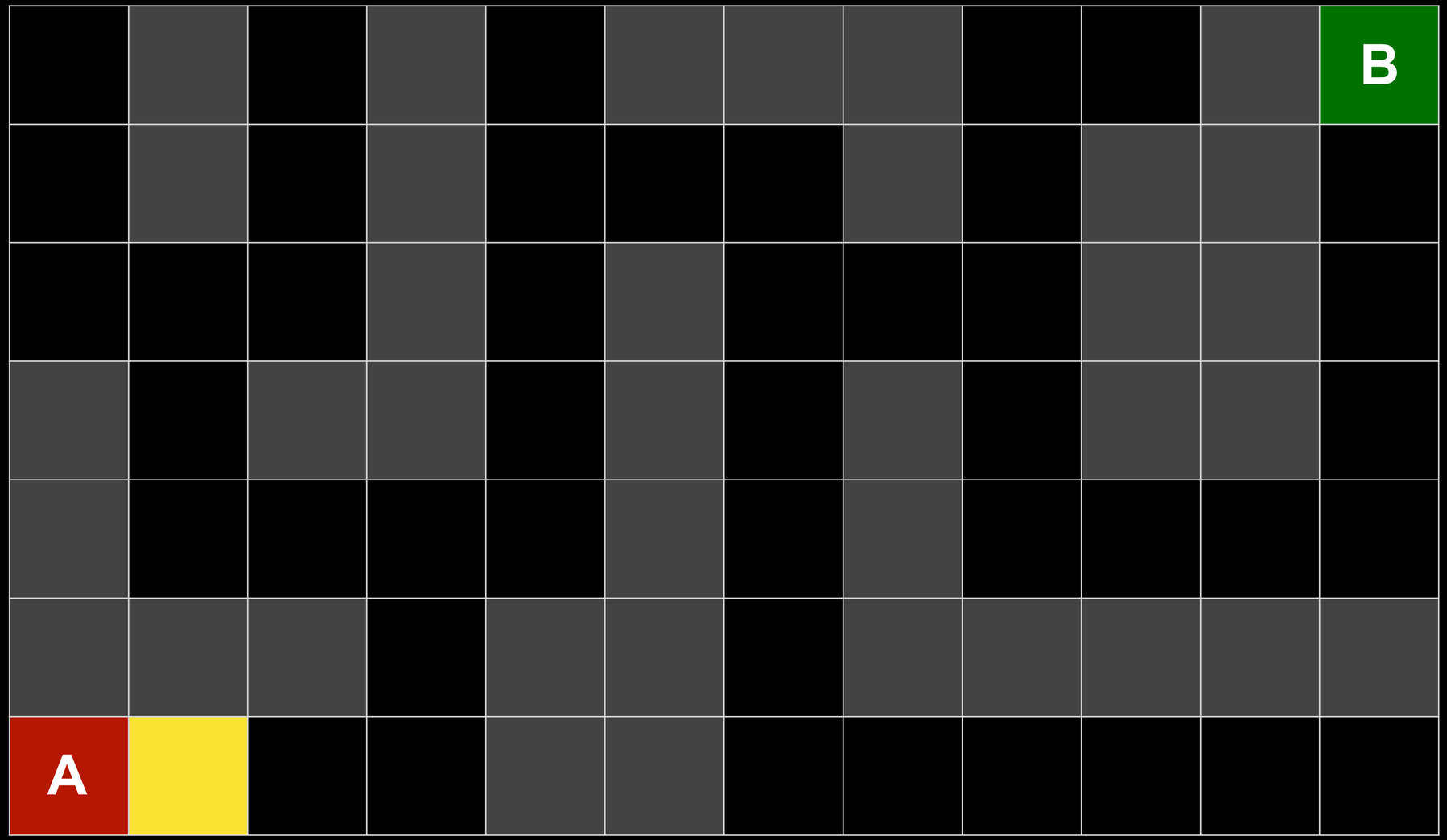

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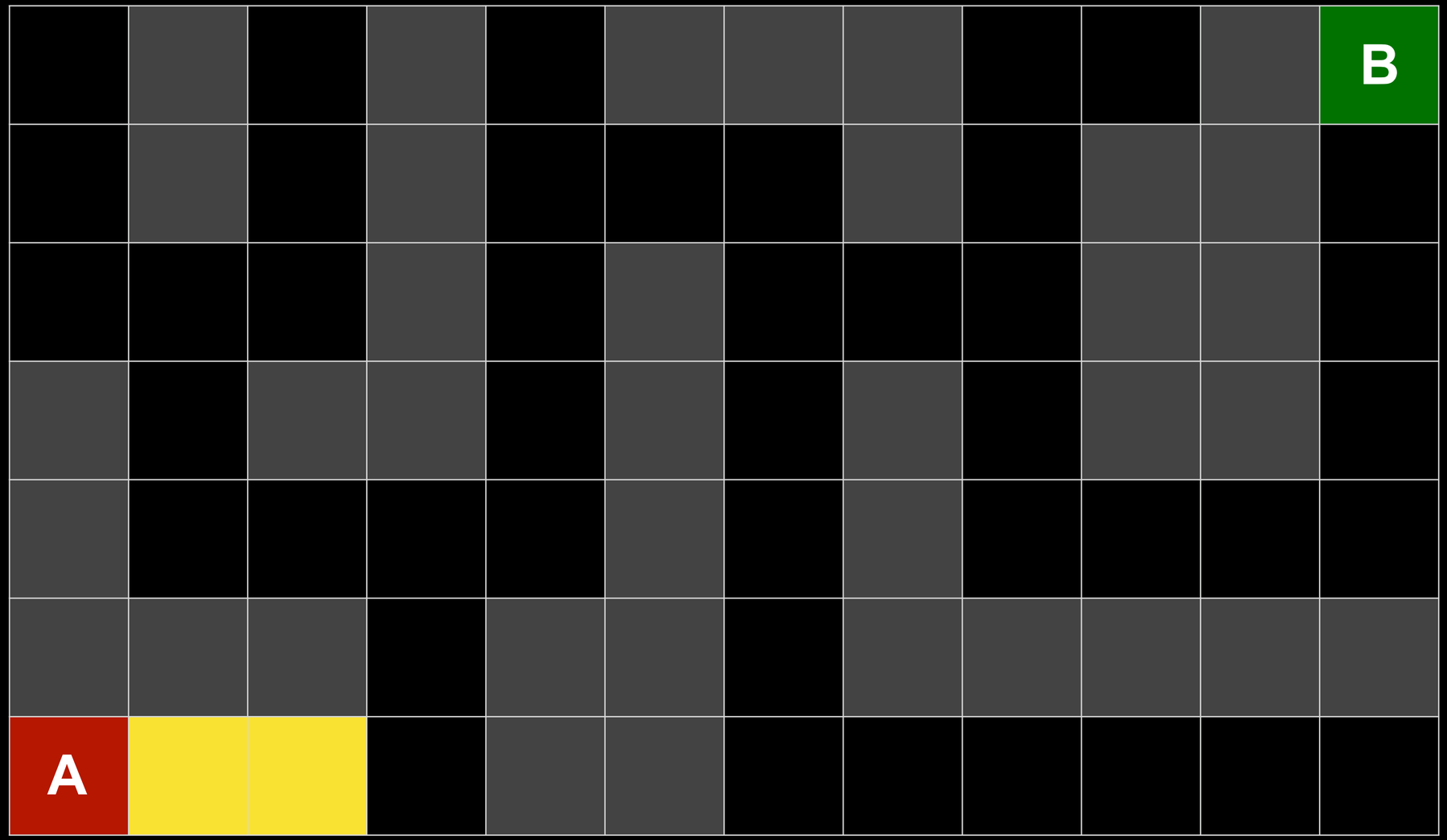

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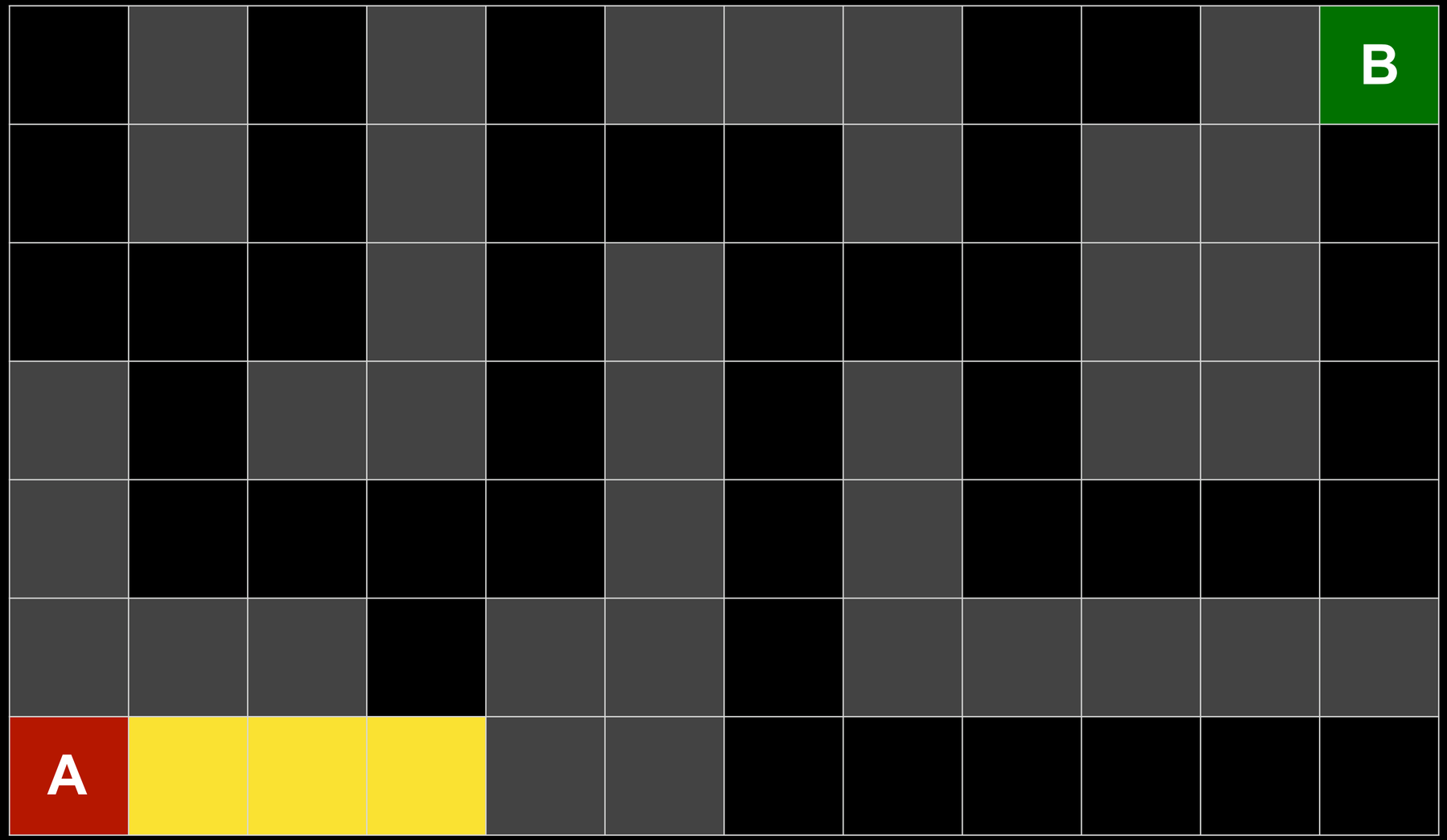

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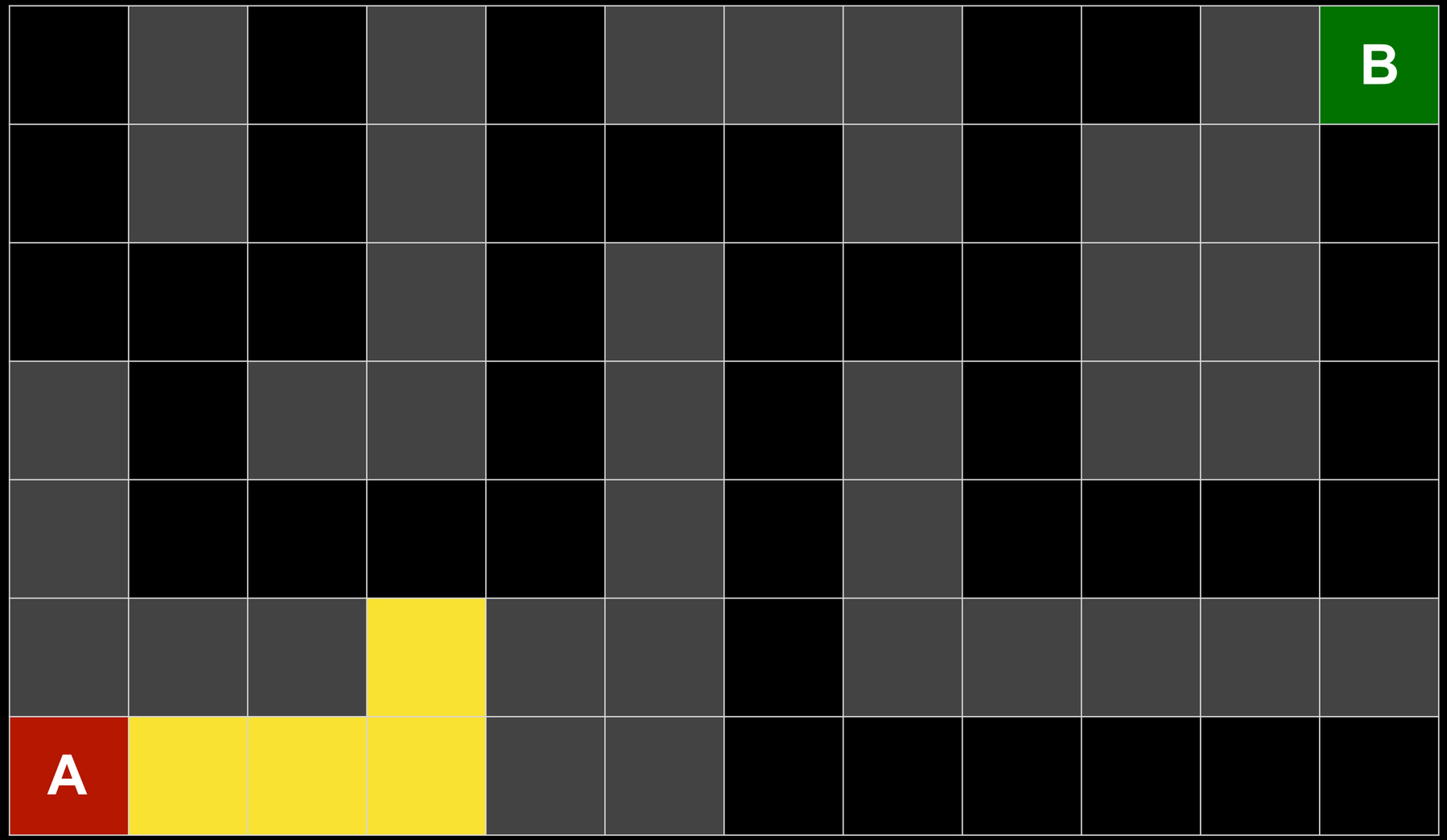

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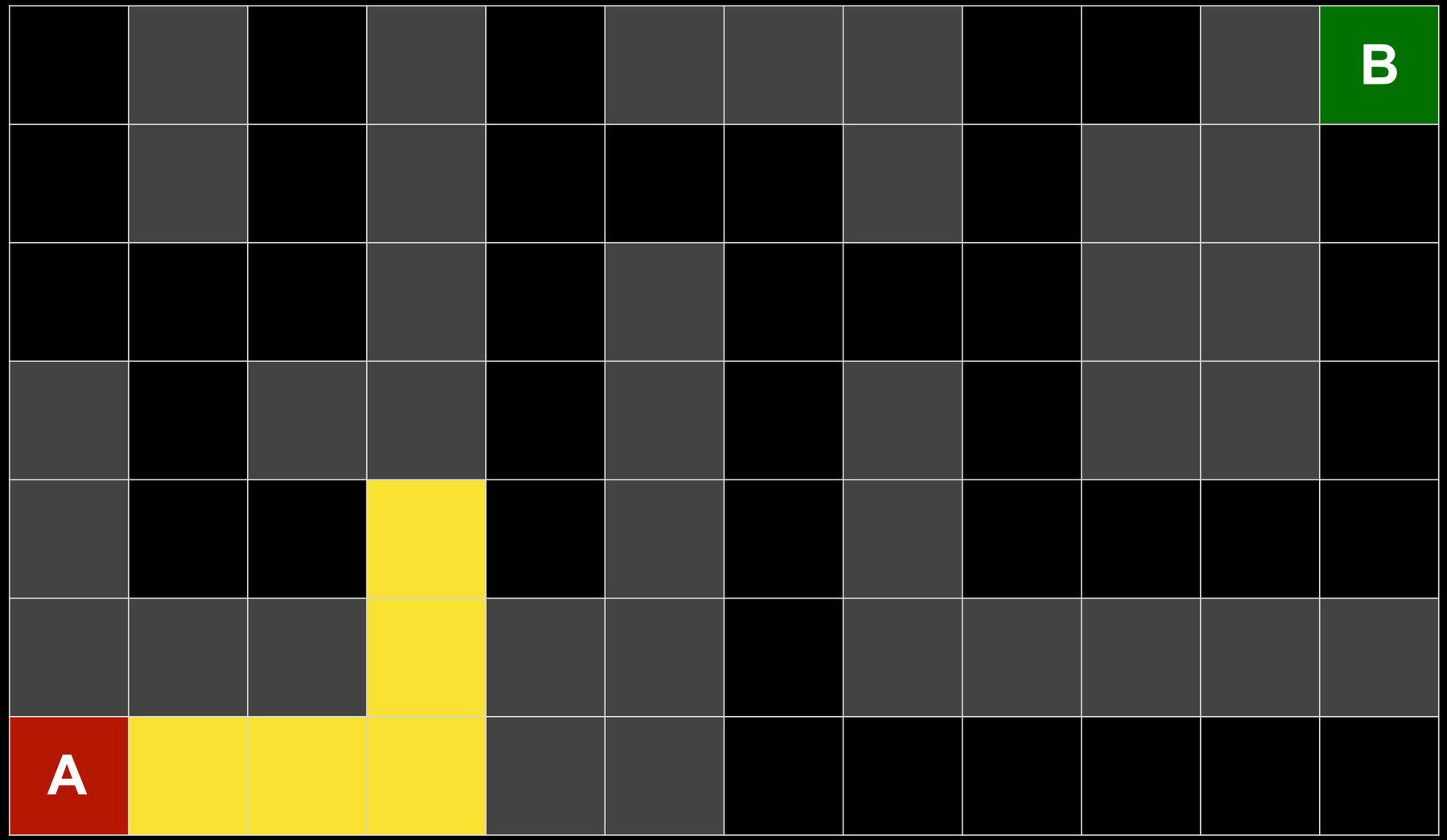

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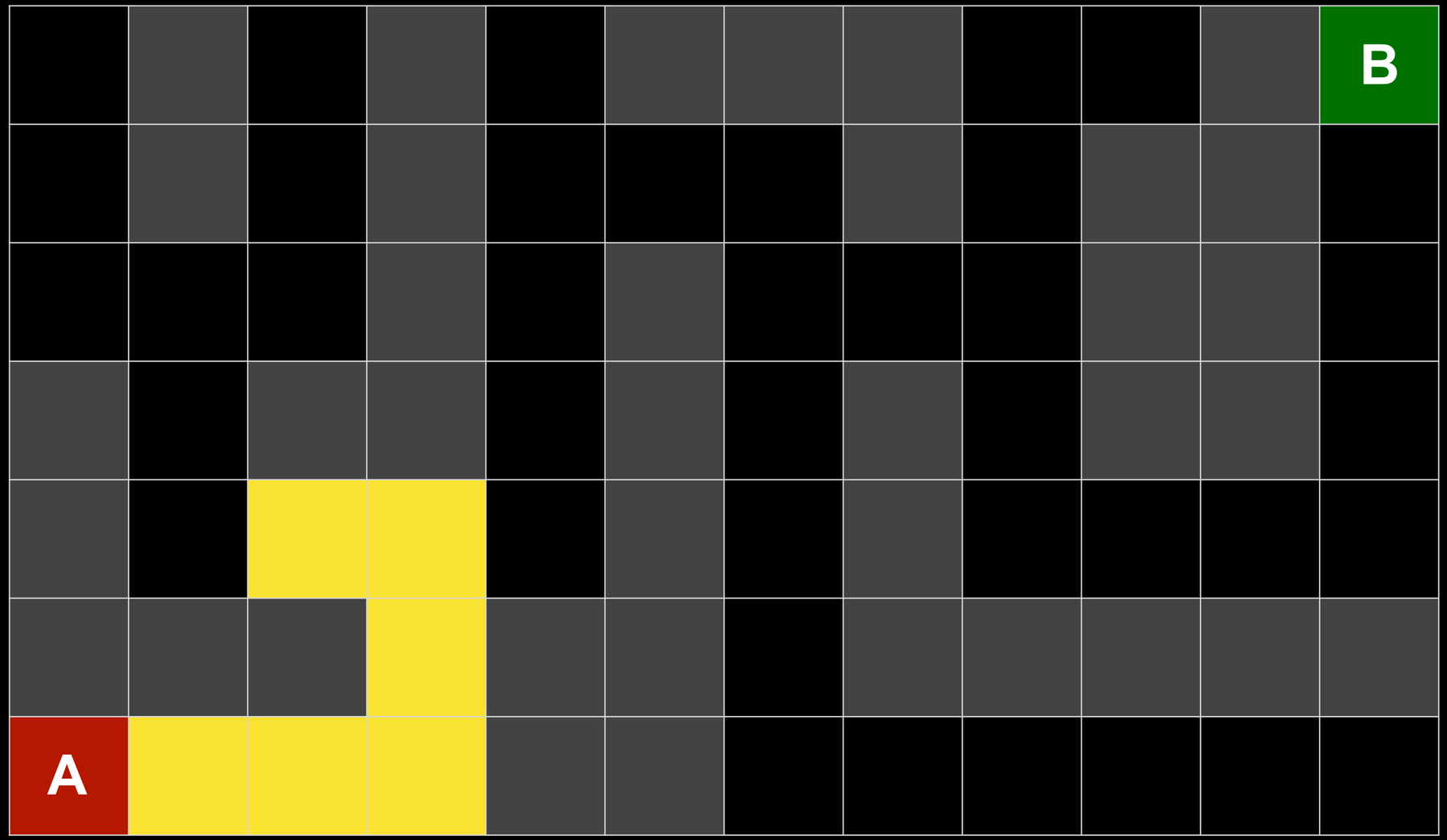

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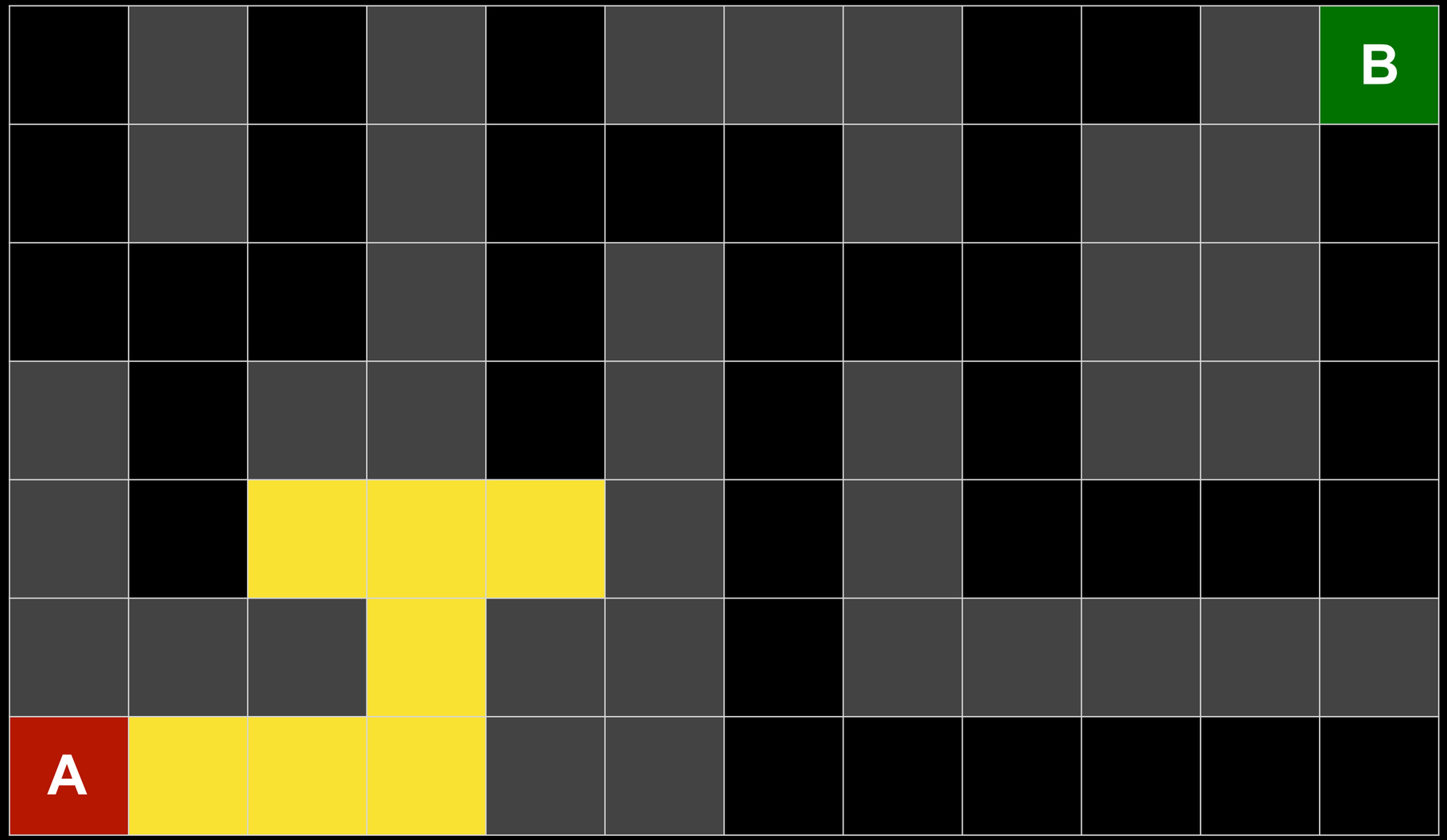

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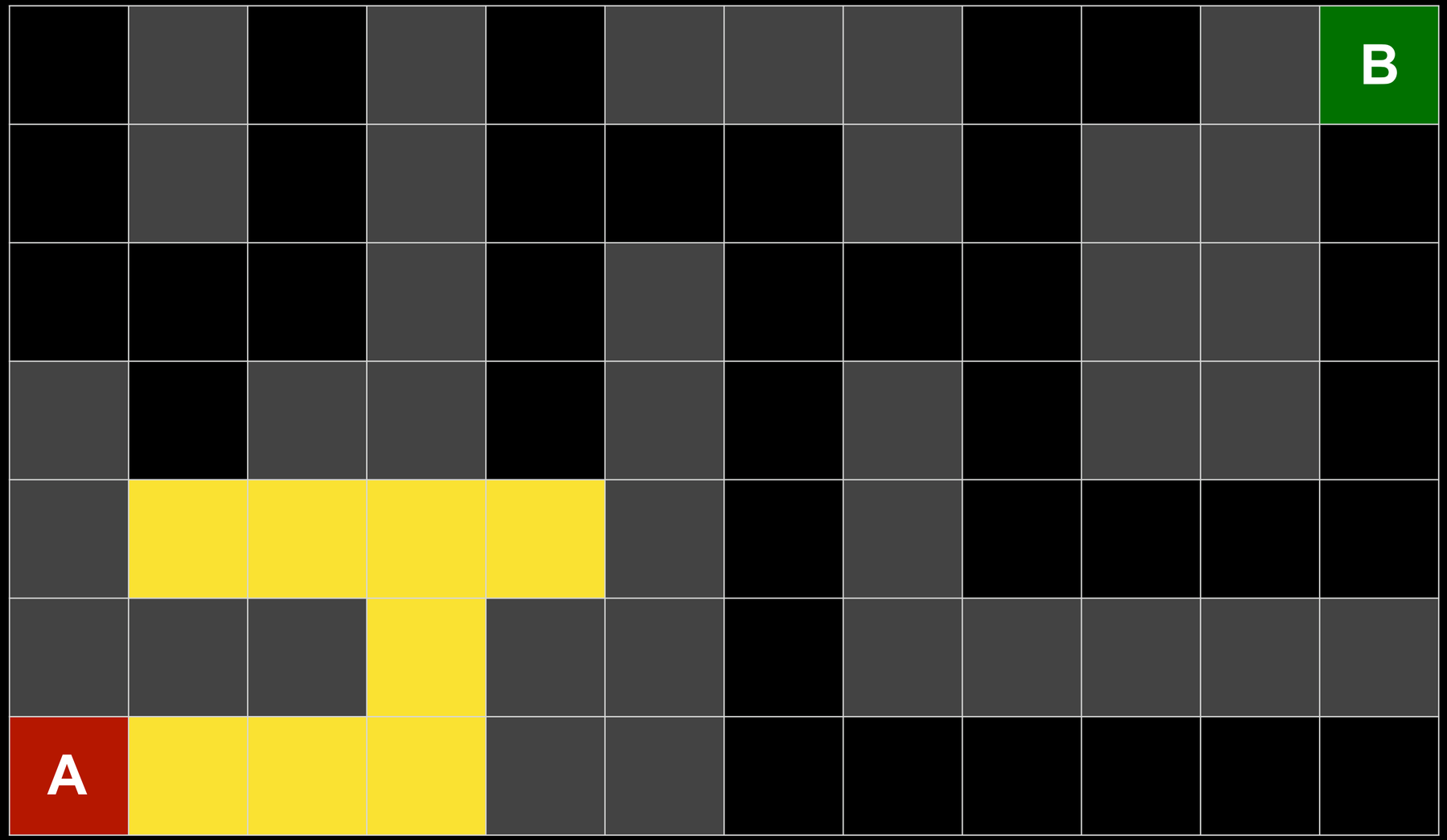

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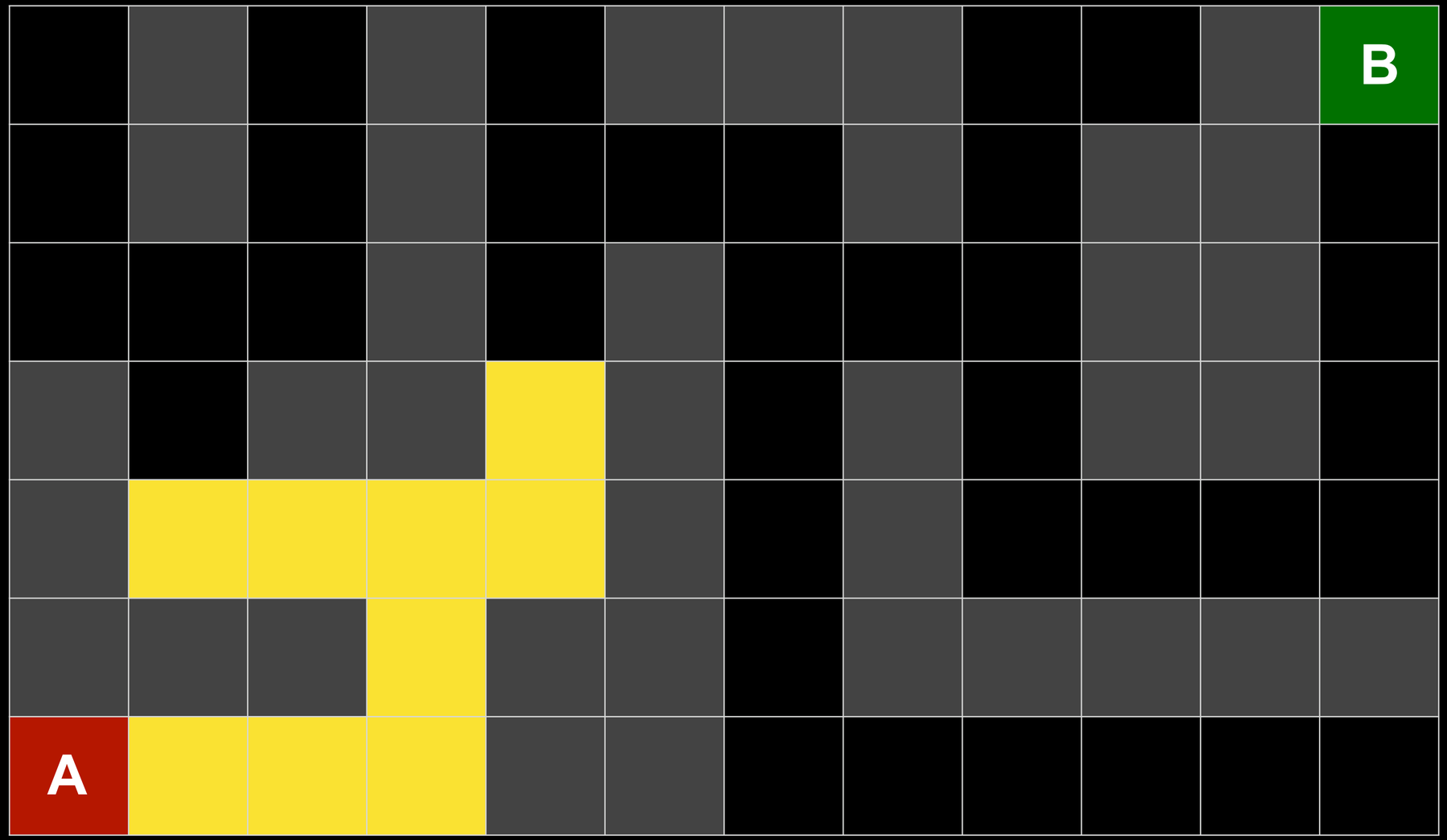

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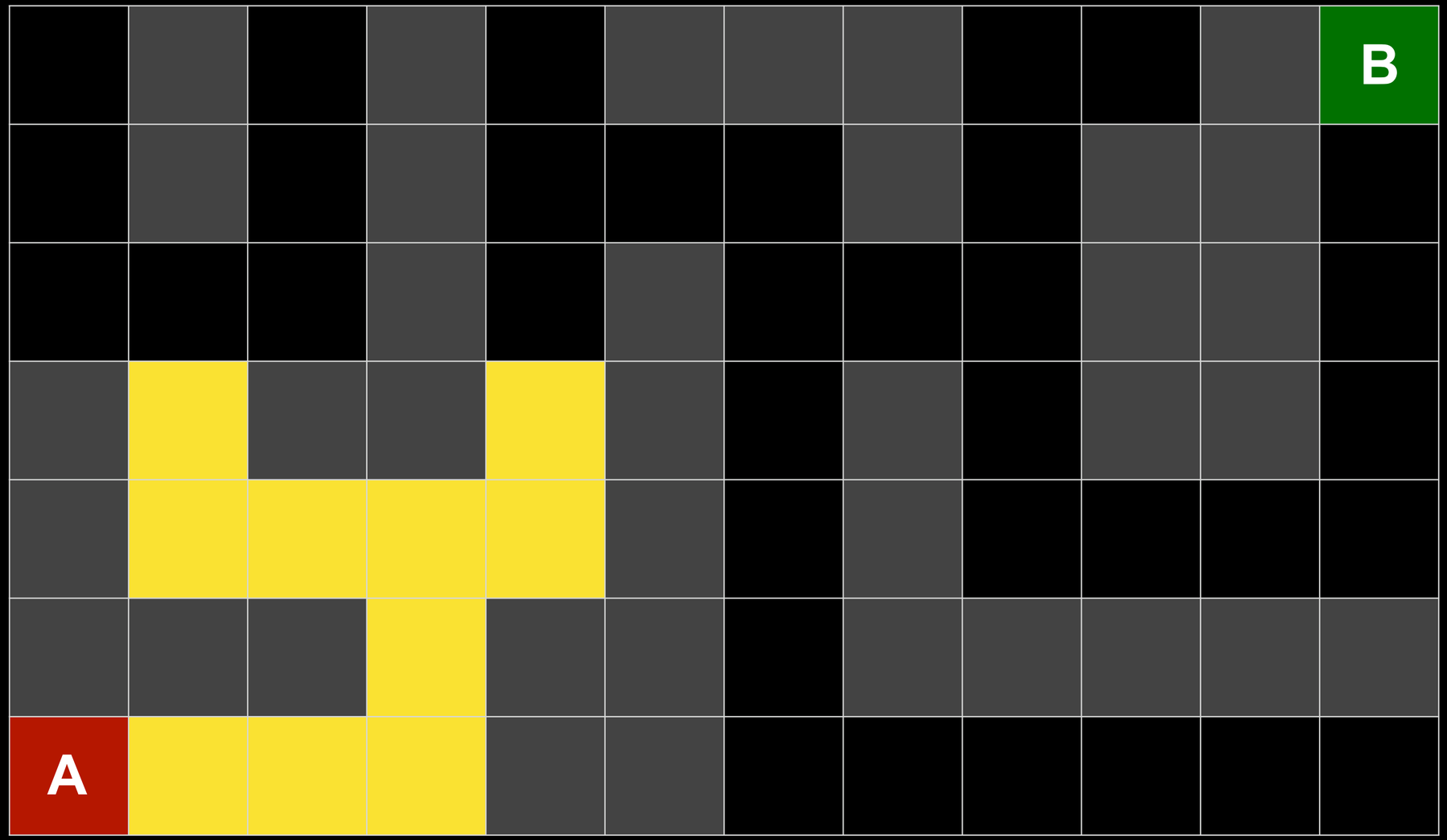

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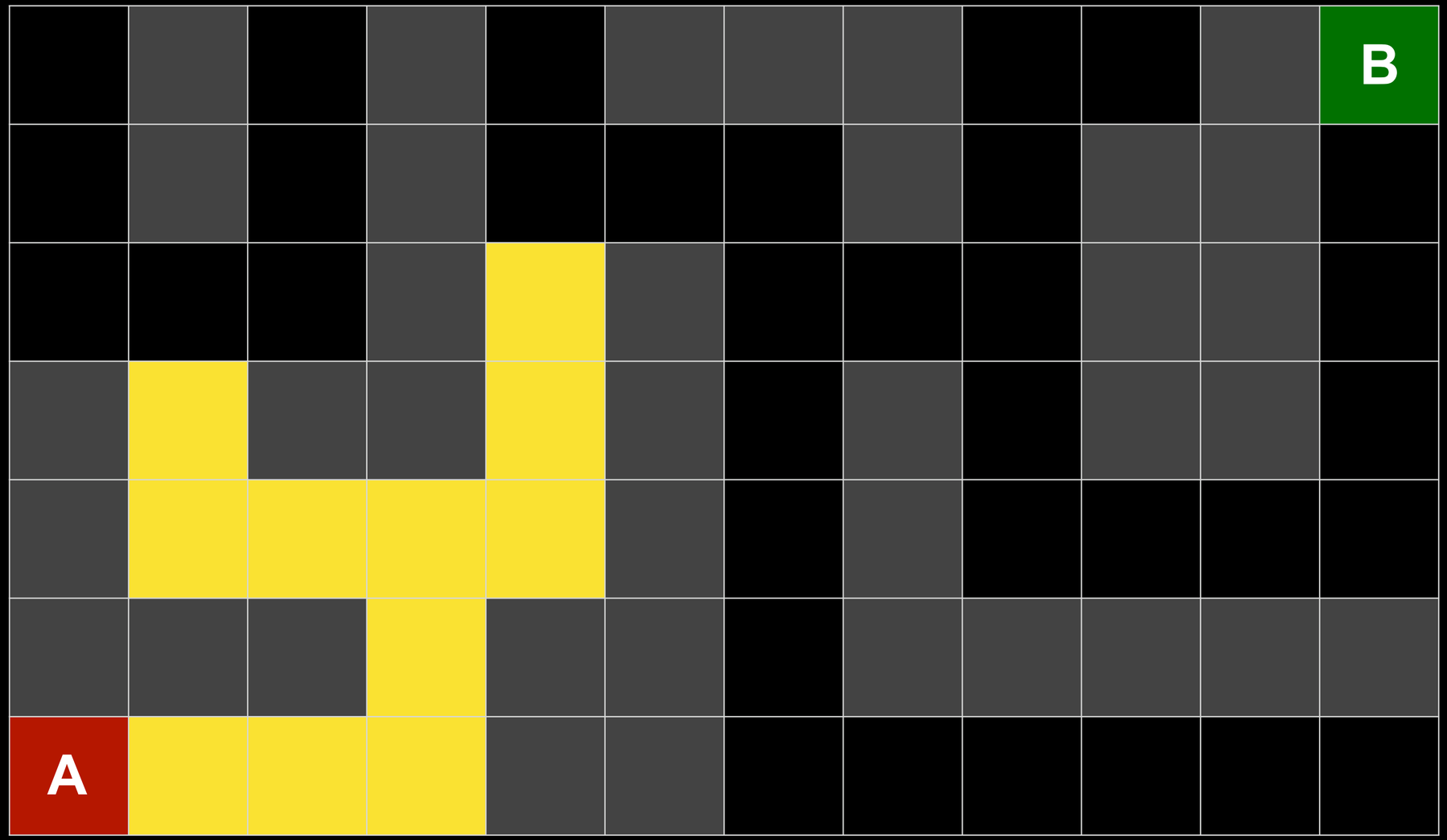

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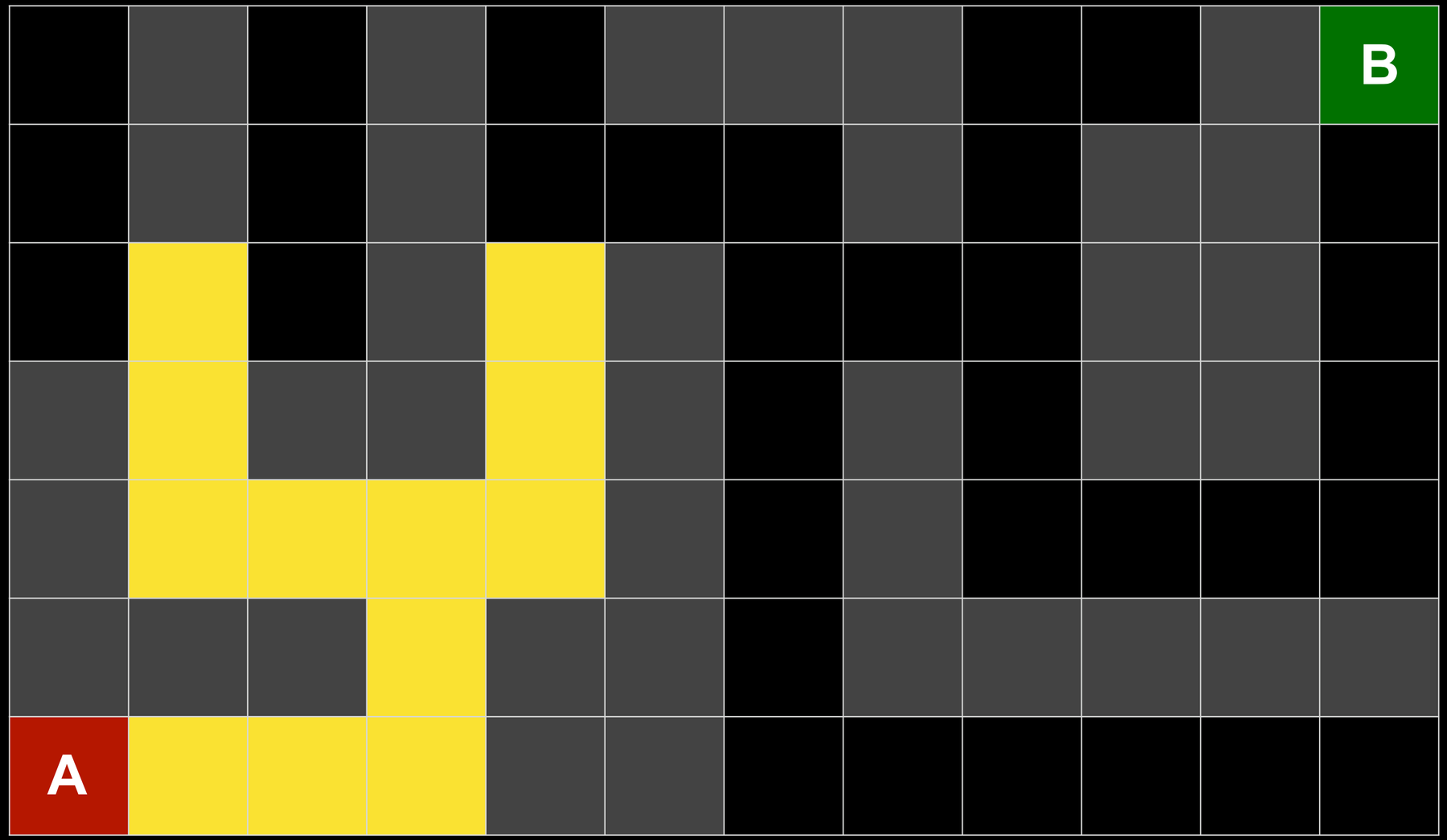

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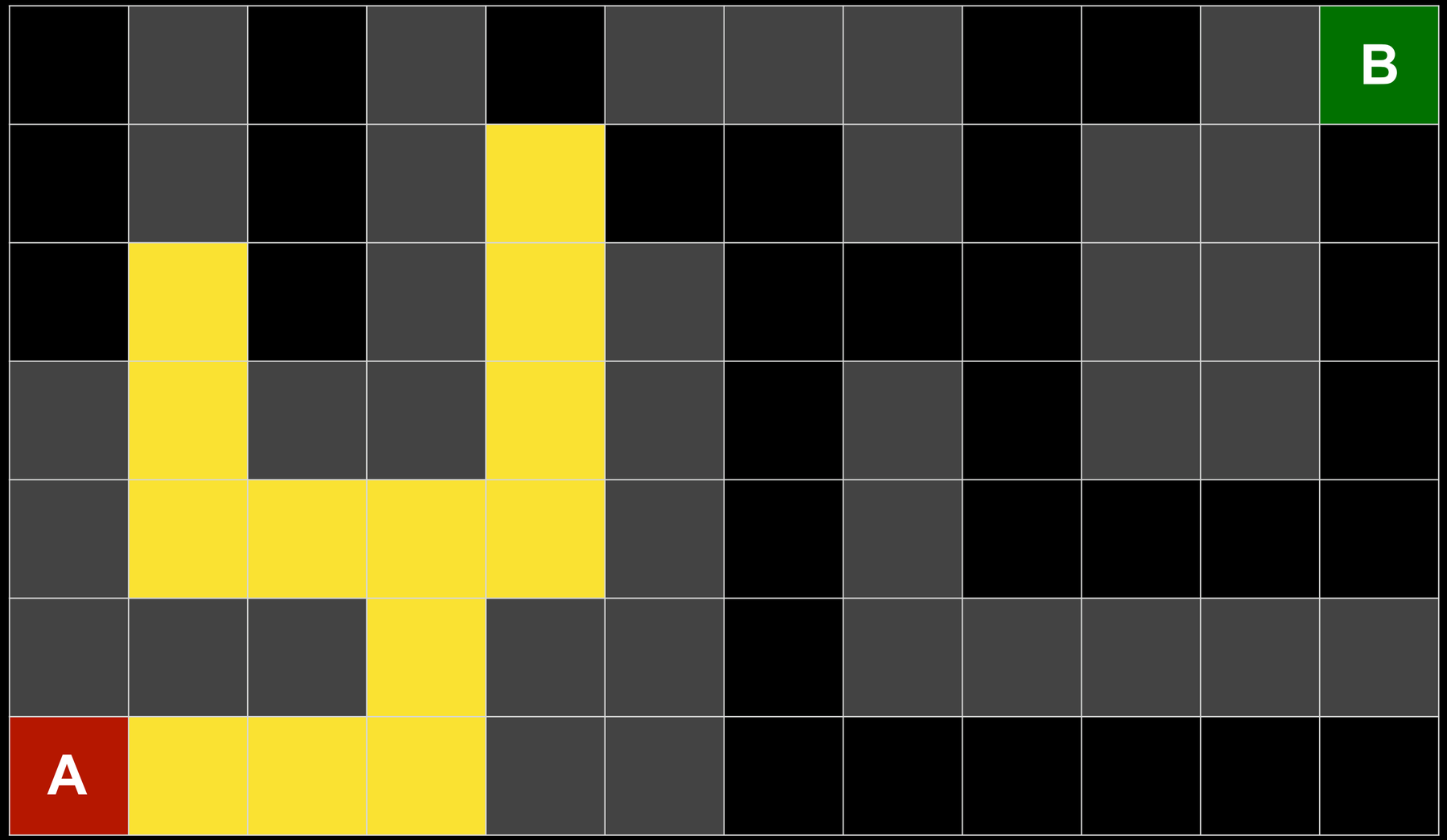

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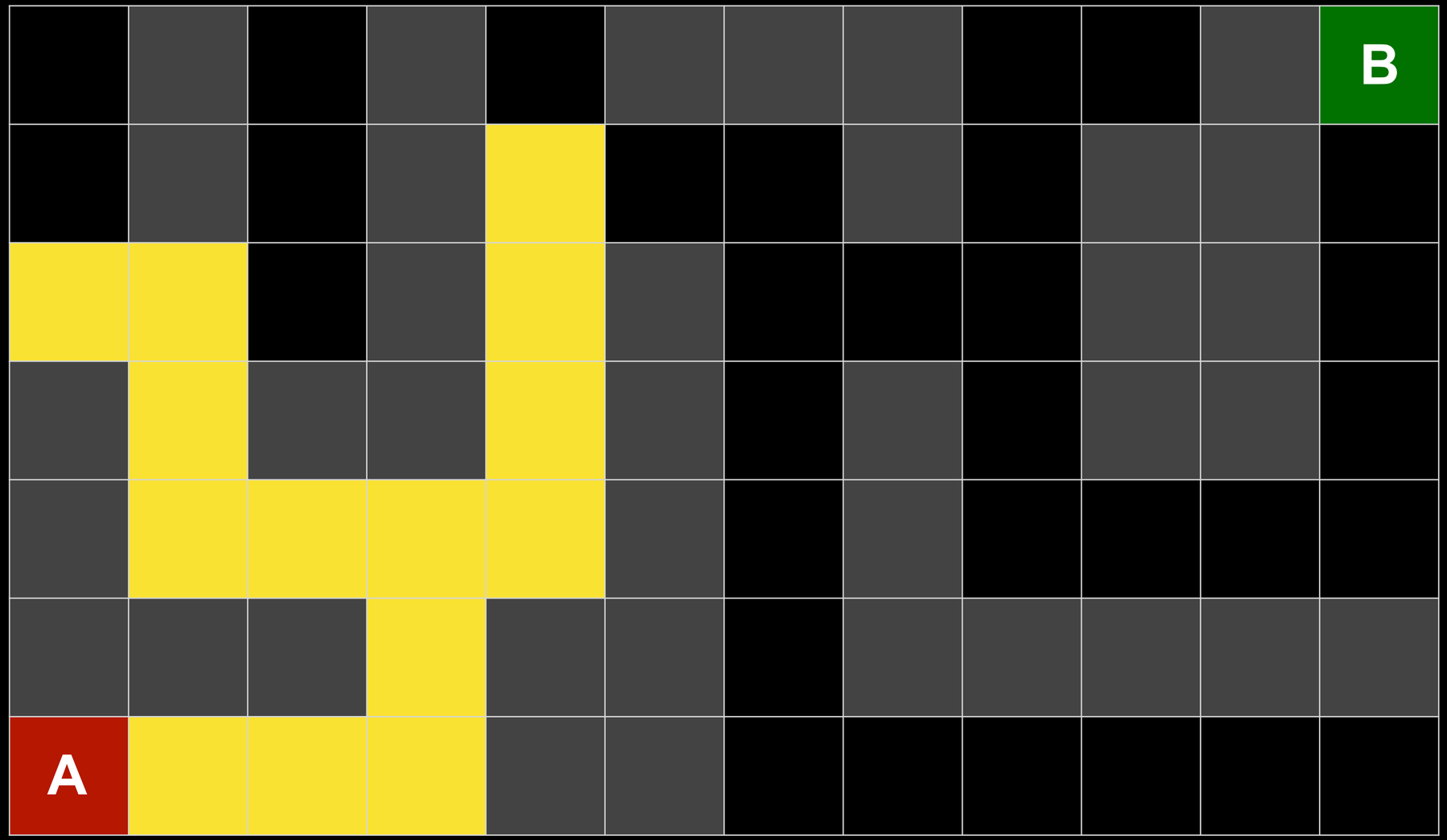

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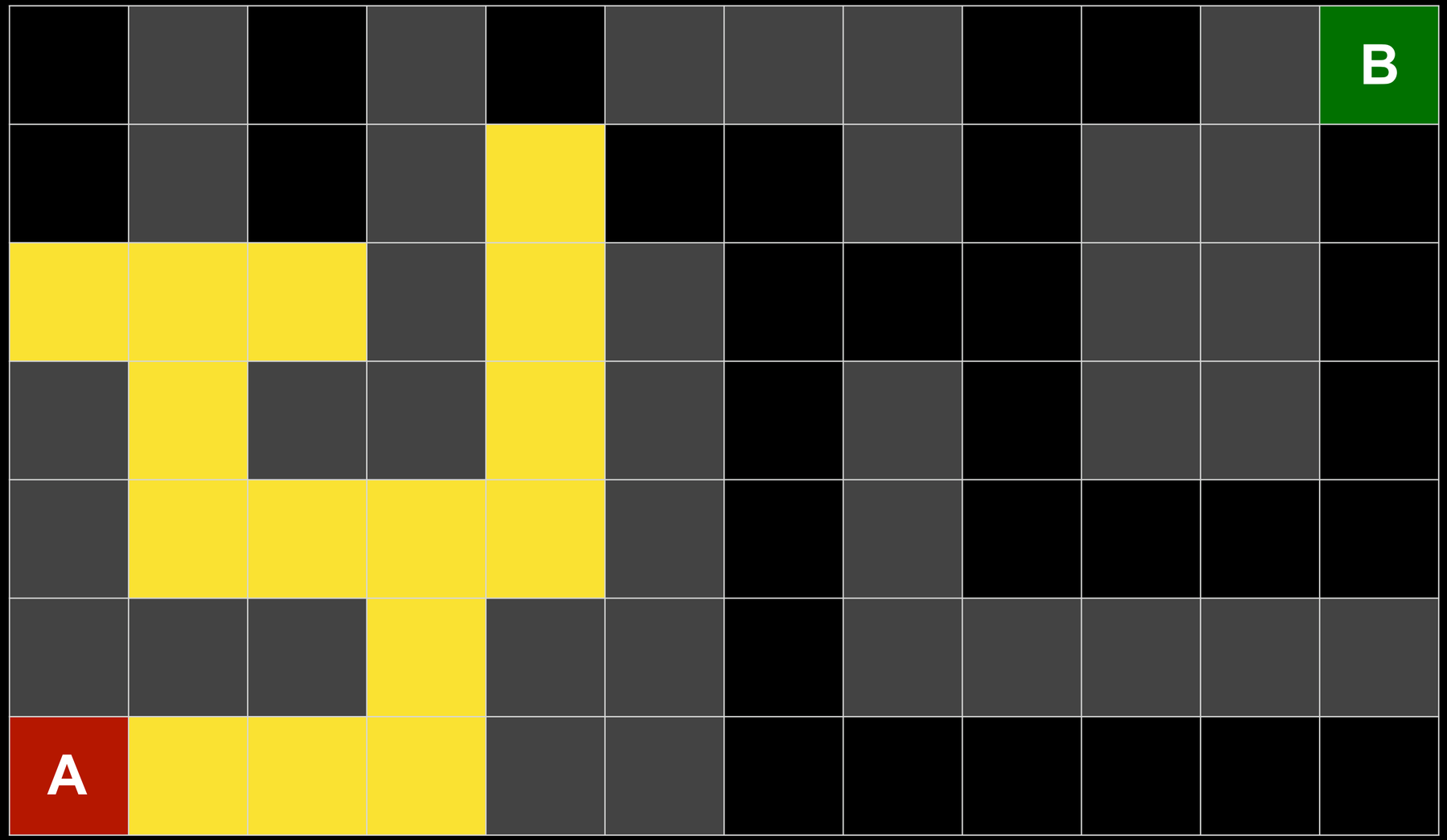

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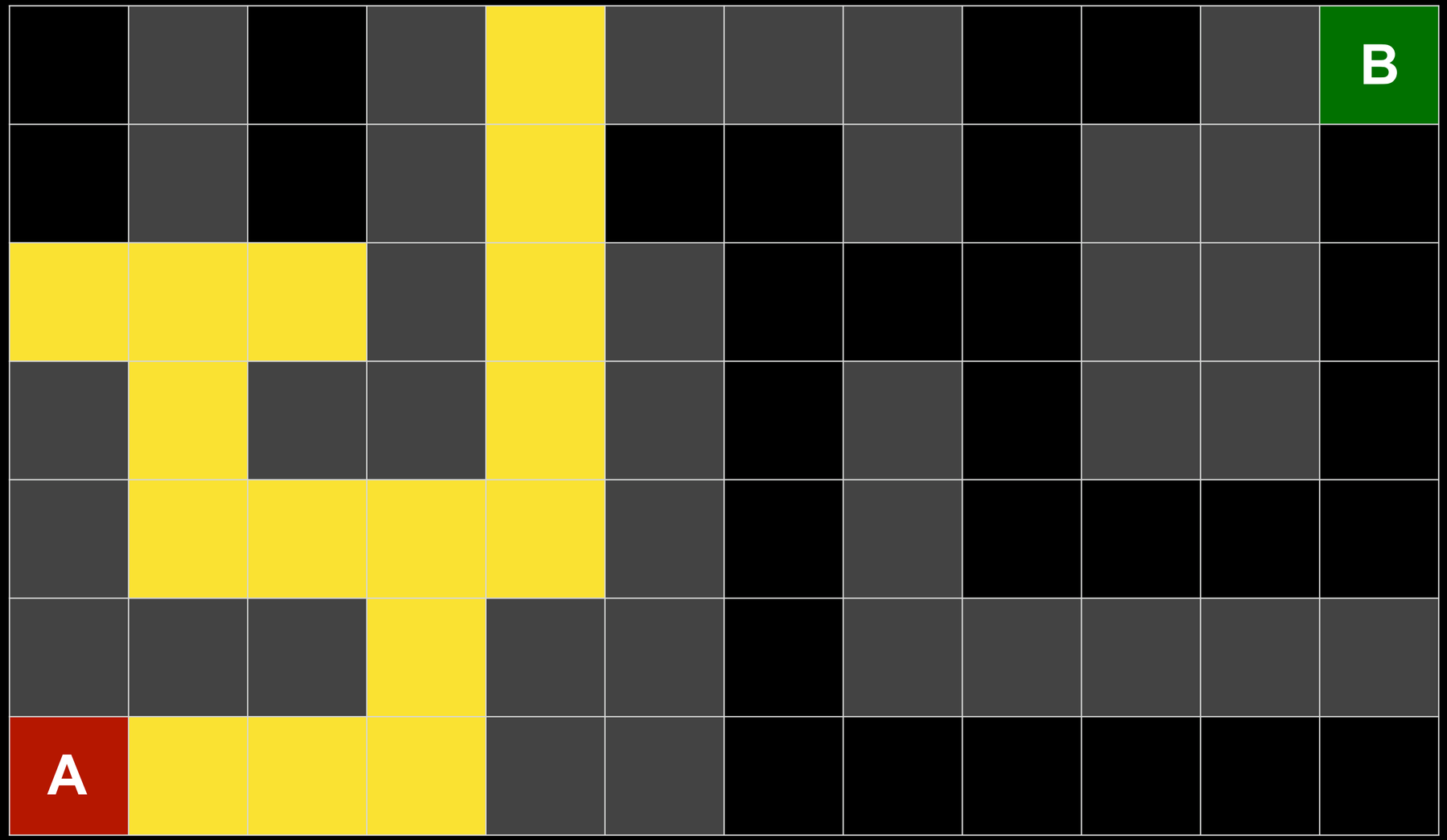

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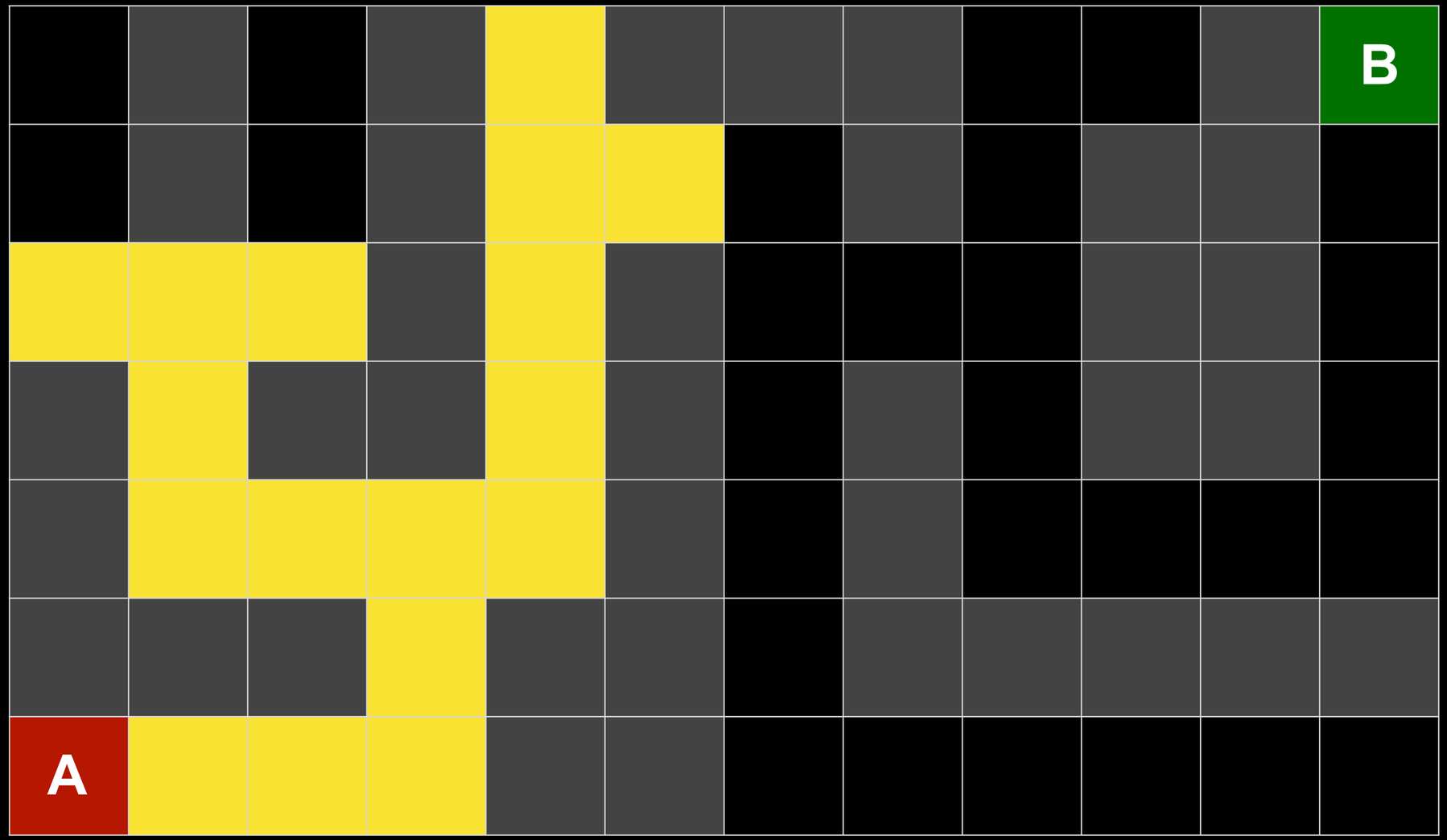

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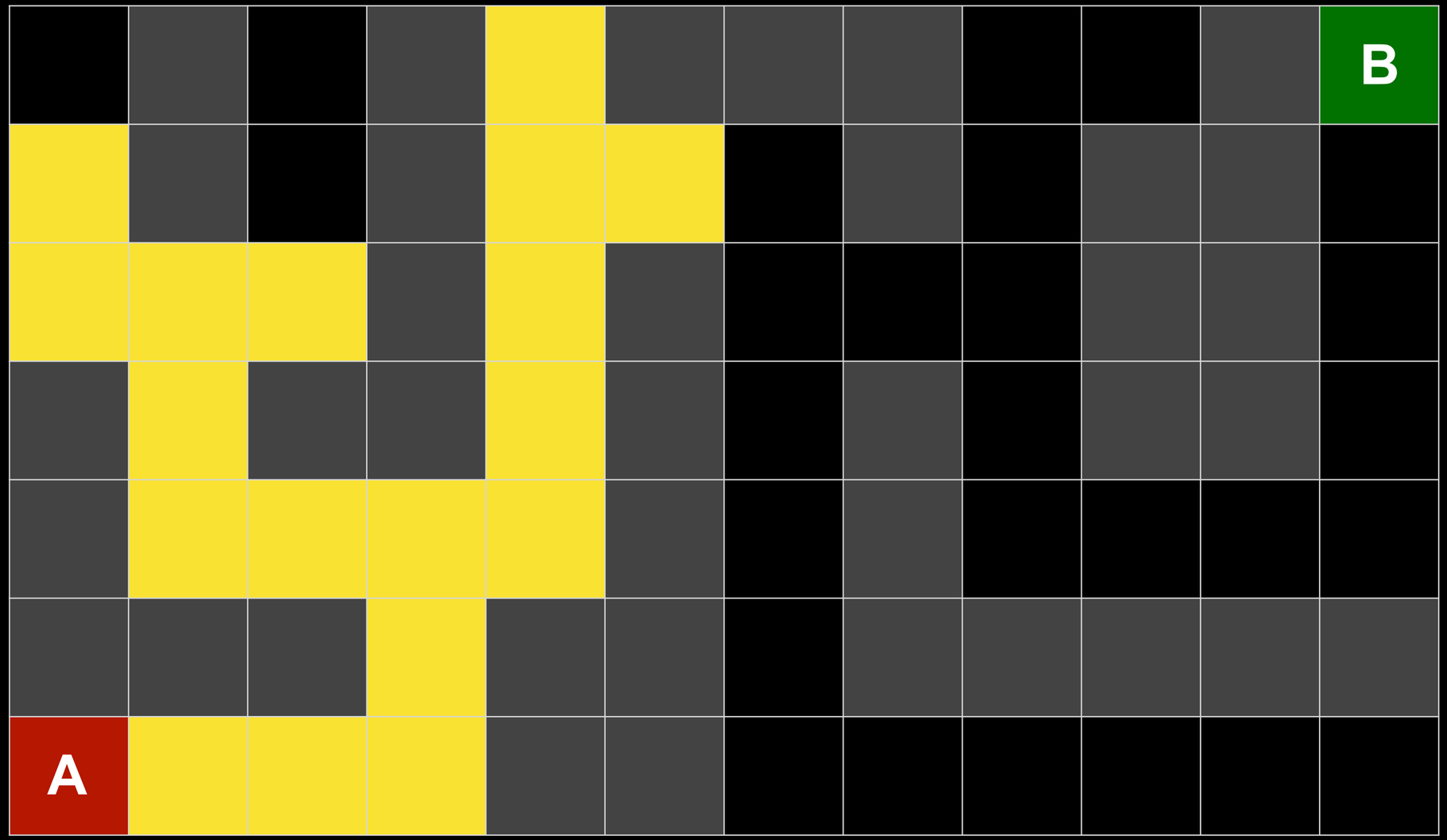

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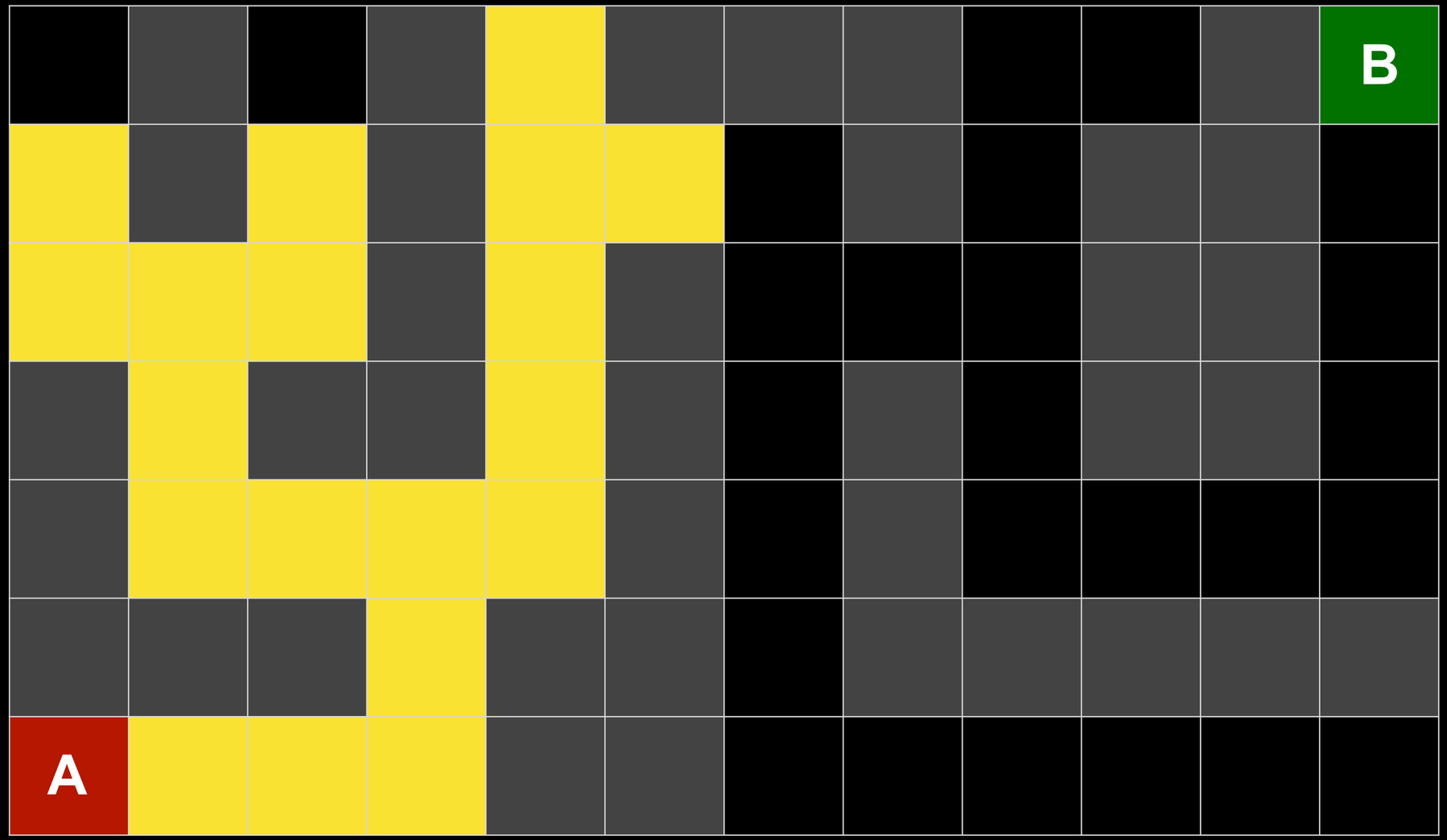

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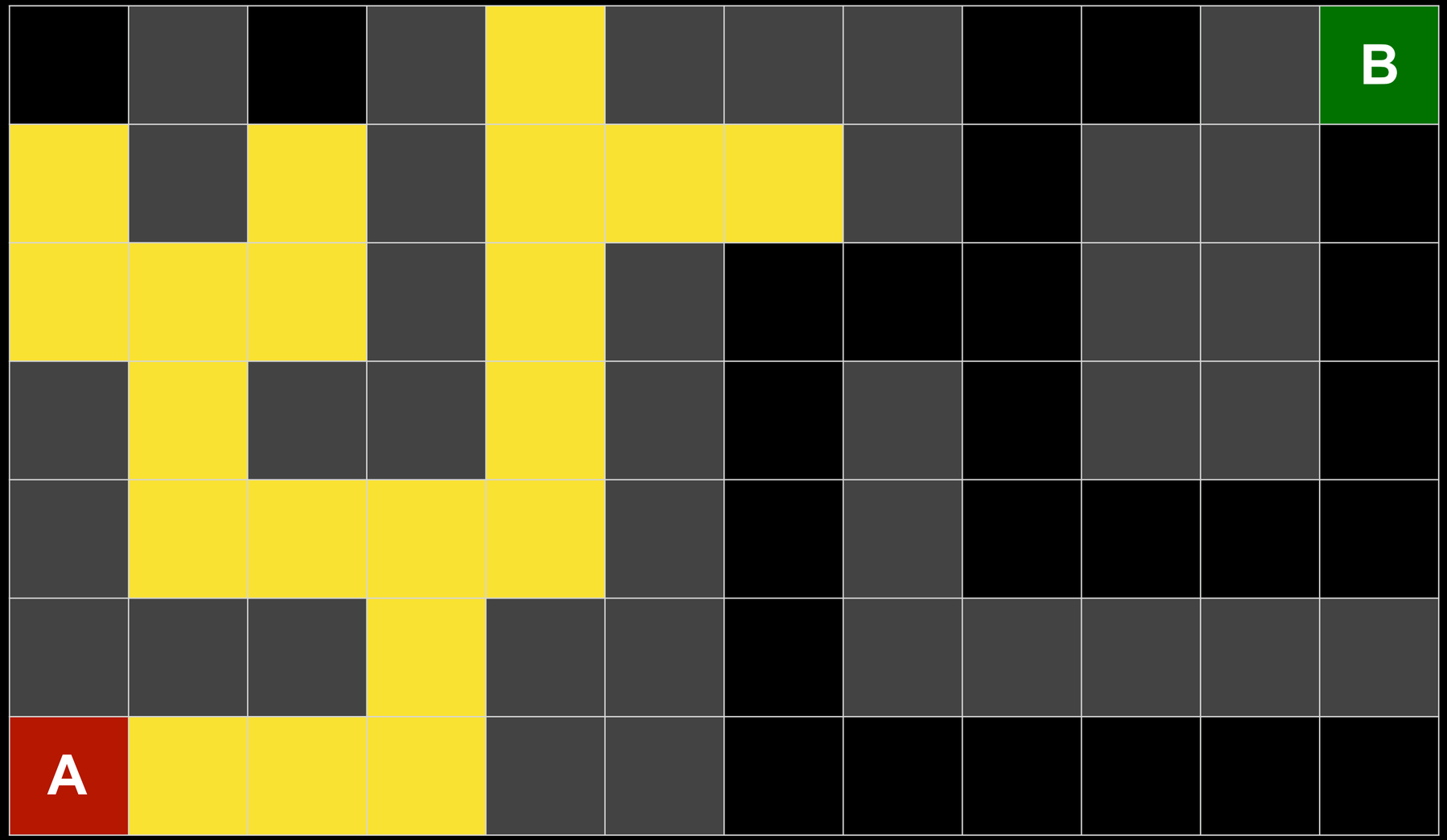

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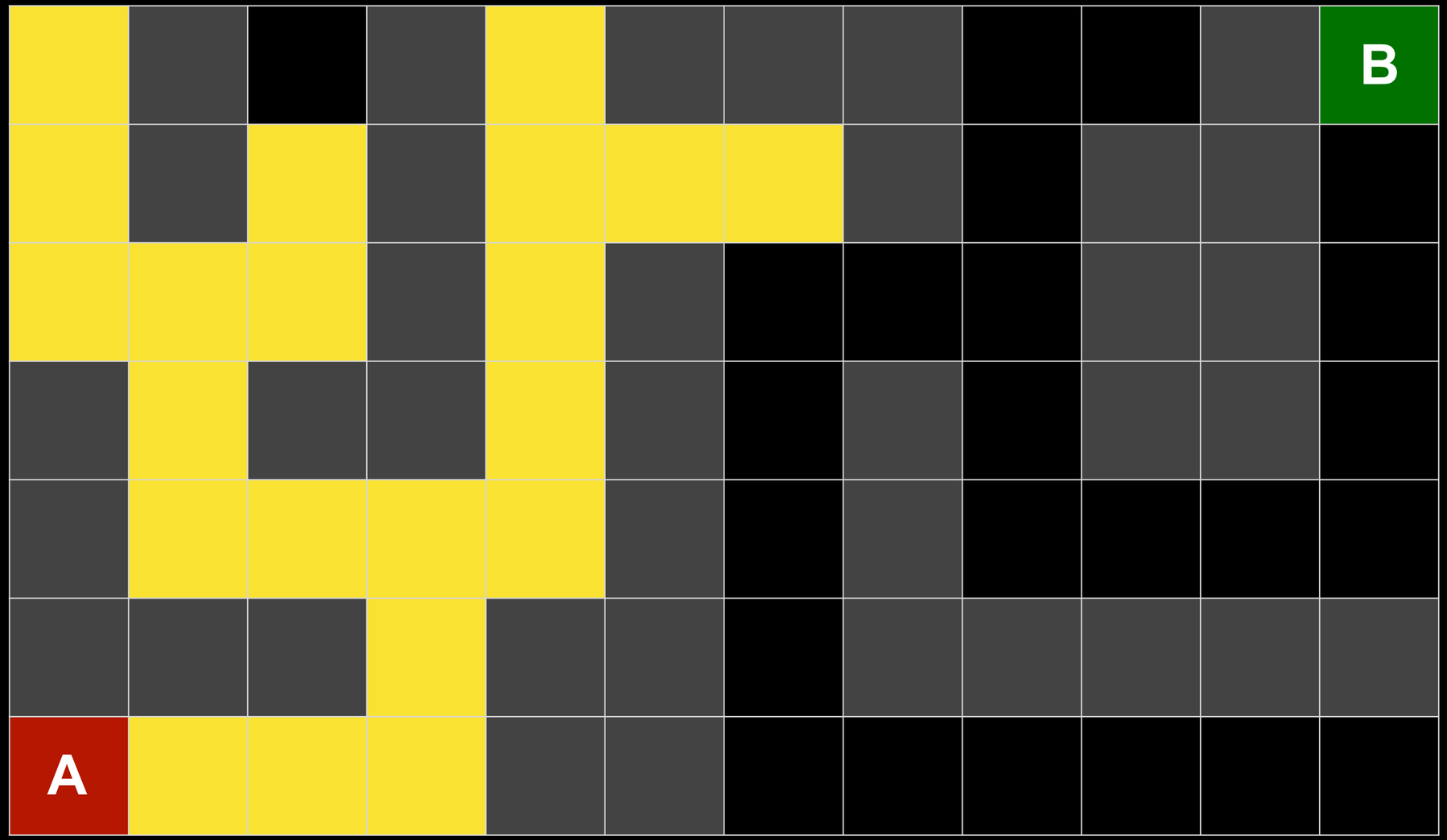

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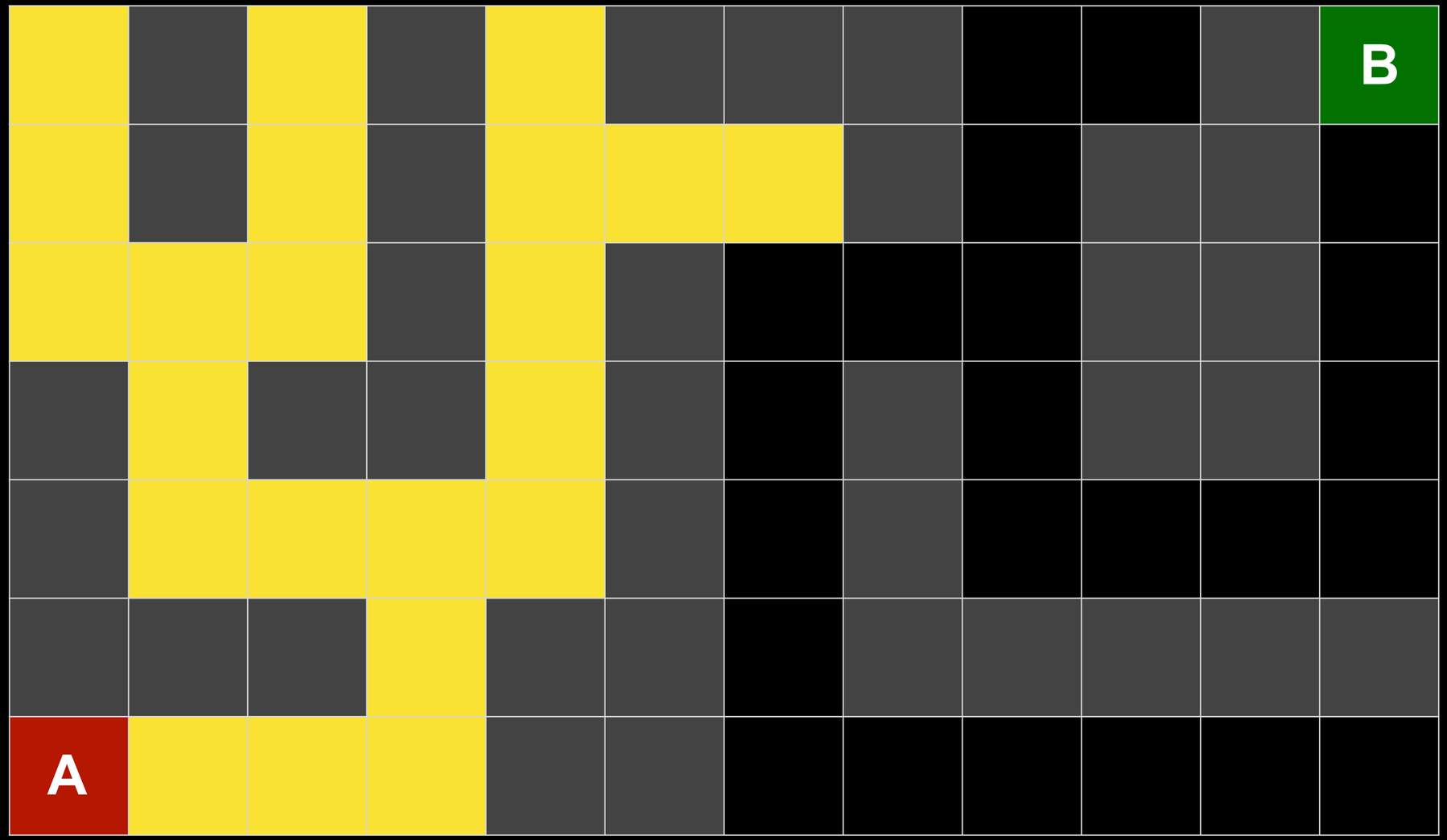

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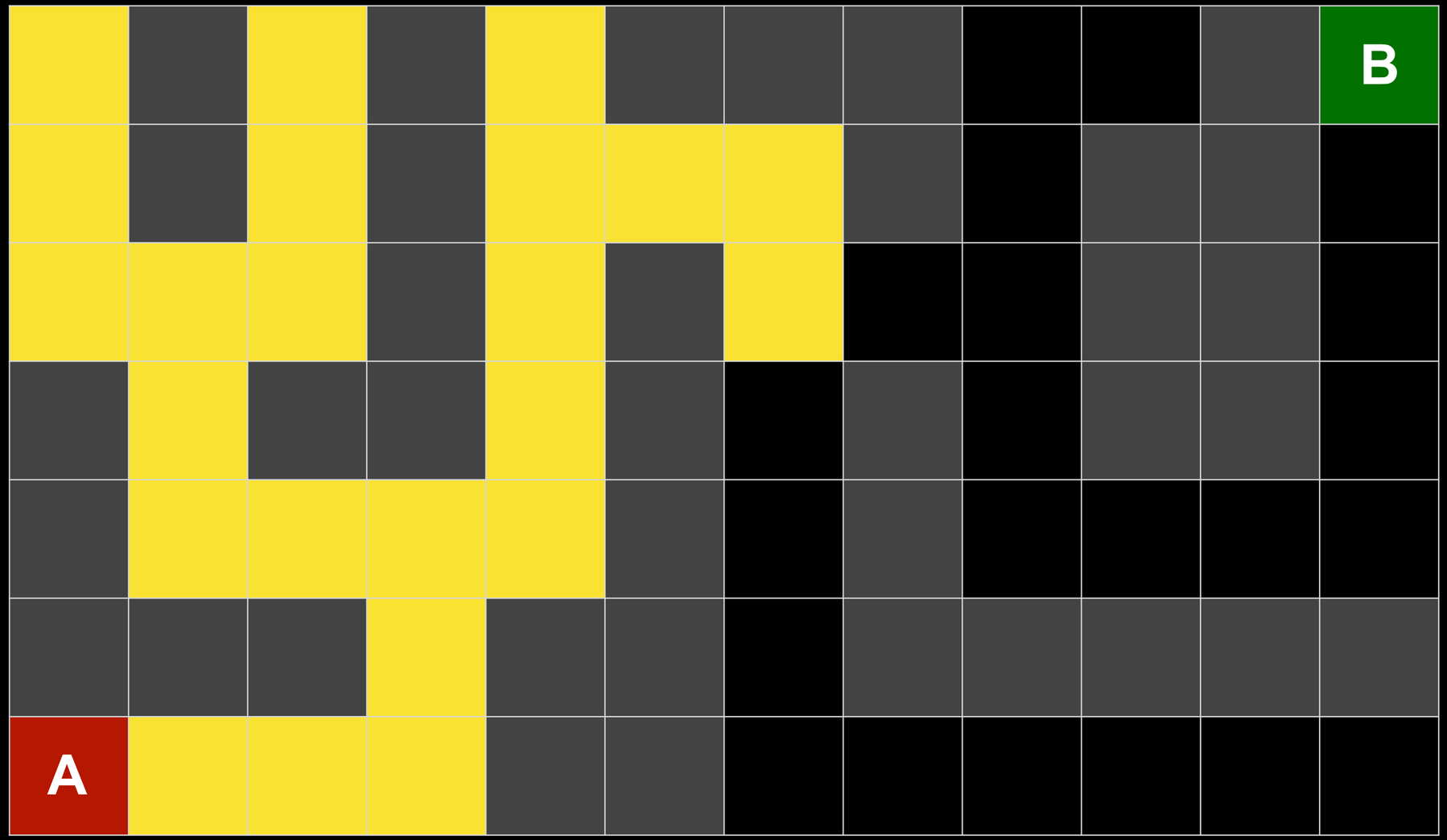

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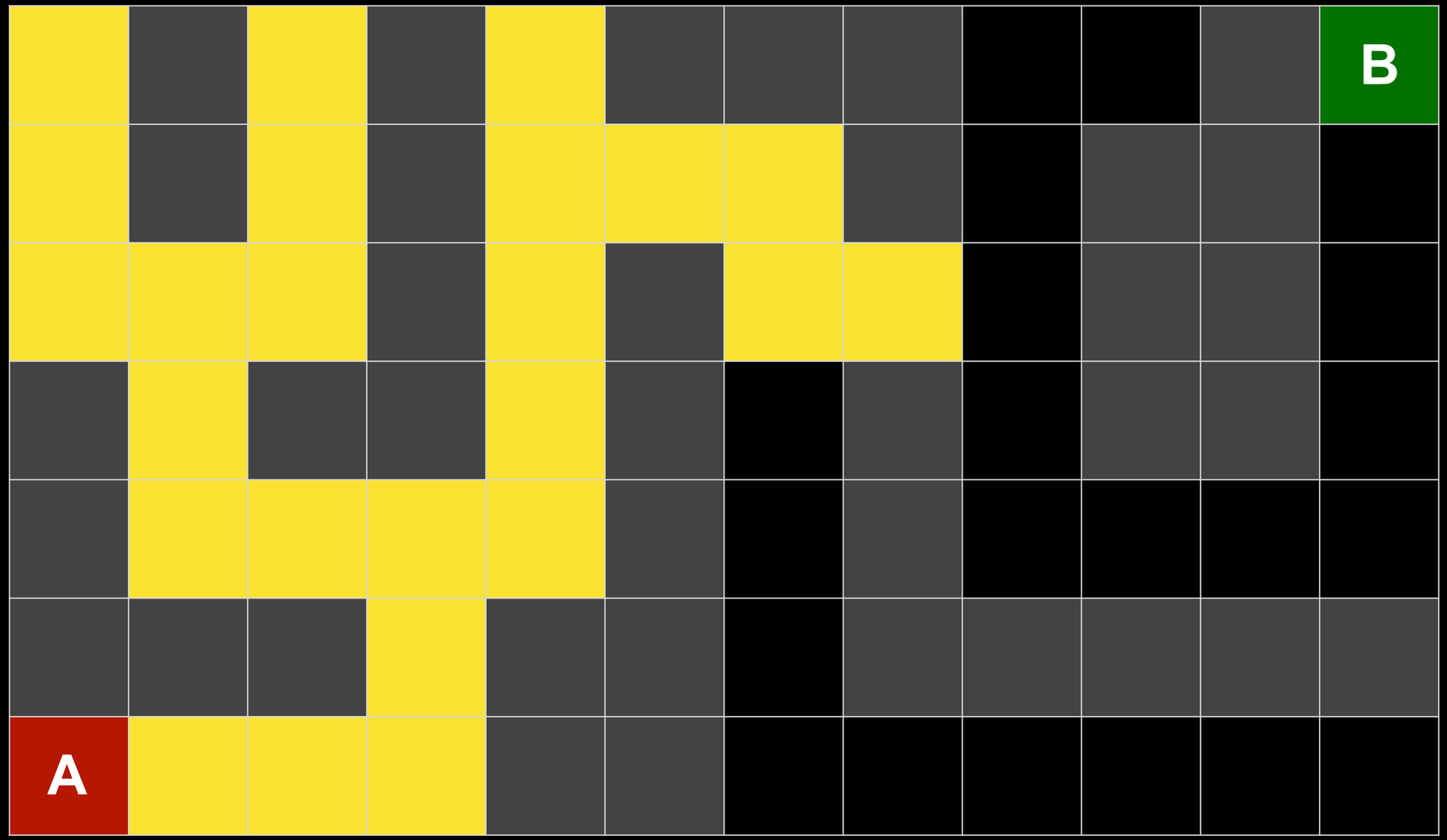

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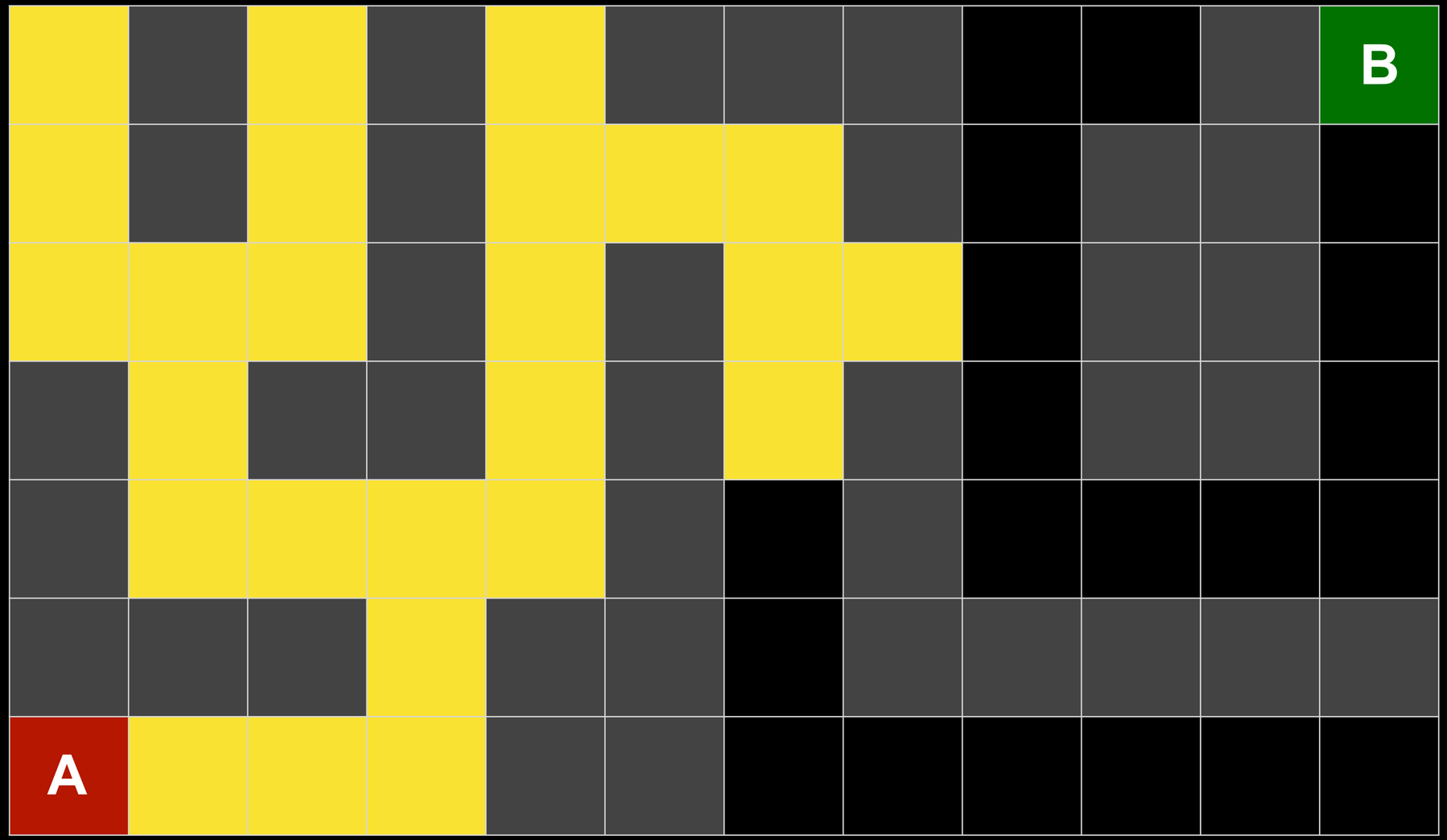

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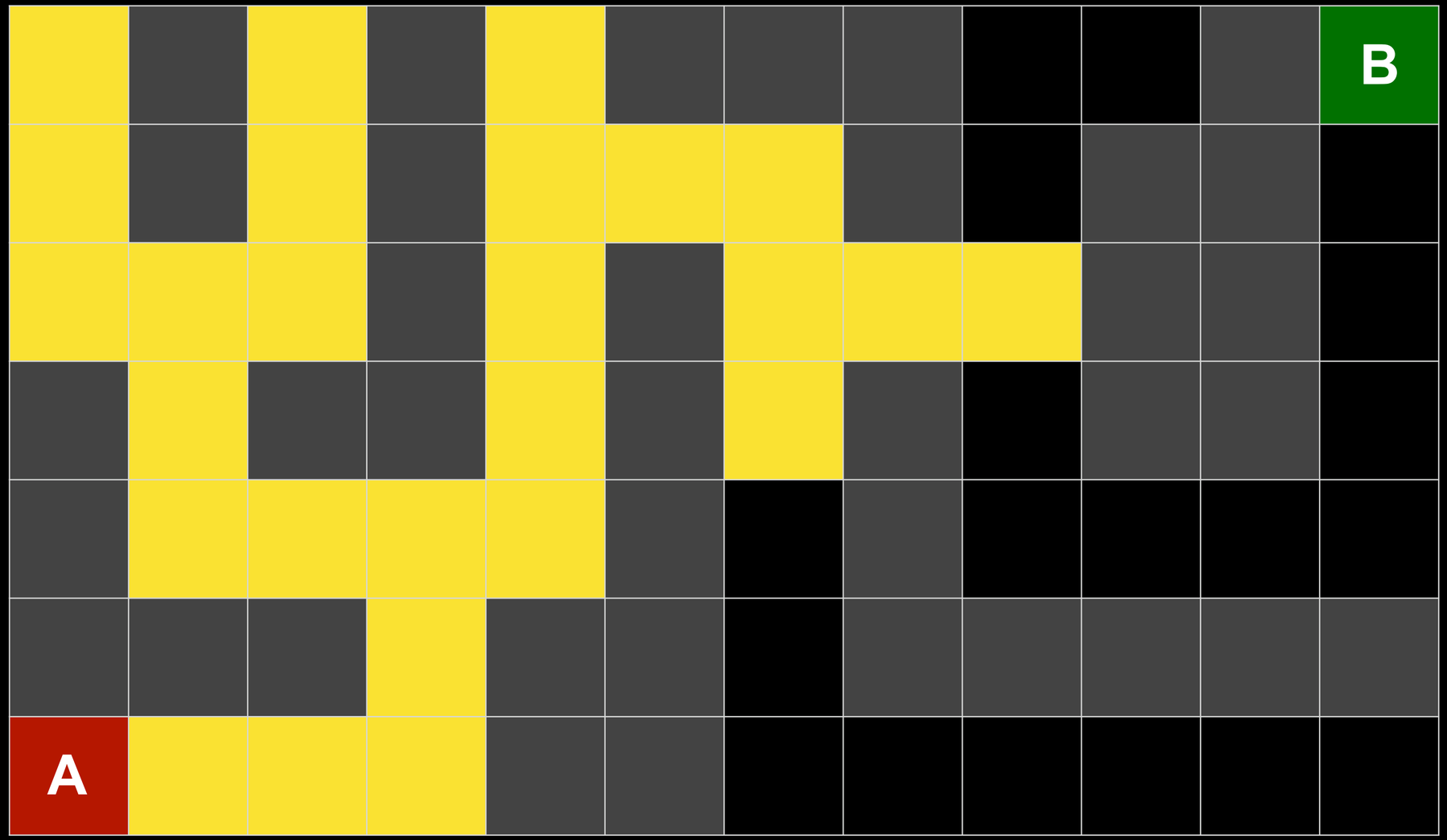

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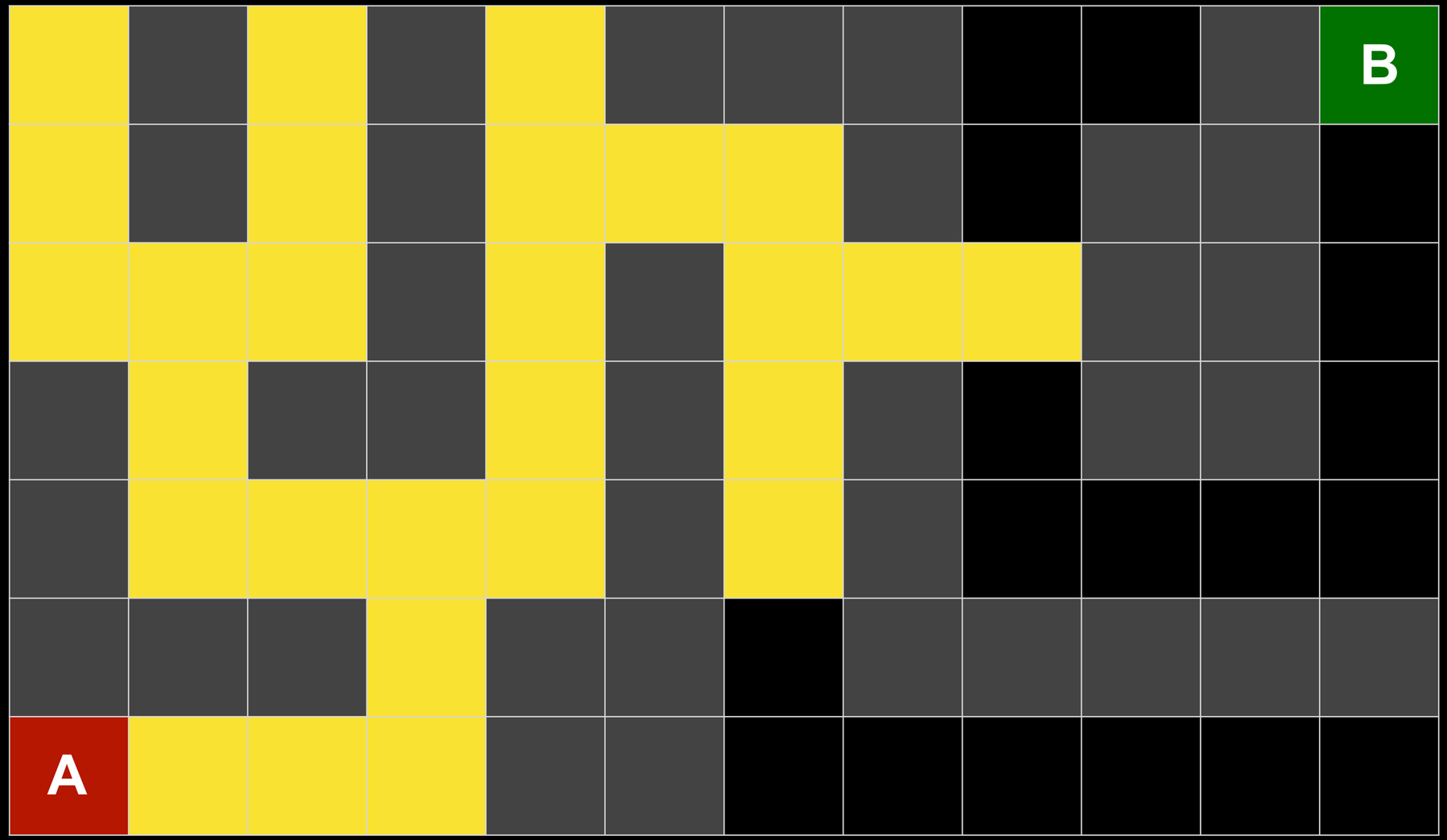

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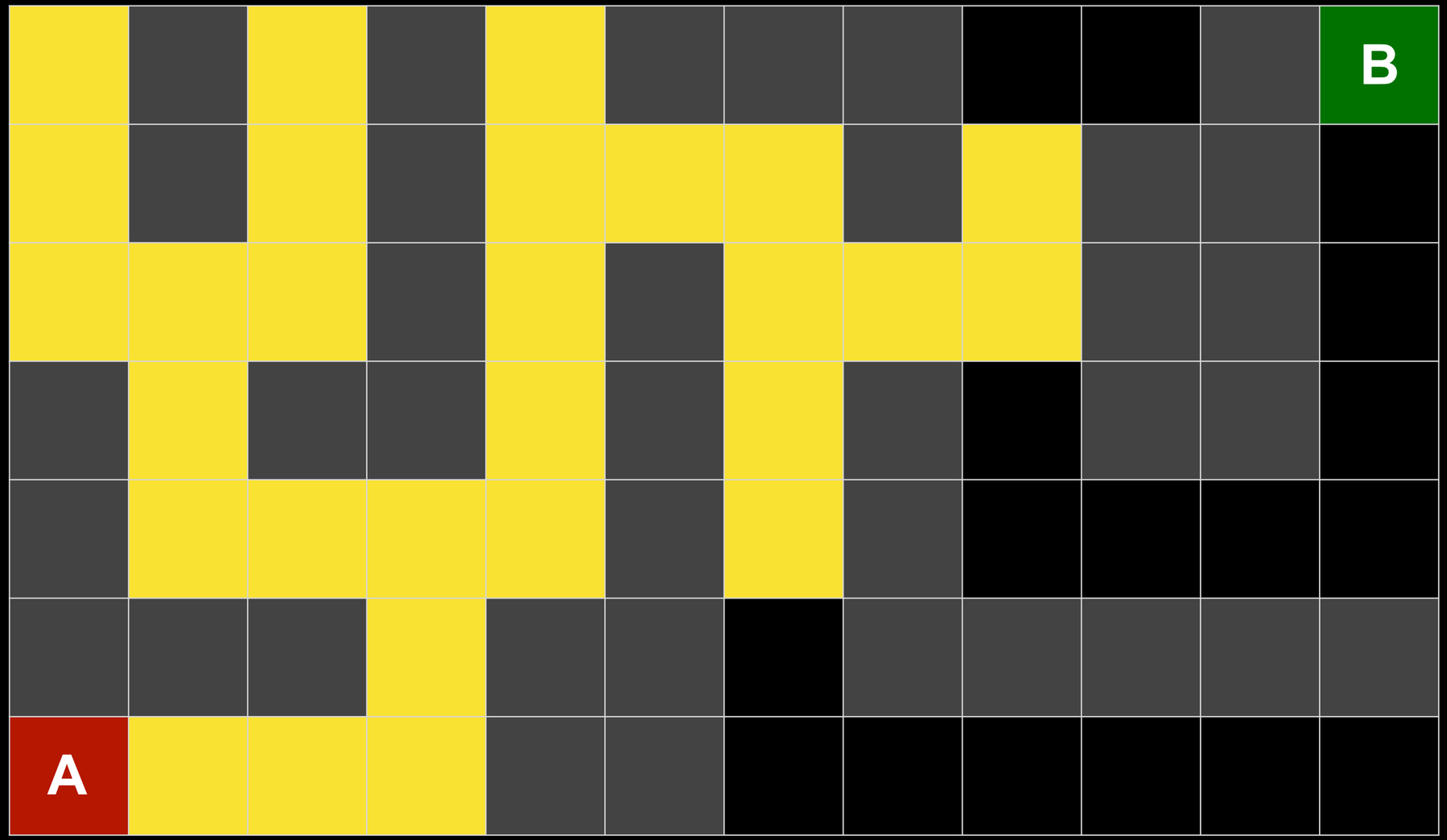

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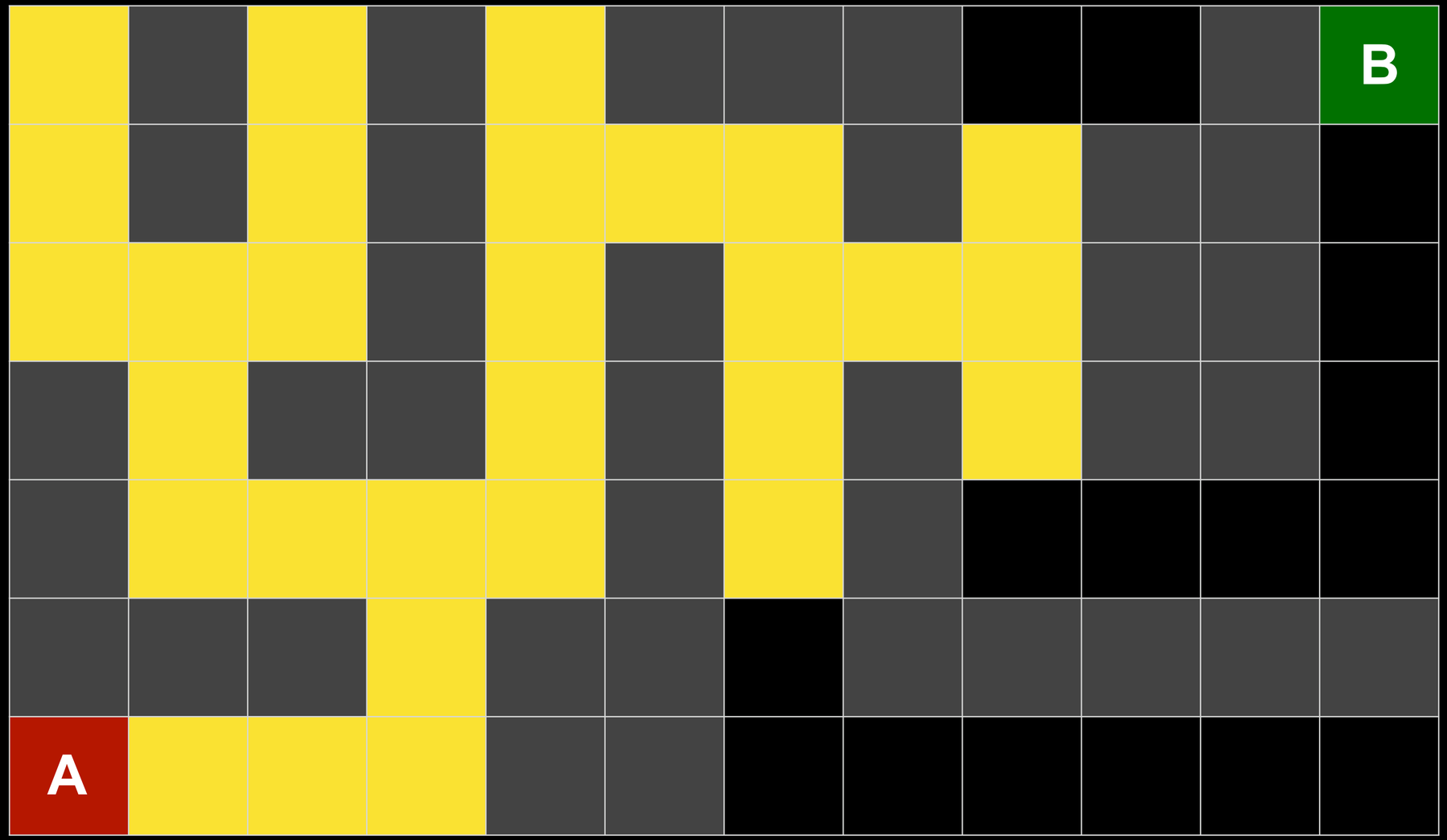

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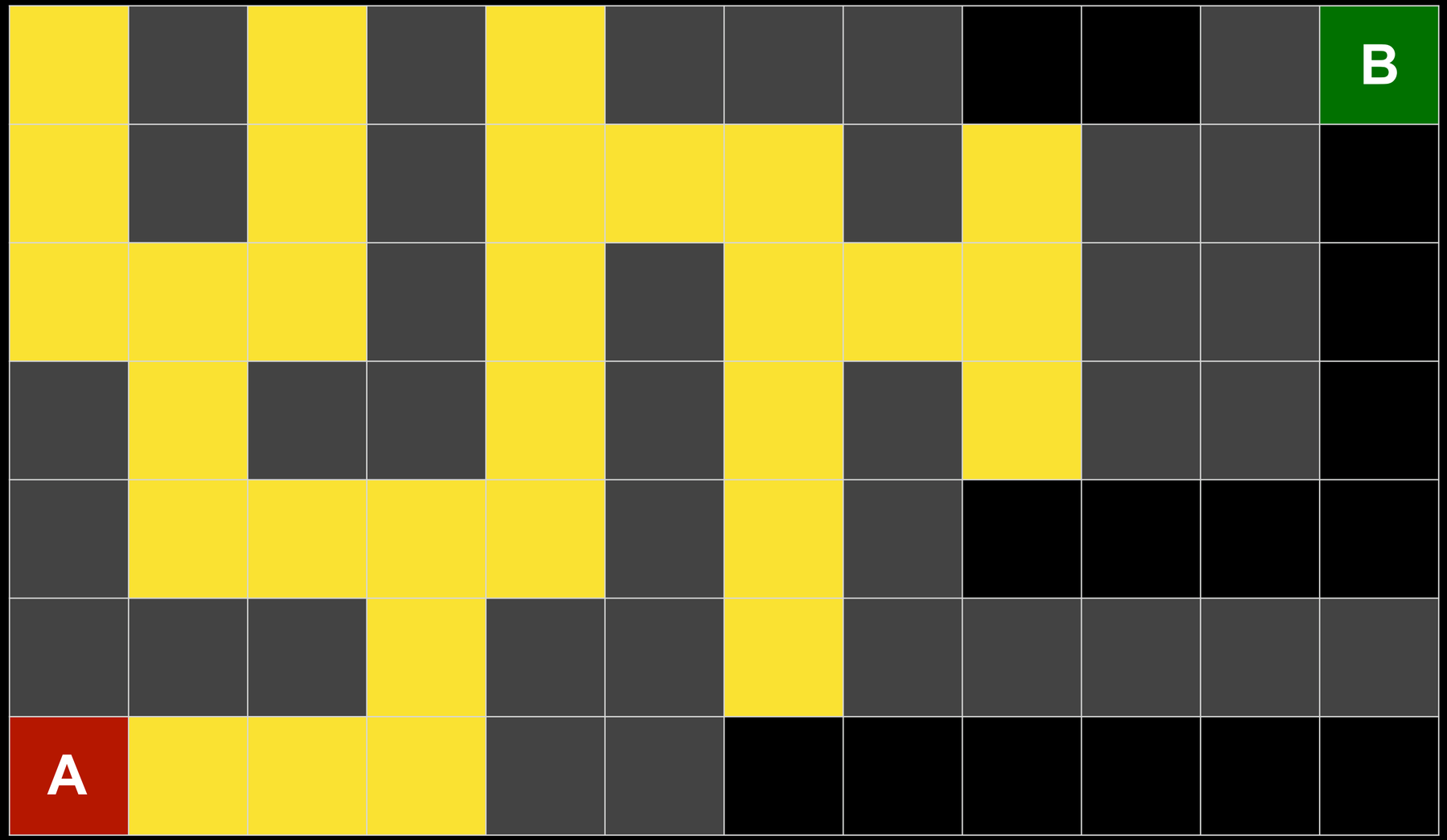

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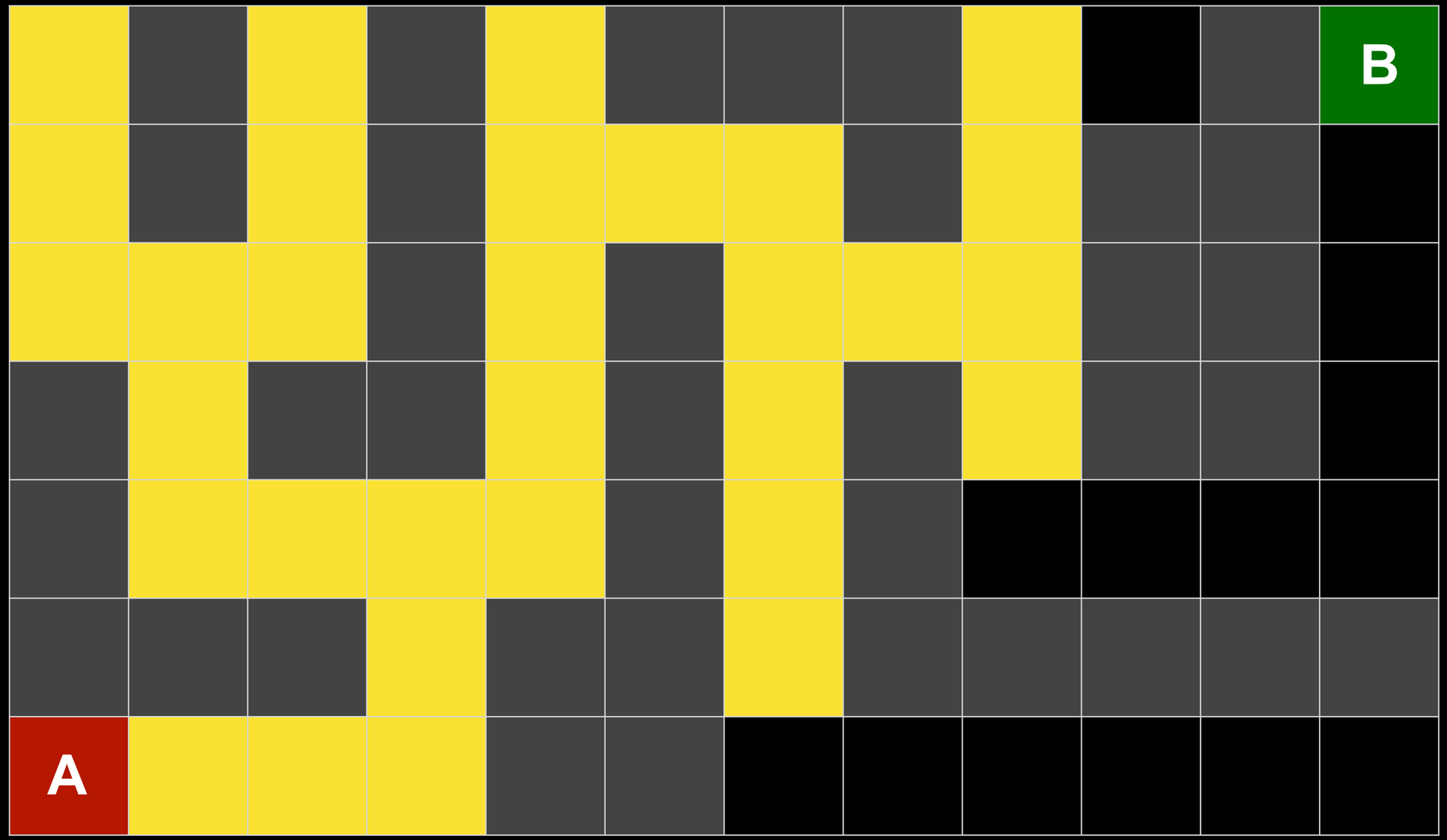

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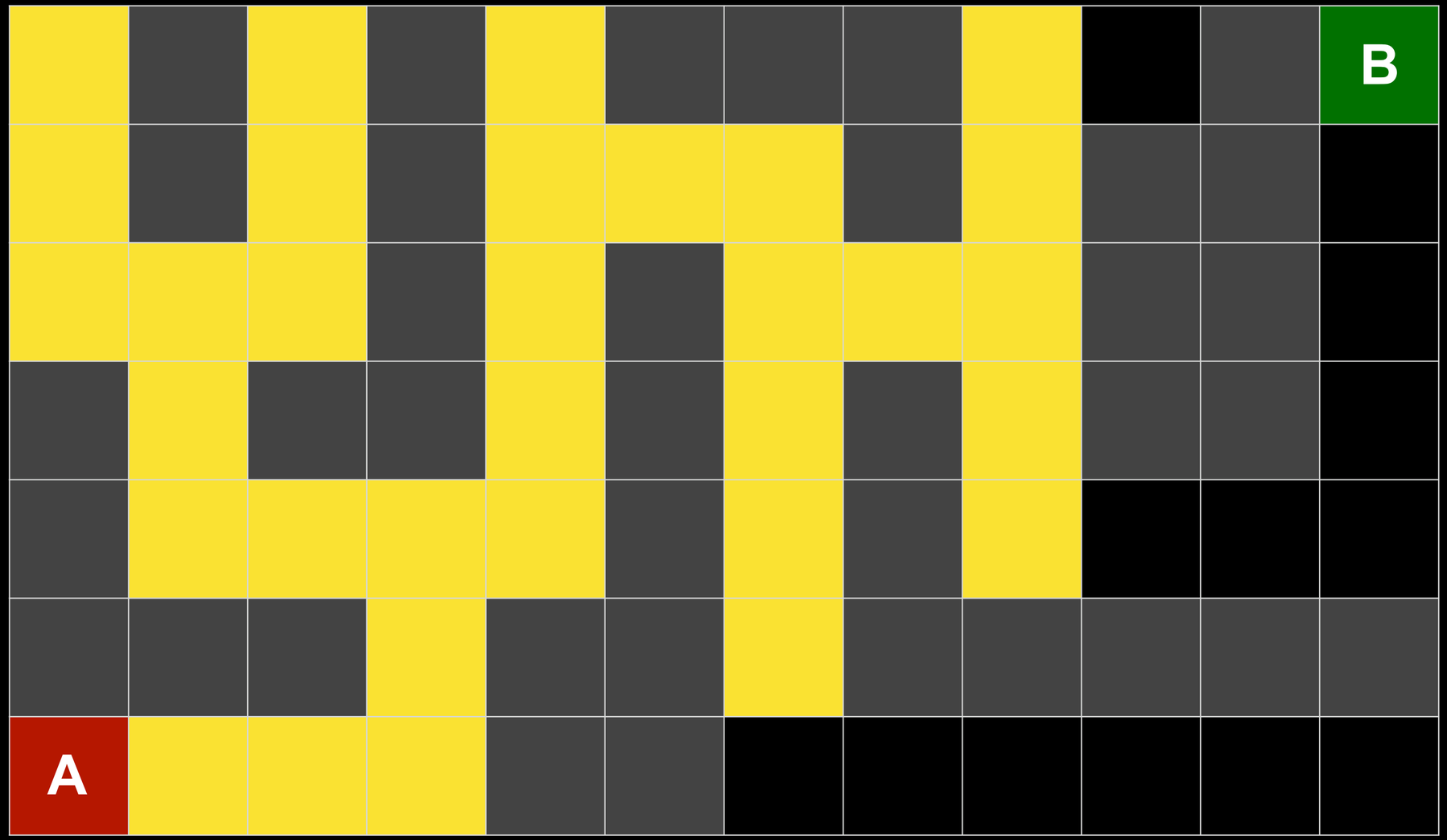

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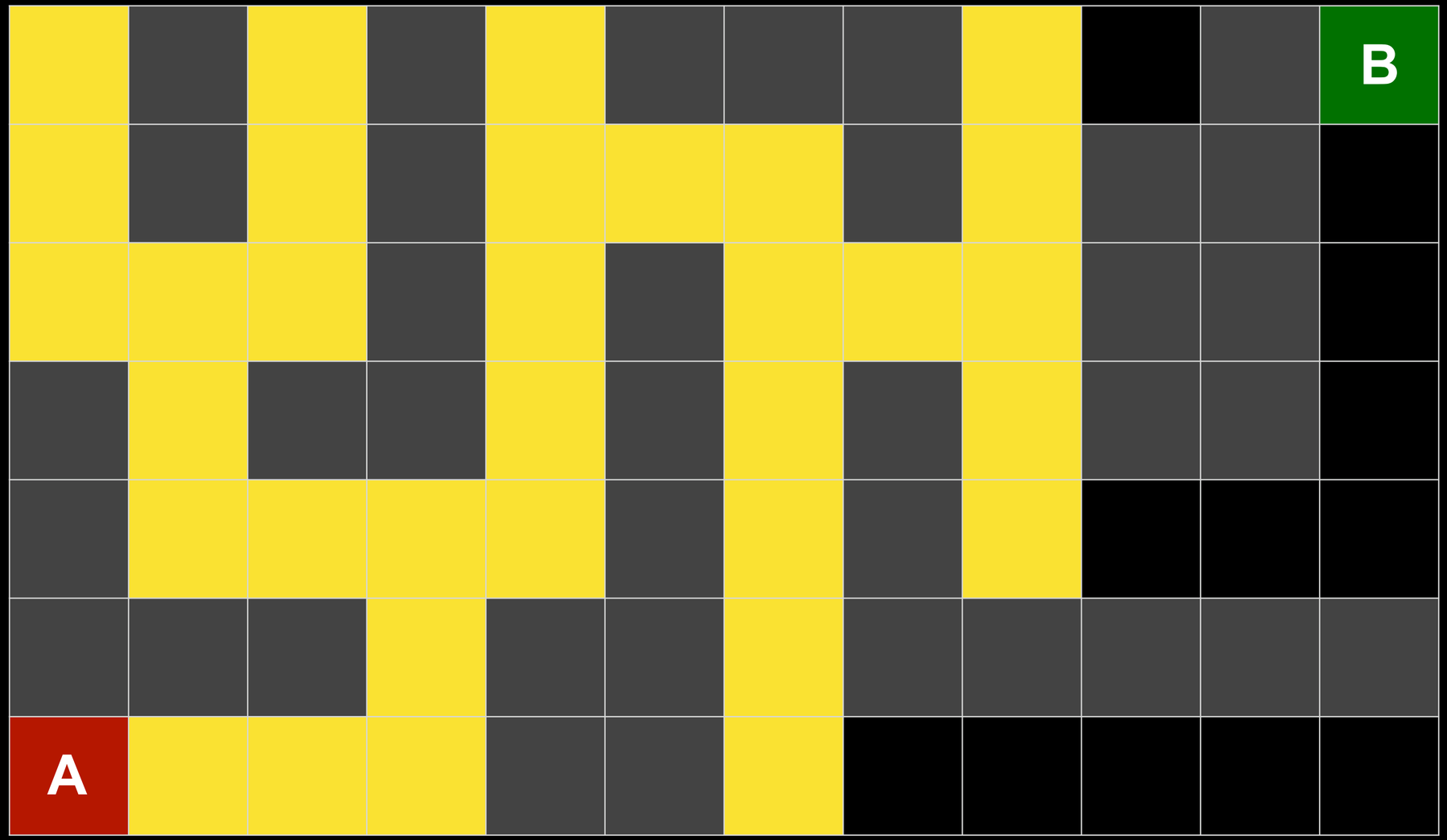

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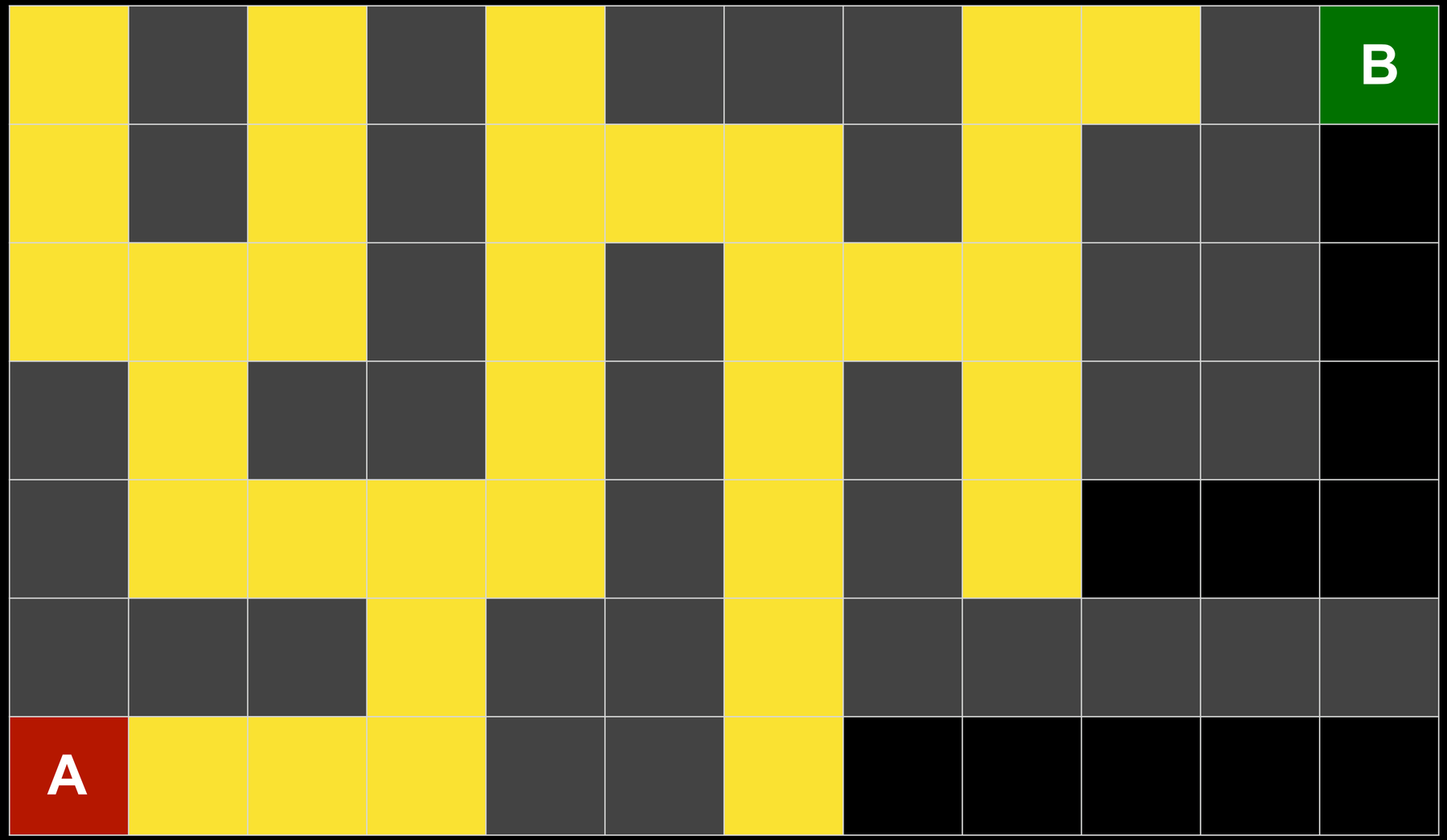

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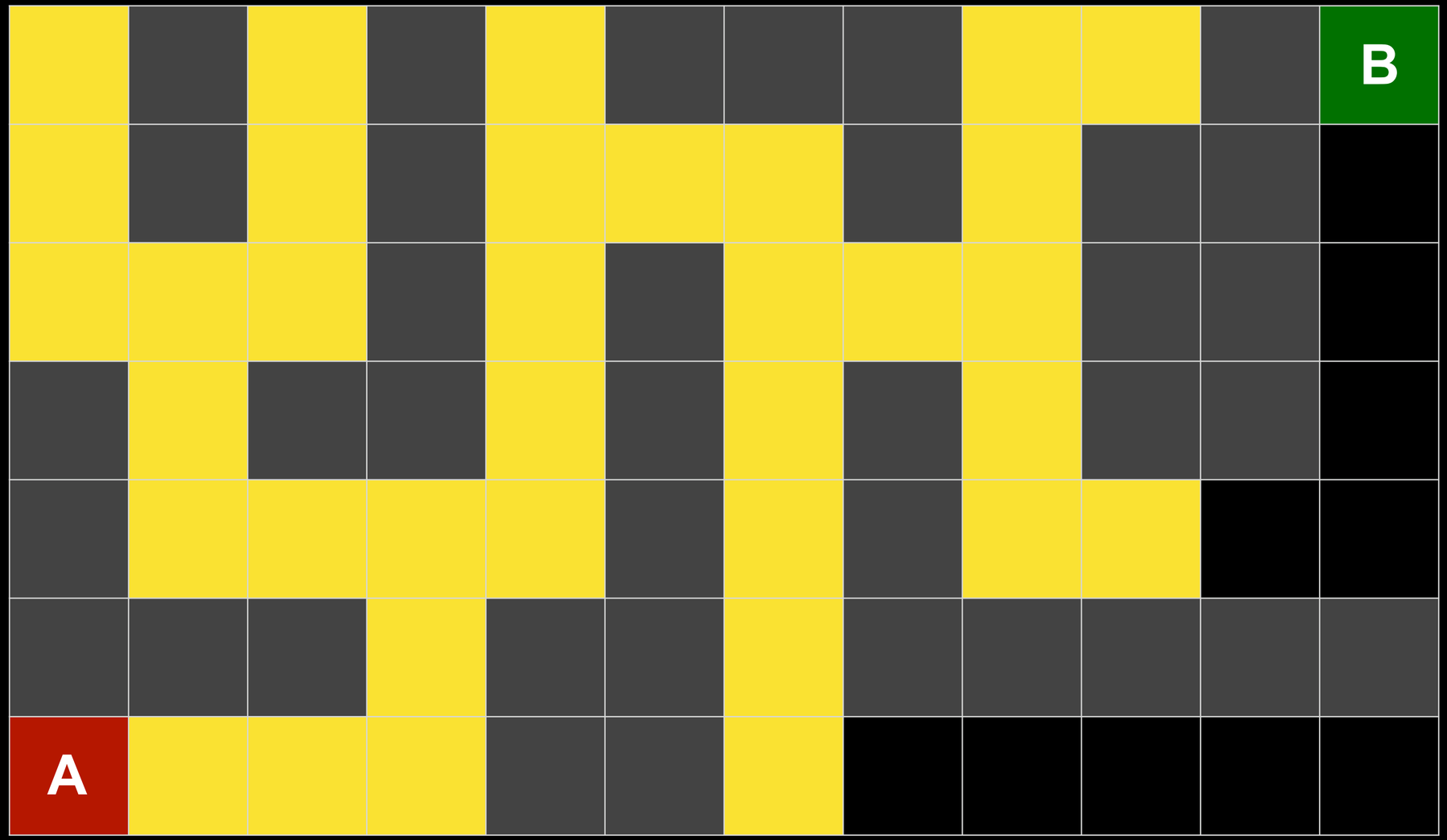

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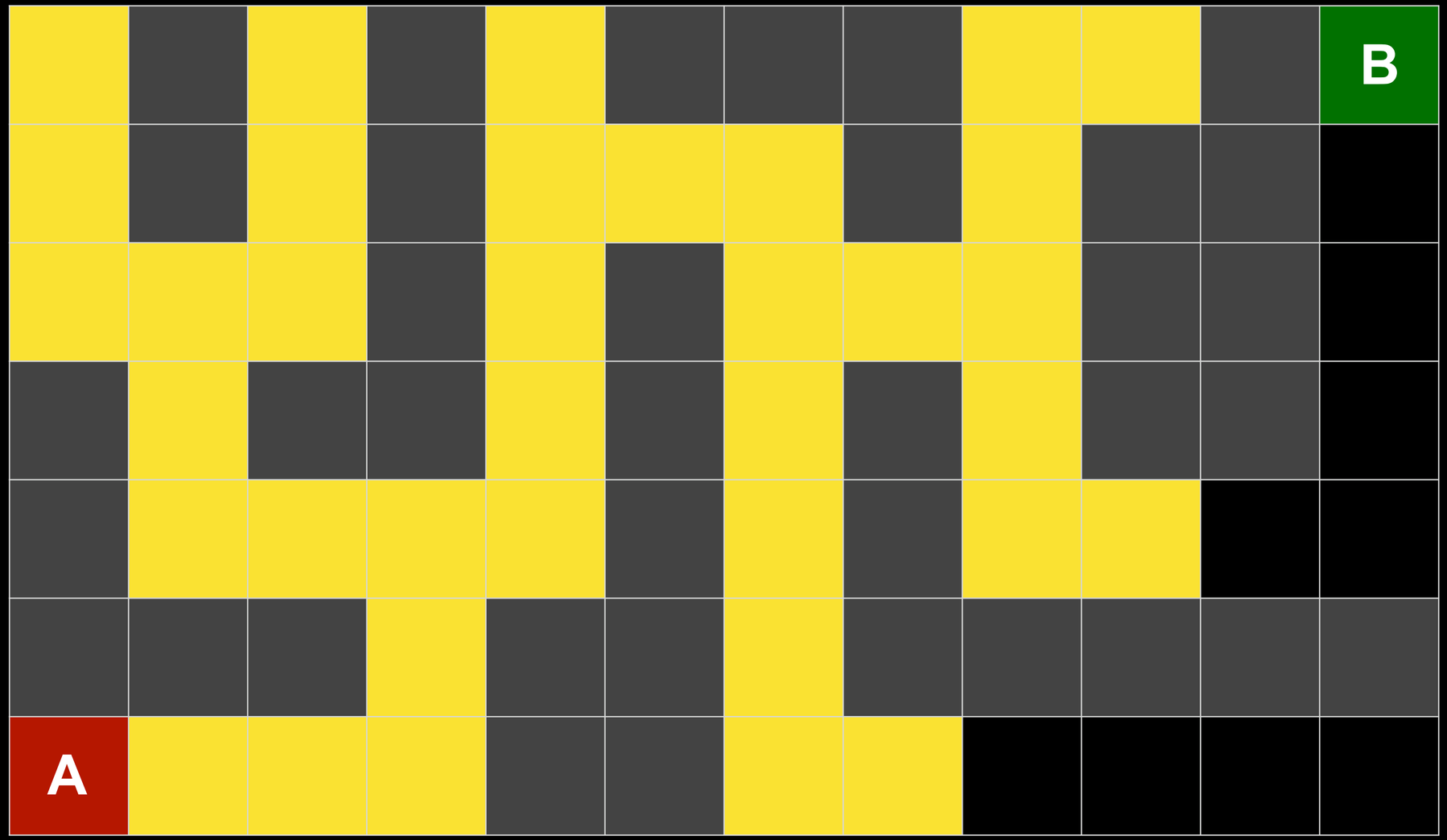

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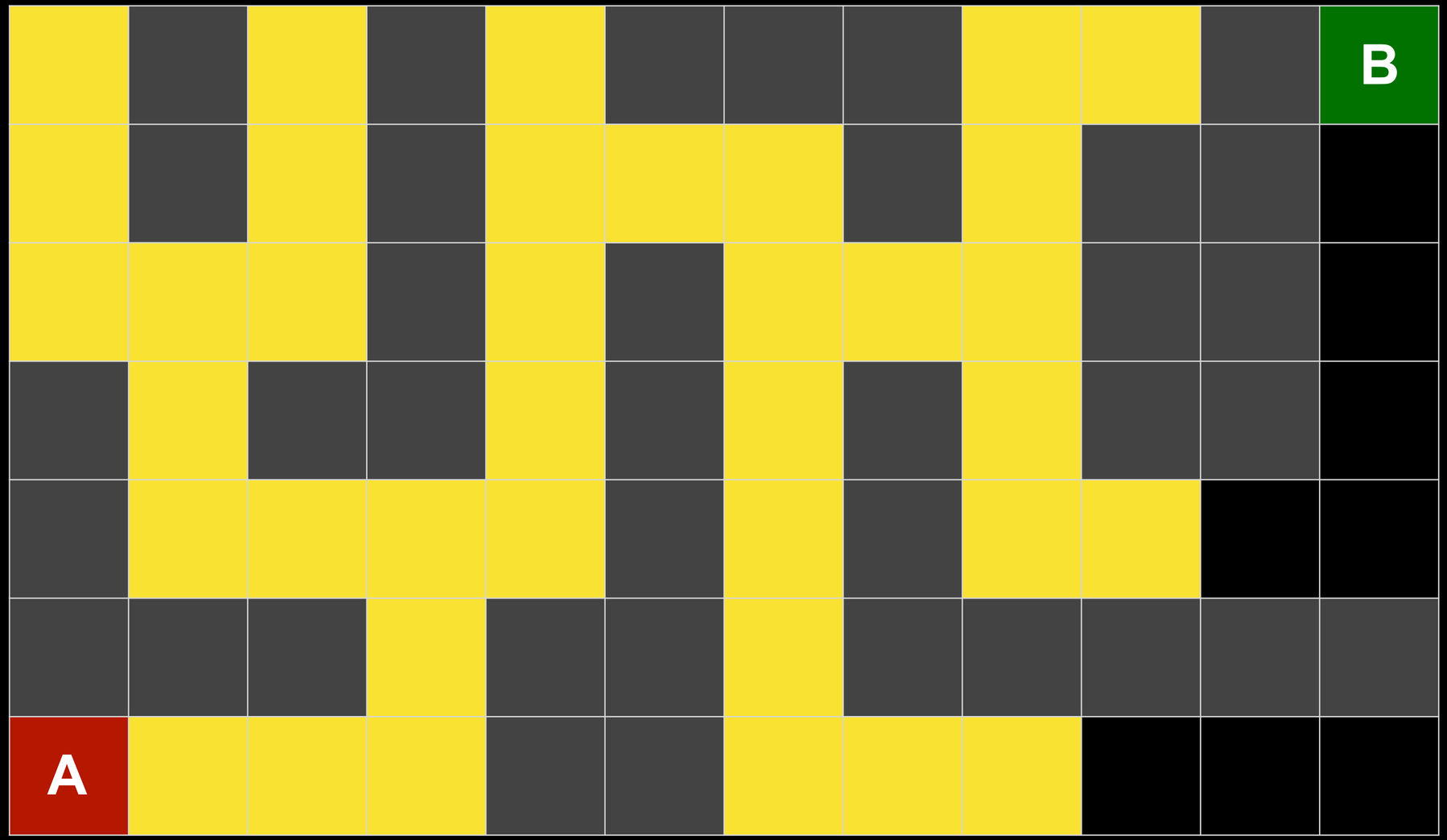

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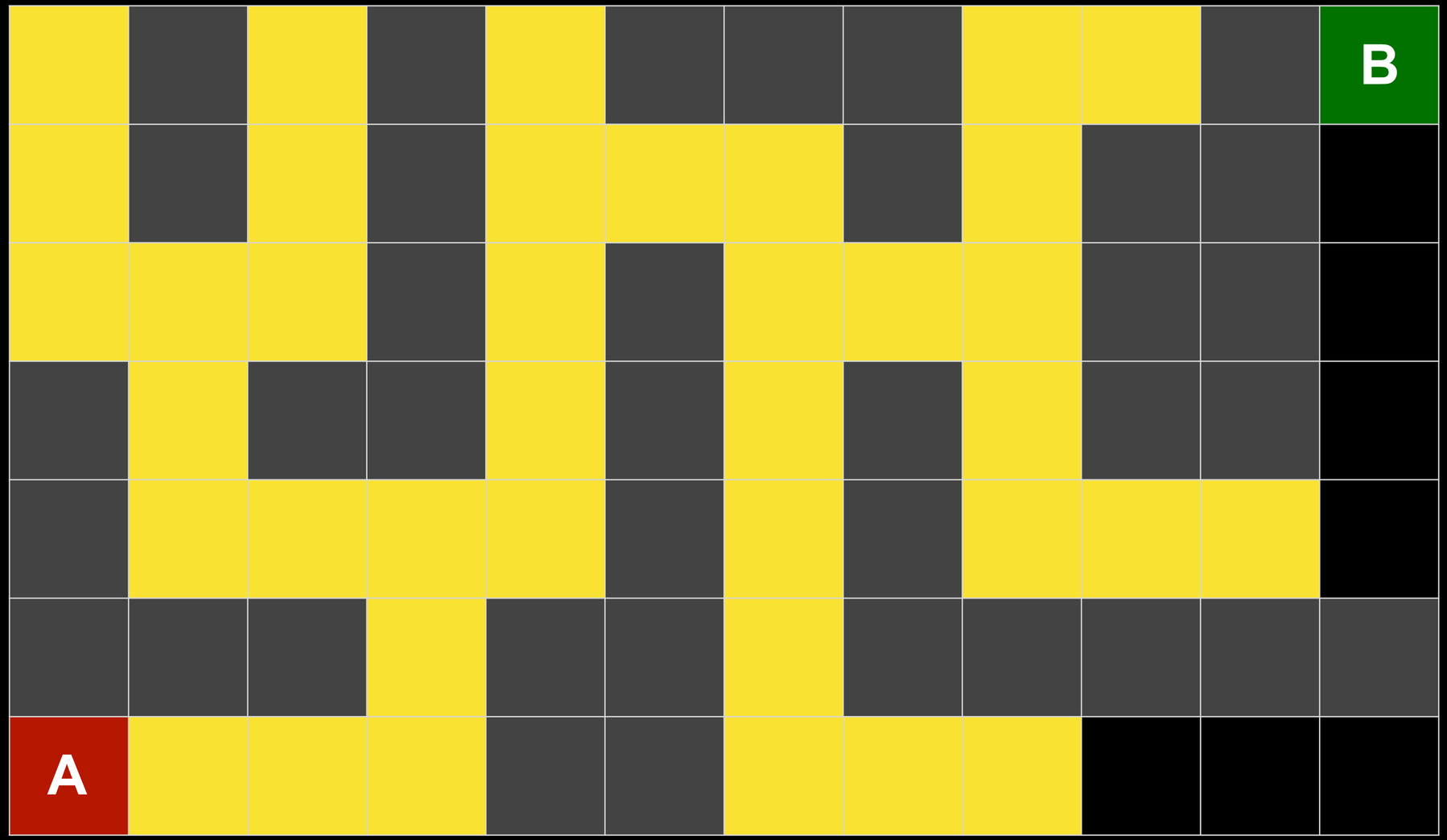

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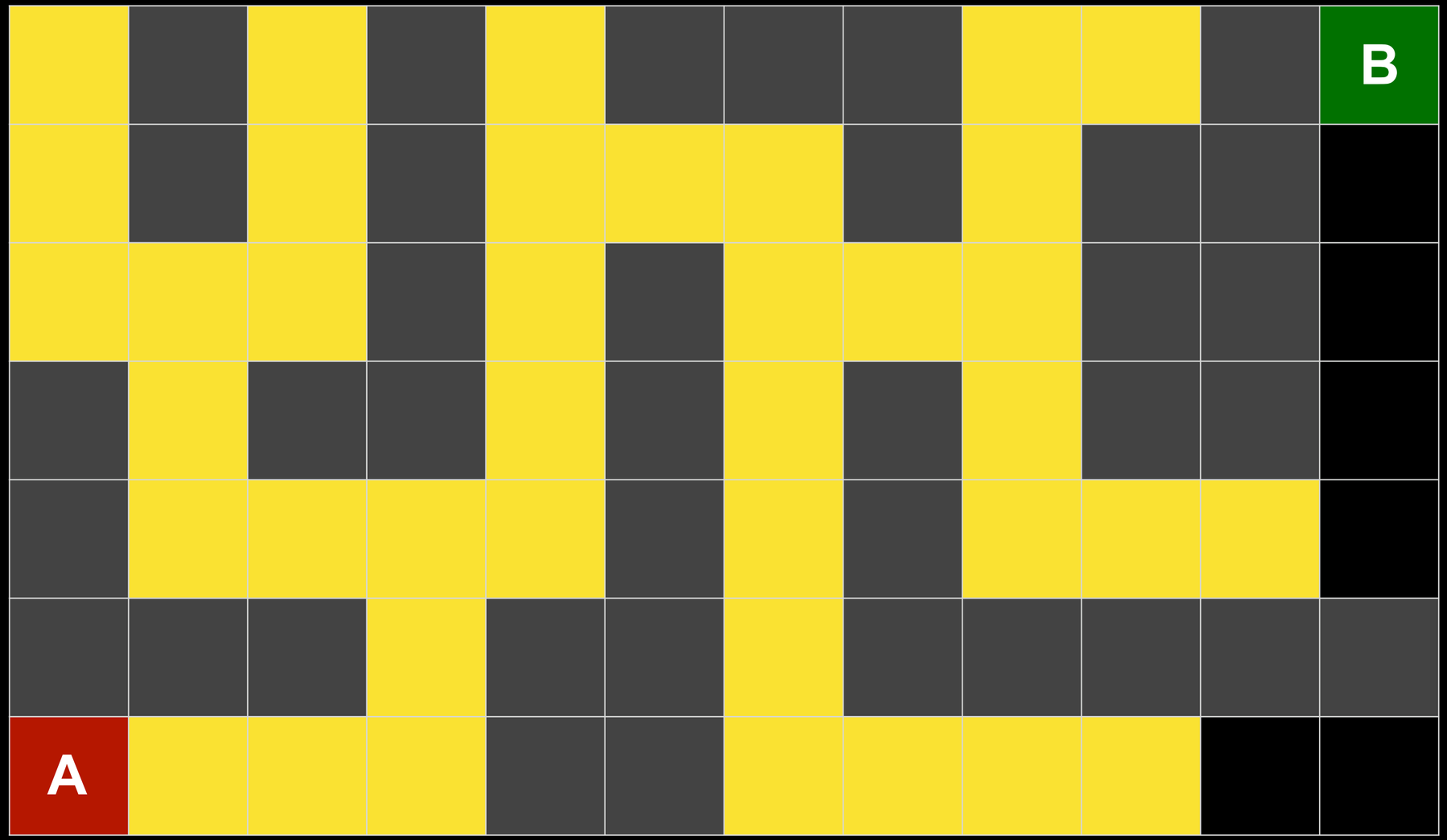

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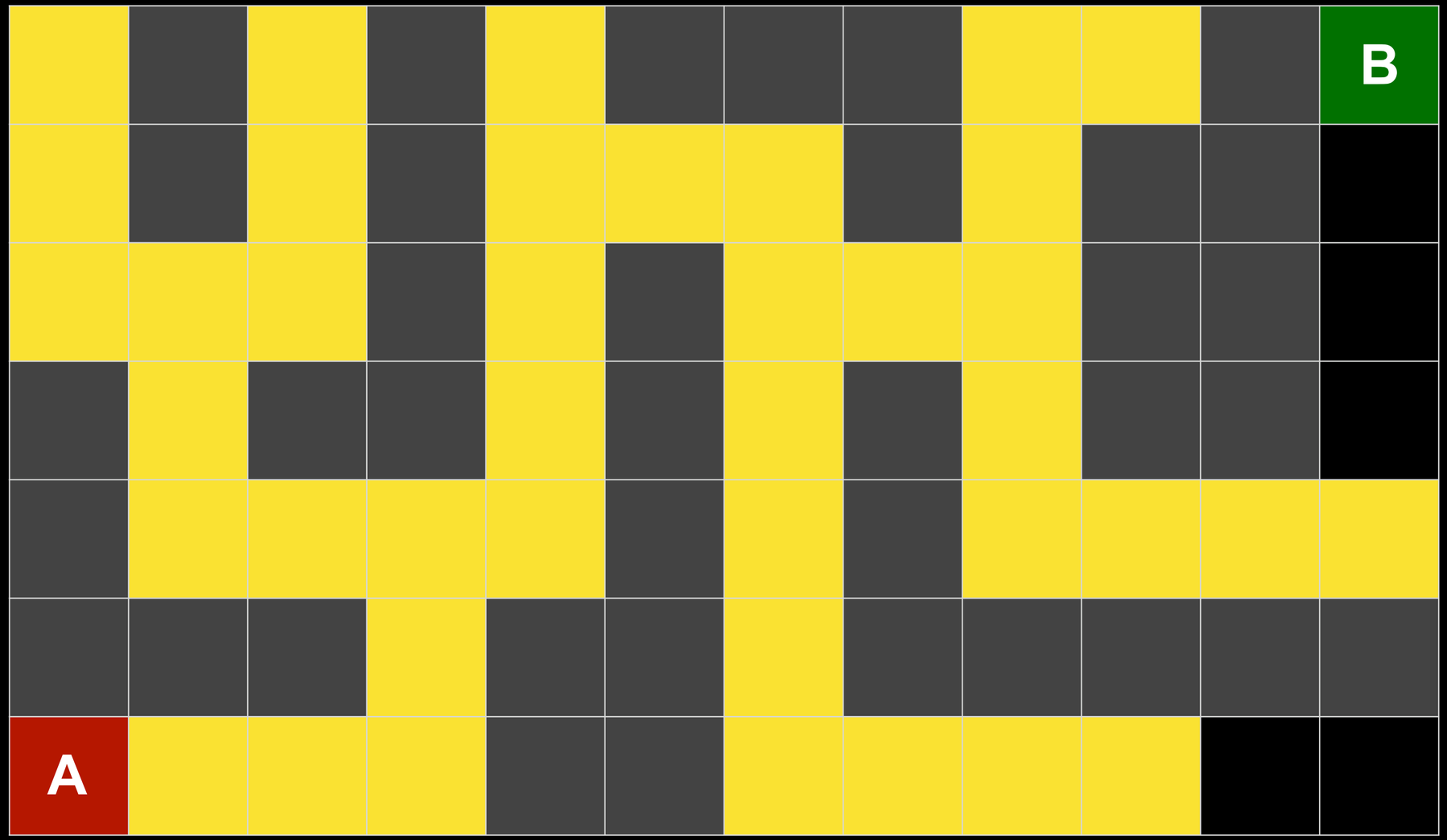

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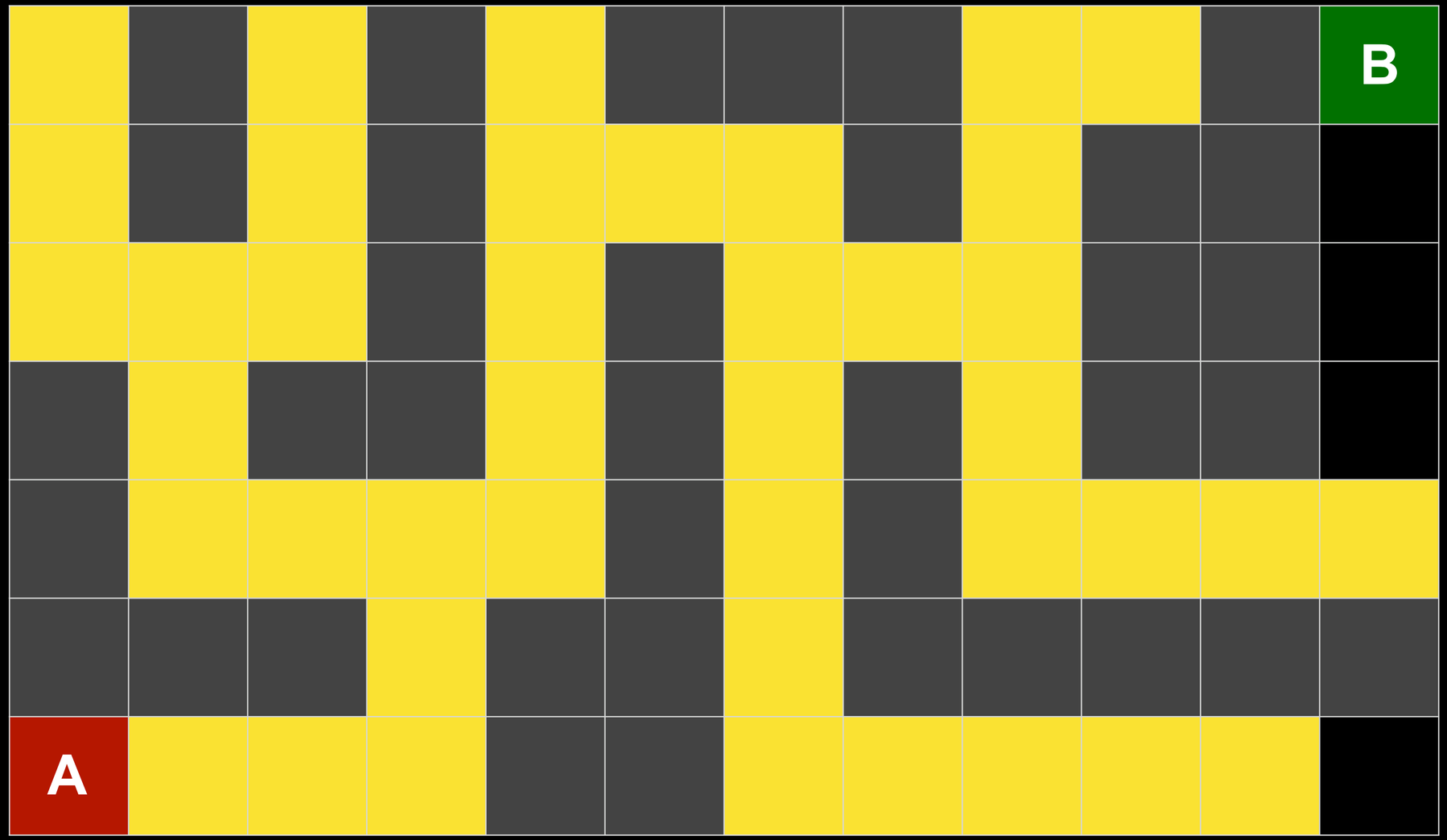

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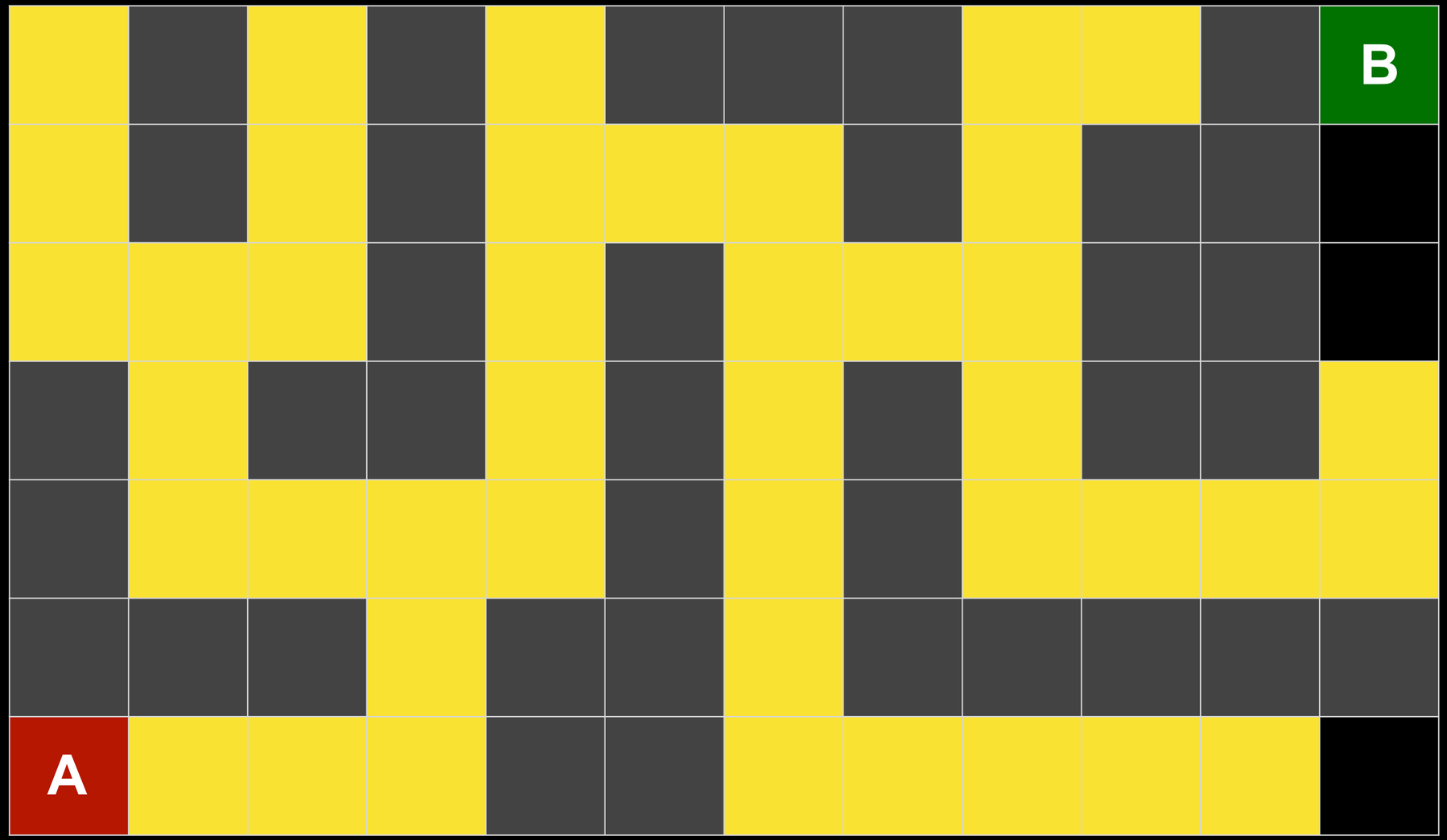

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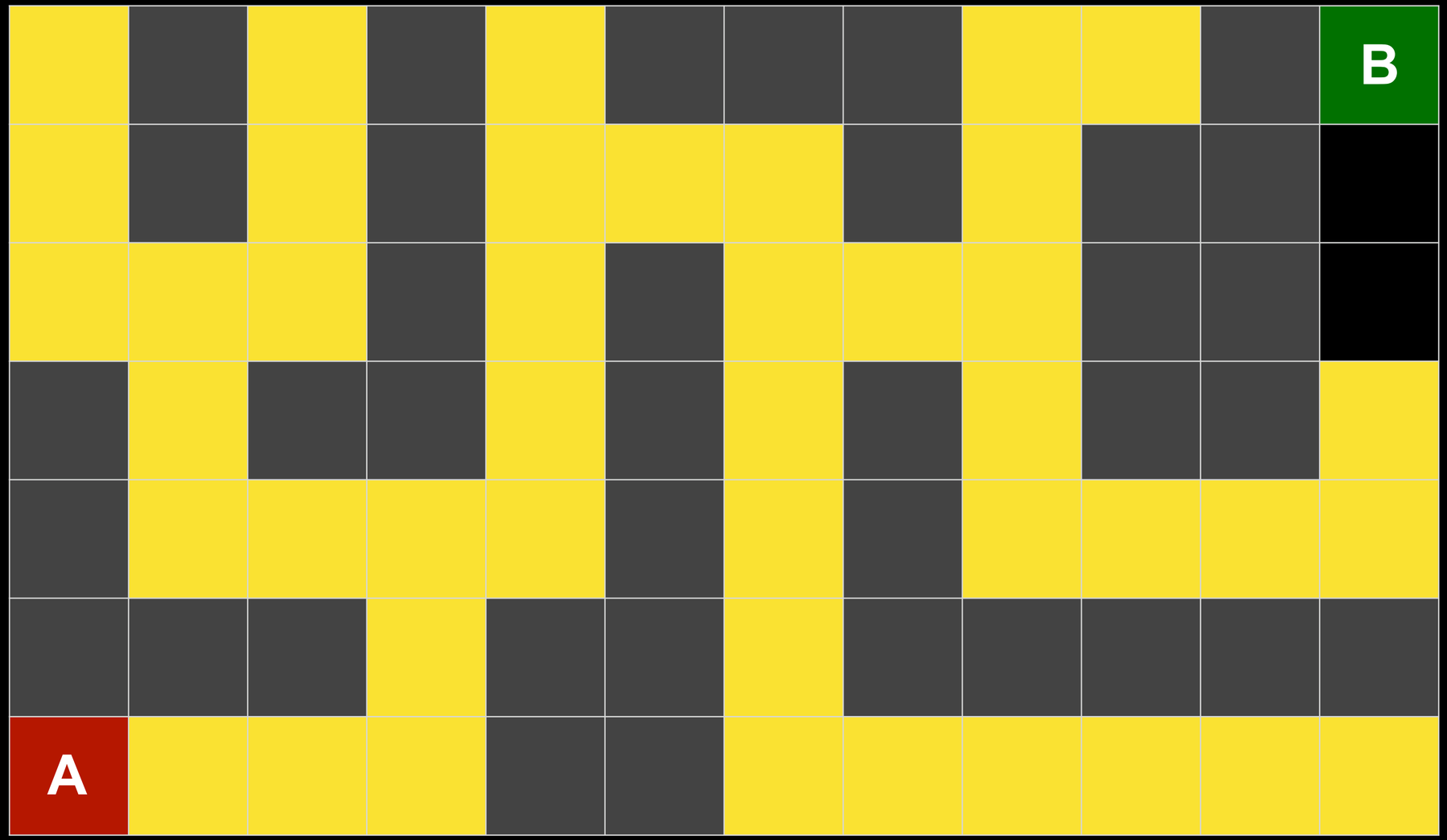

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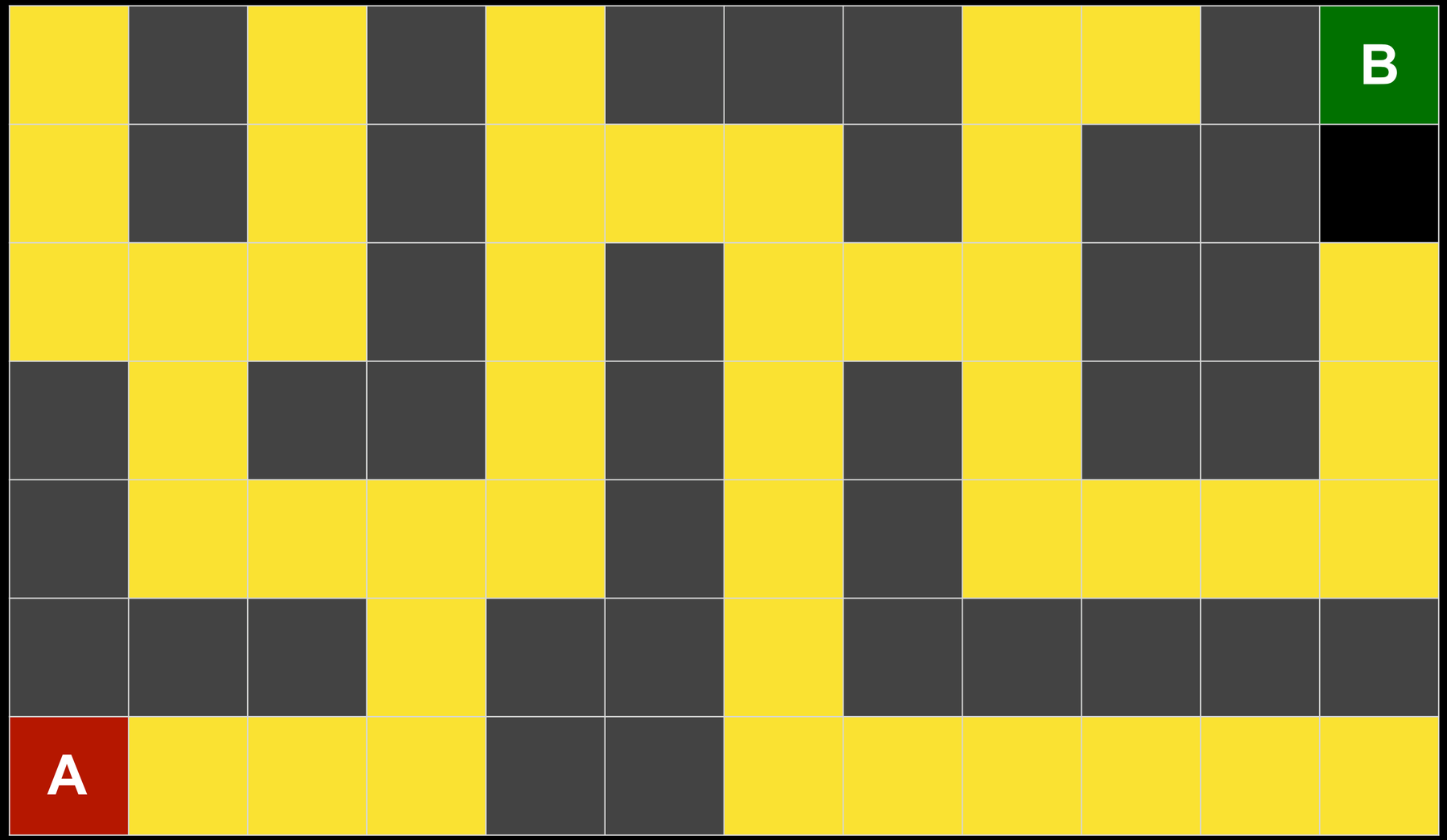

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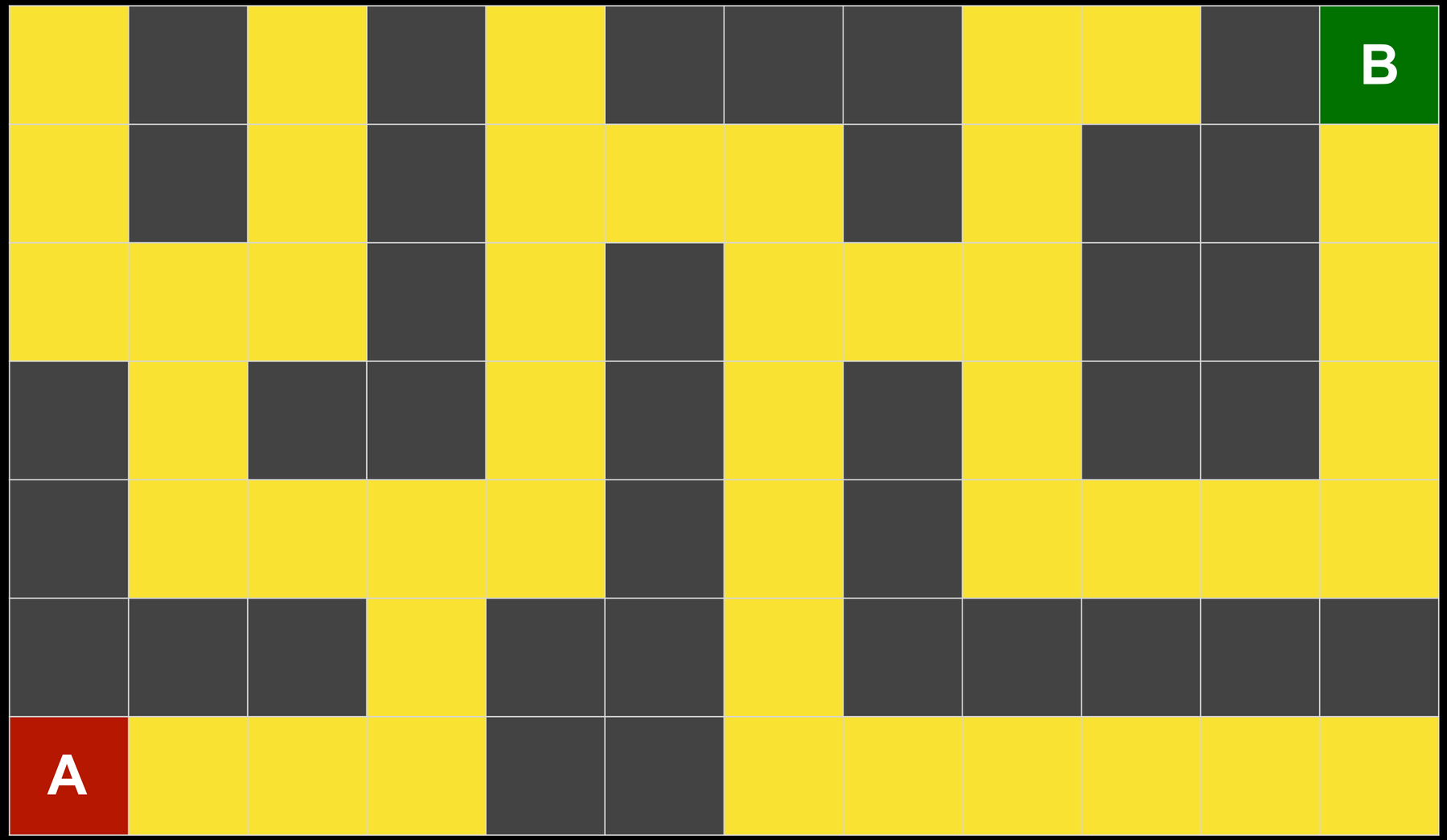

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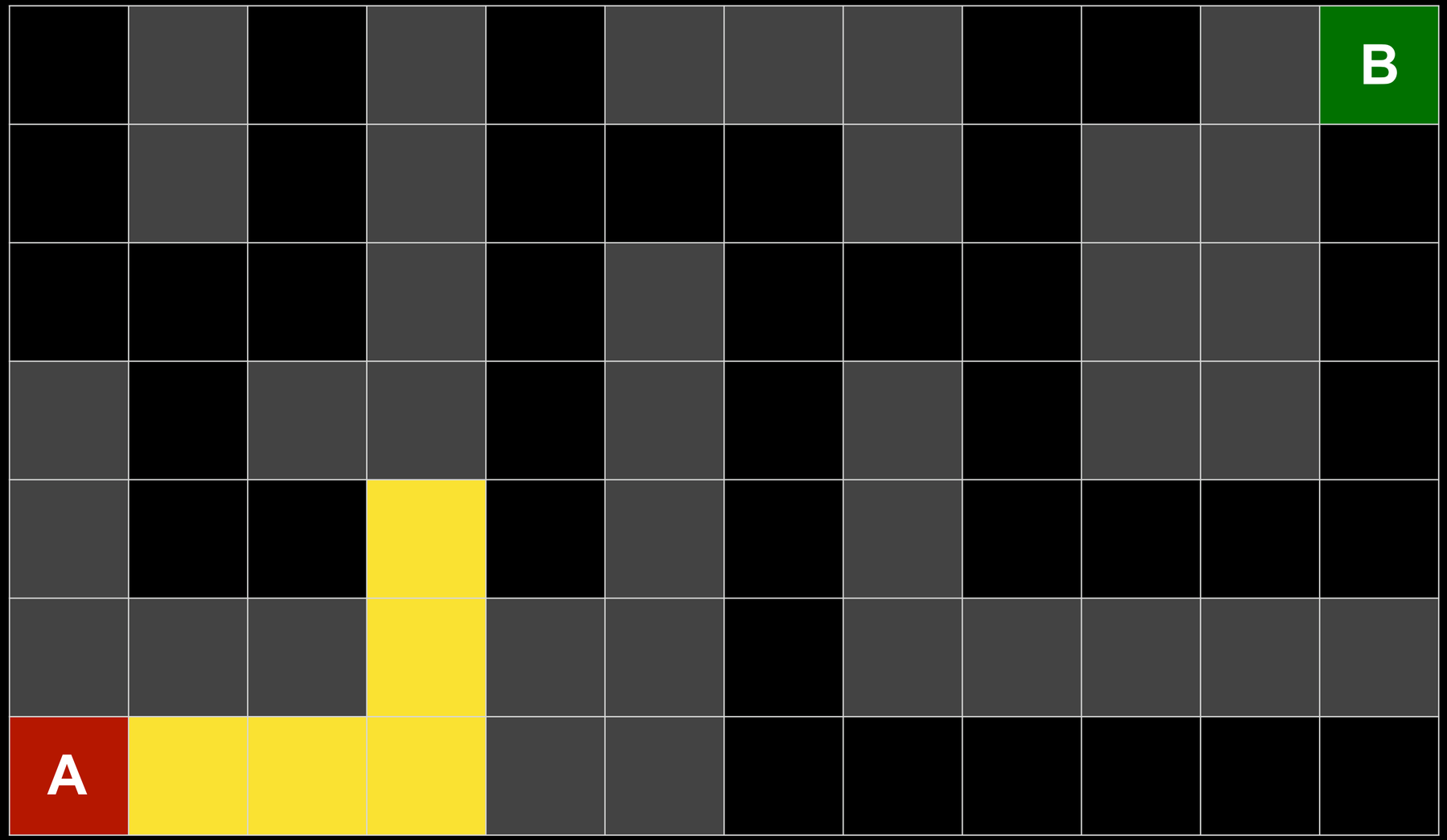


uninformed search

search strategy that uses no problem-specific knowledge

informed search

search strategy that uses problem-specific knowledge to find solutions more efficiently

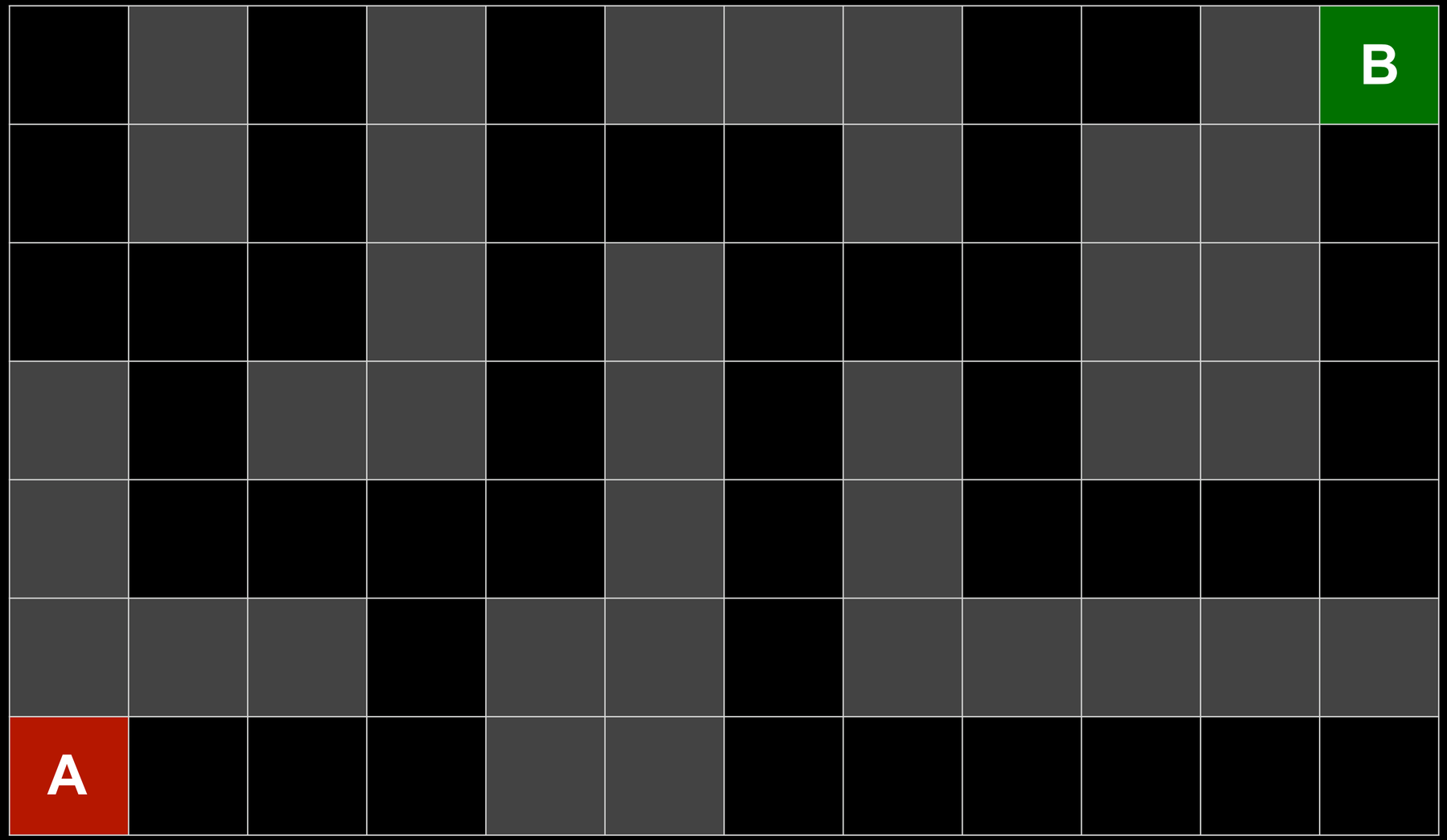
Lecture 1.3

GBFS and A*

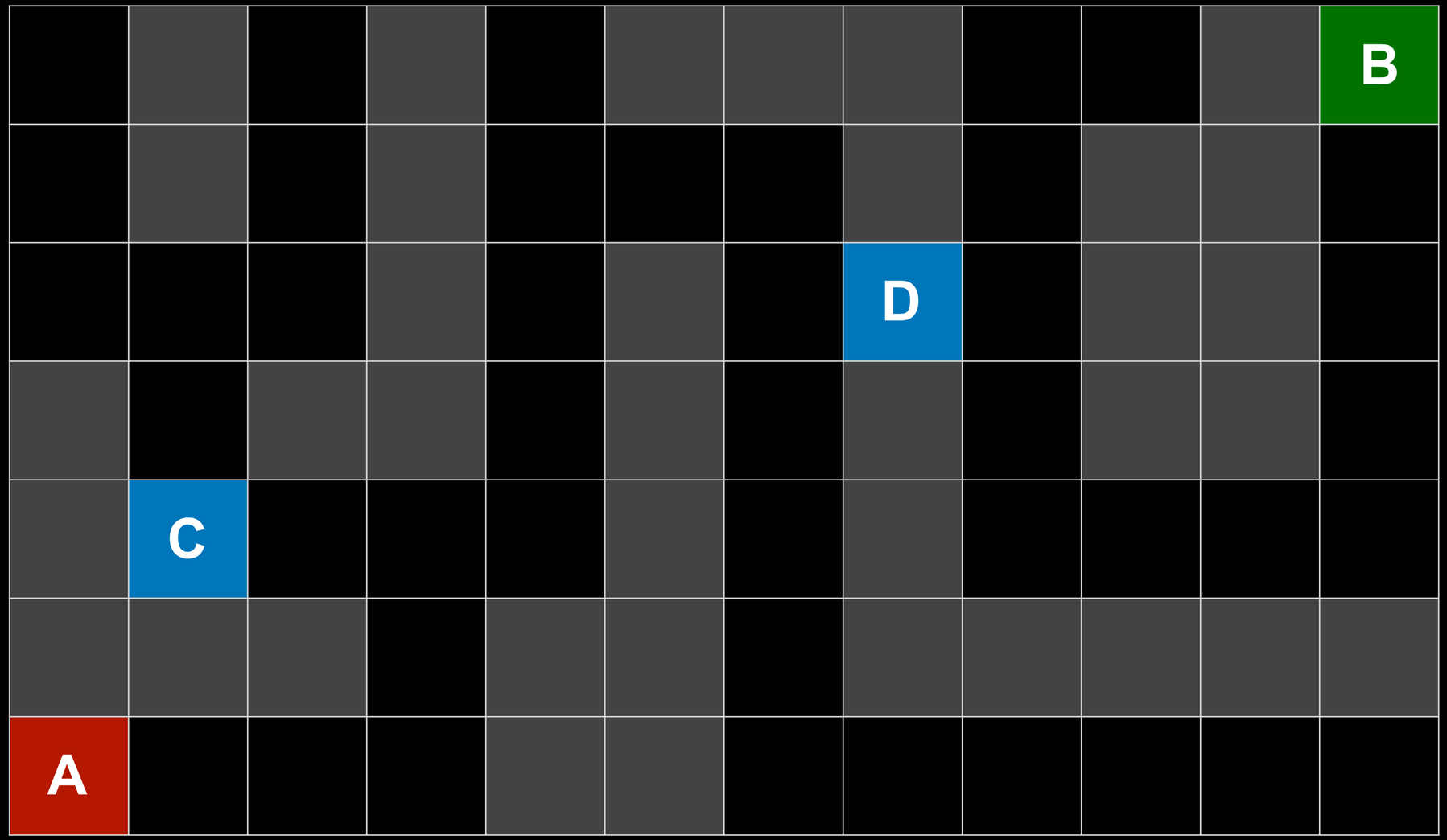
greedy best-first search

search algorithm that expands the node that is closest to the goal, as estimated by a heuristic function $h(n)$

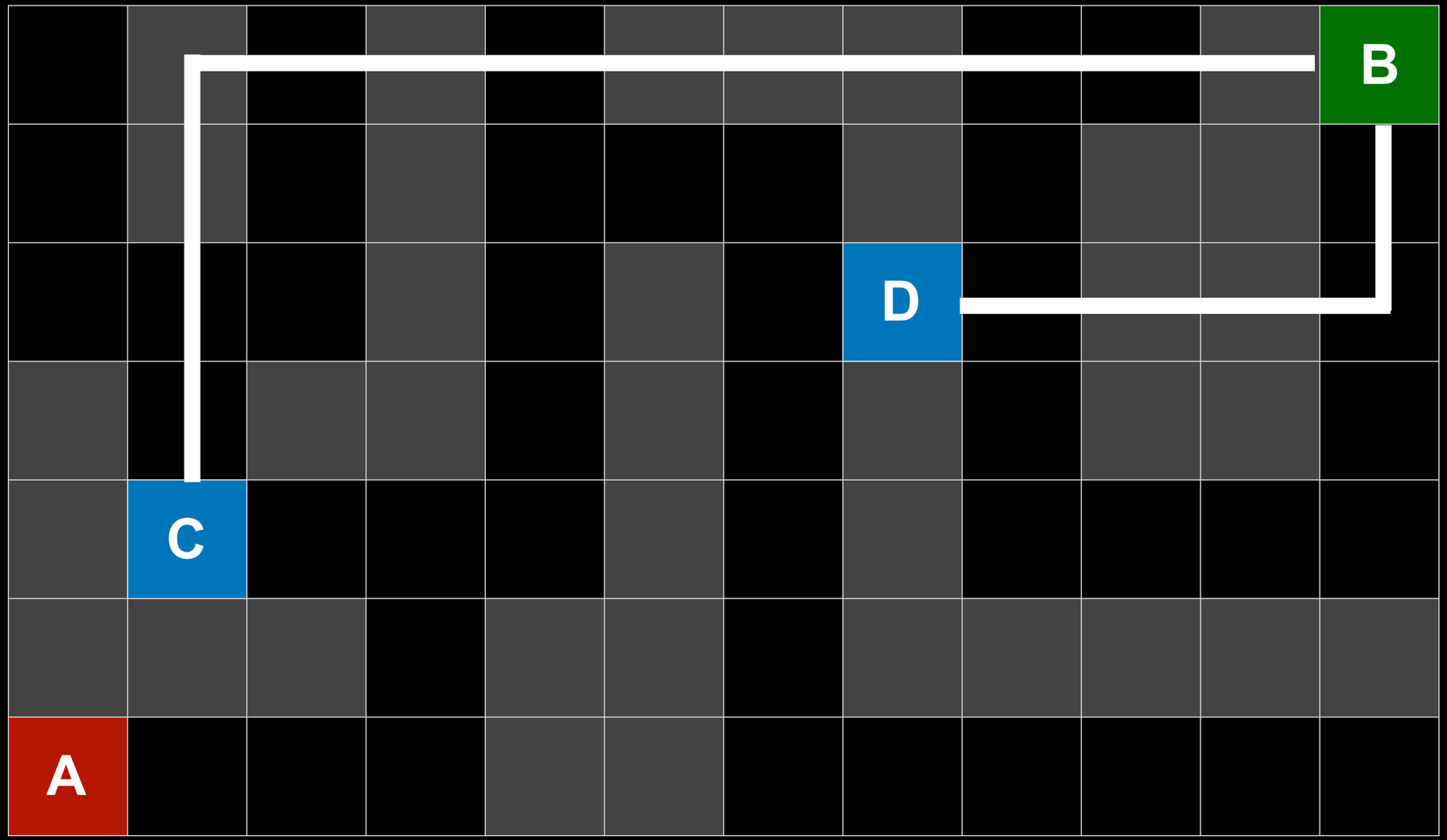
Heuristic function?



Heuristic function?



Heuristic function? Manhattan distance.



Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

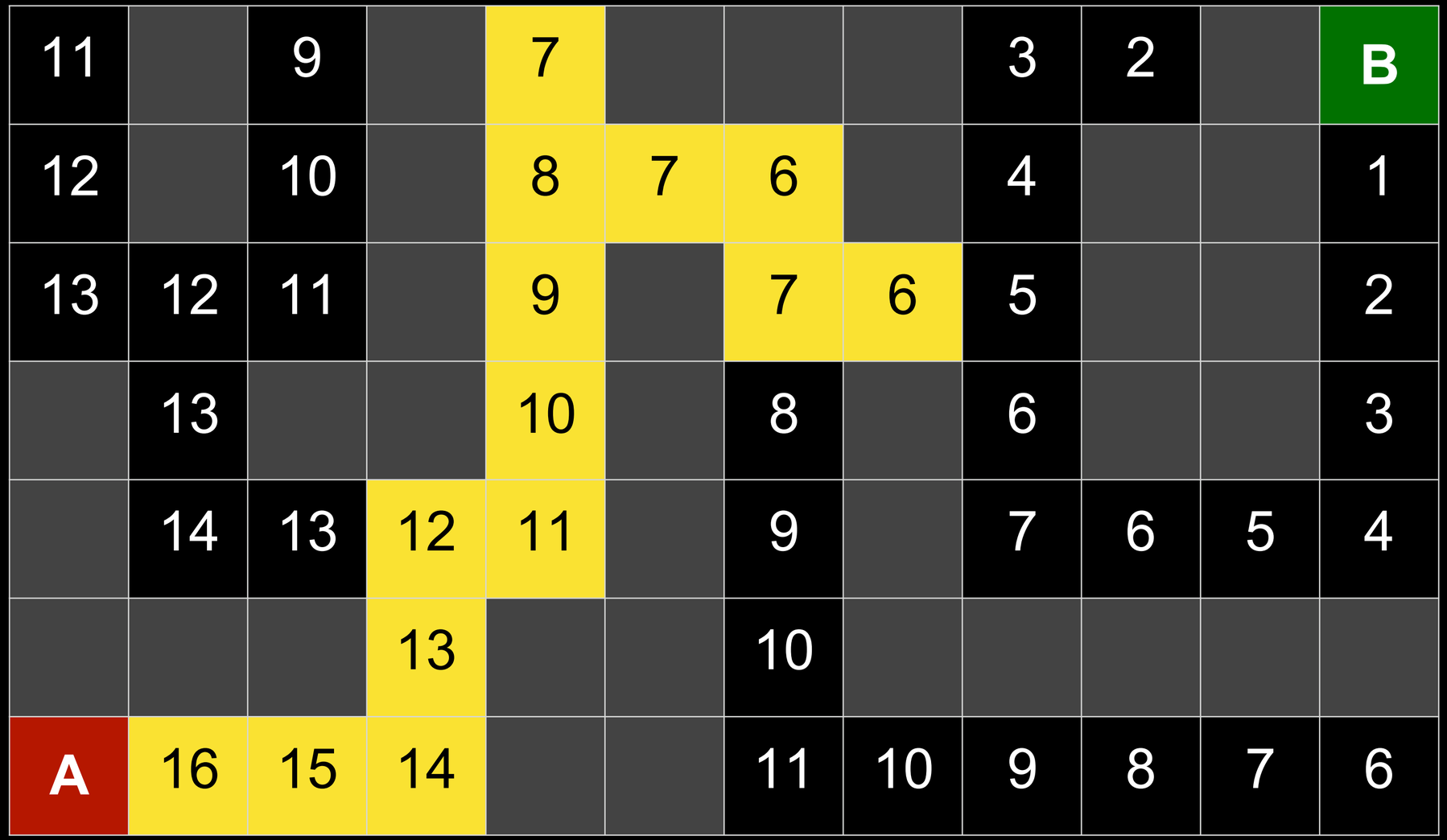
Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

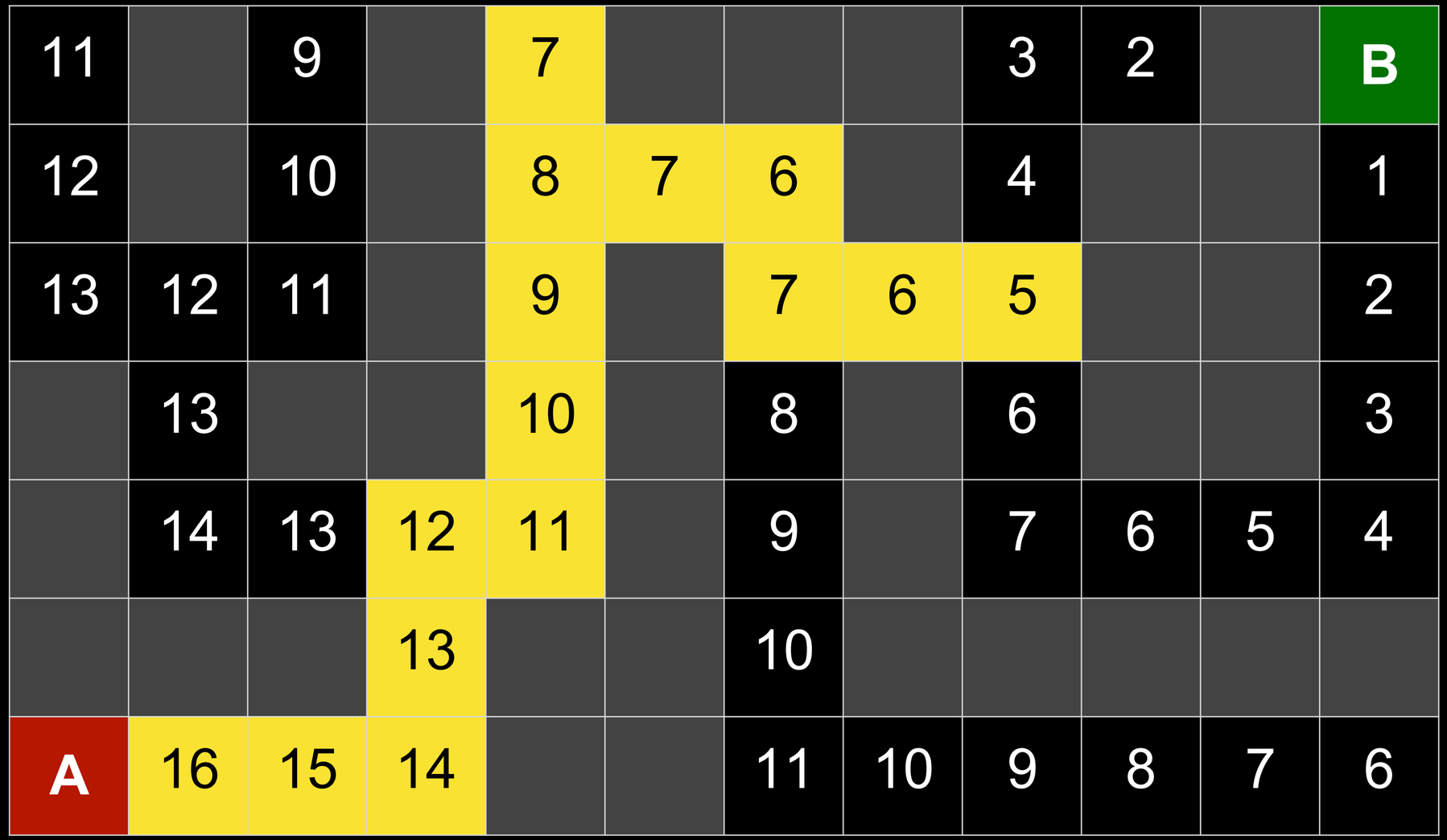
Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

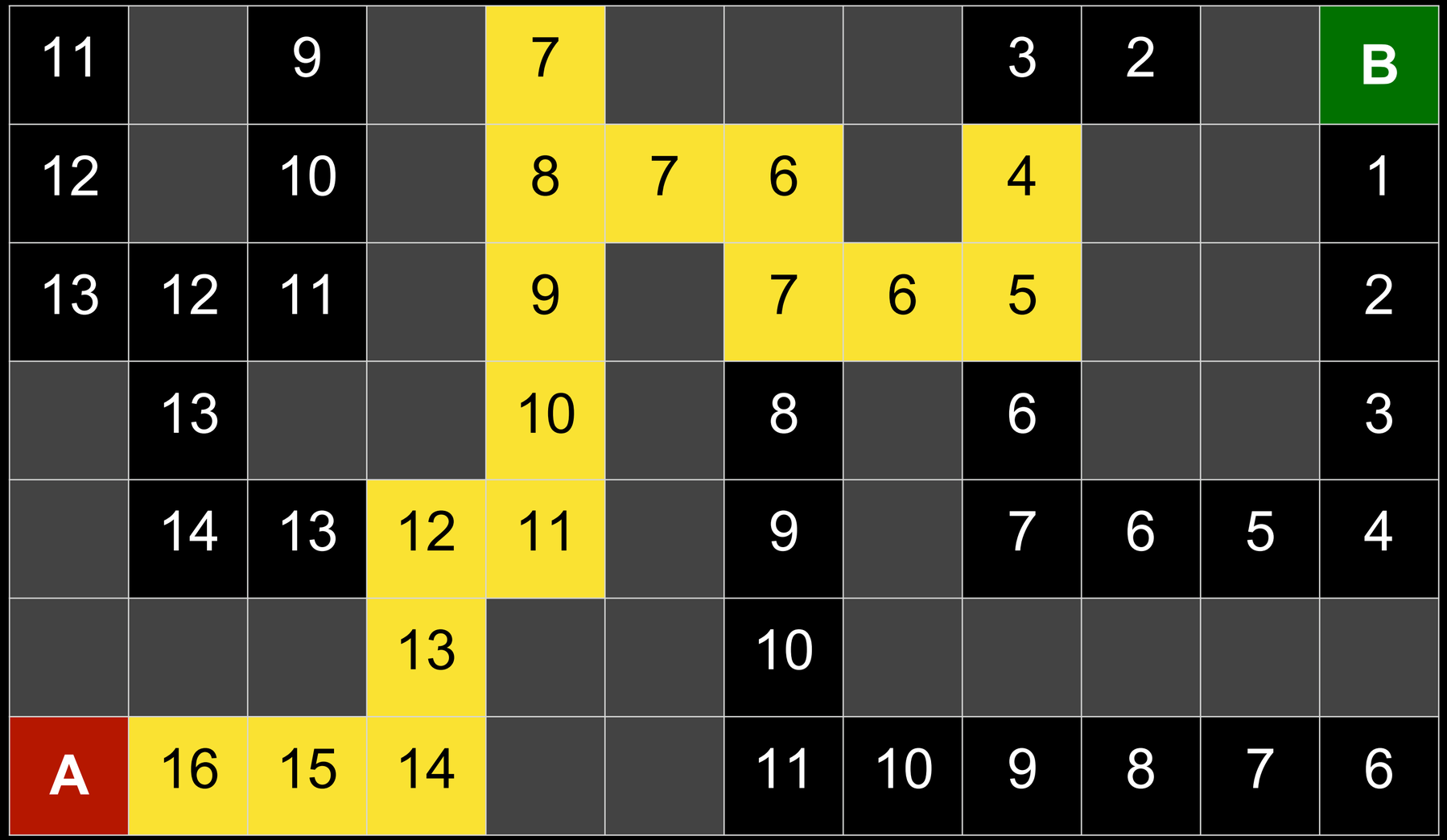
Greedy Best-First Search



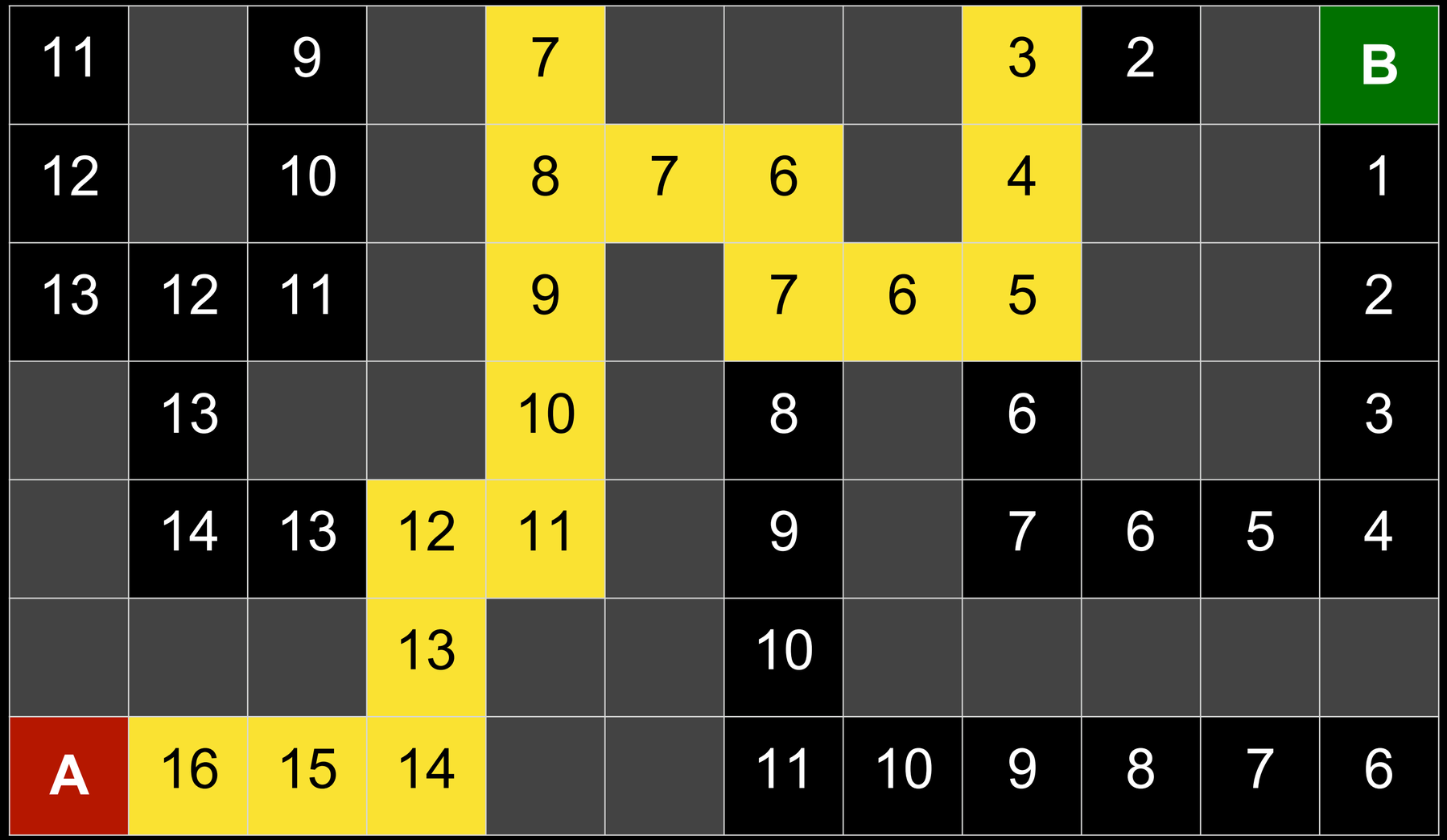
Greedy Best-First Search



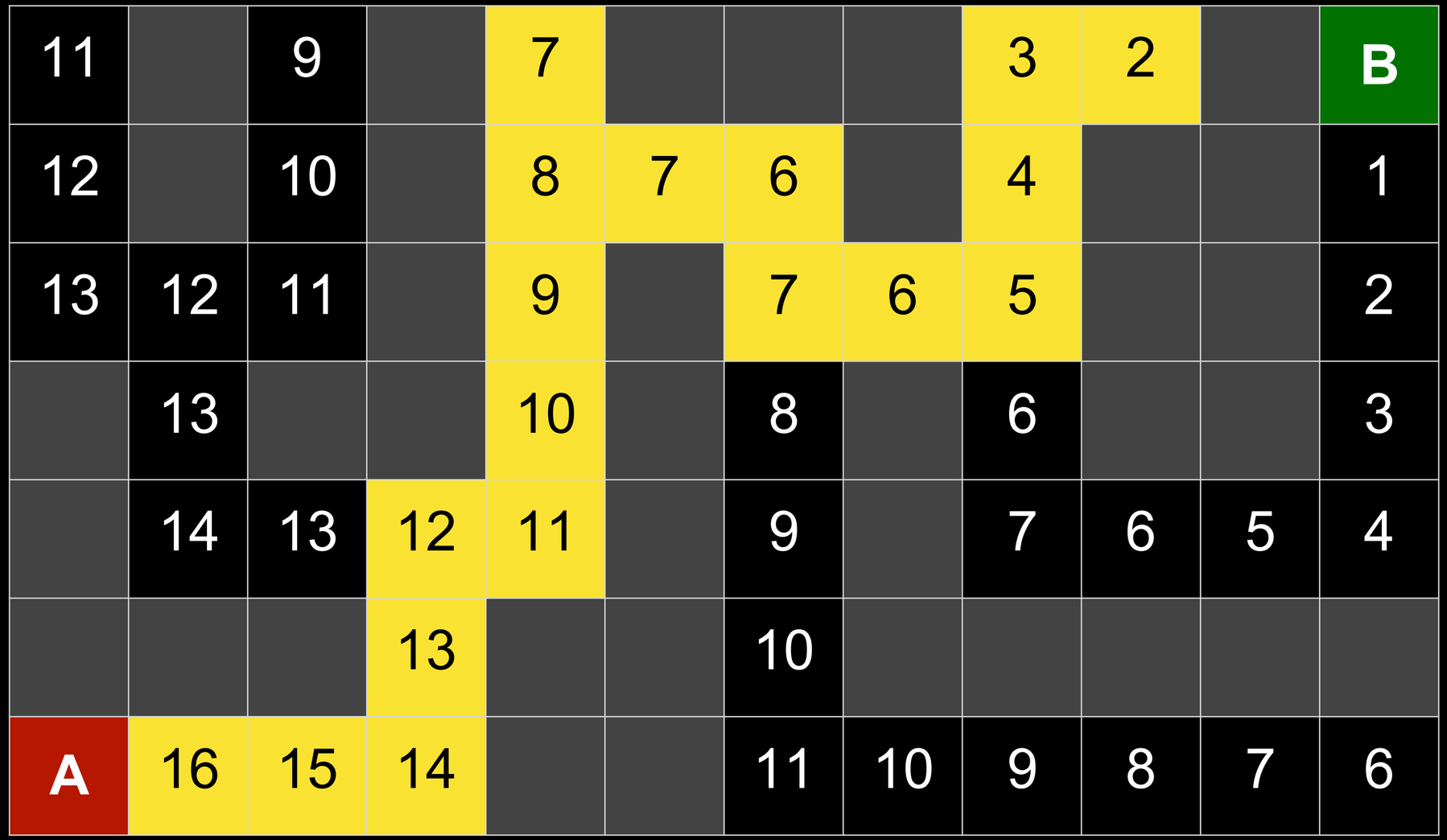
Greedy Best-First Search



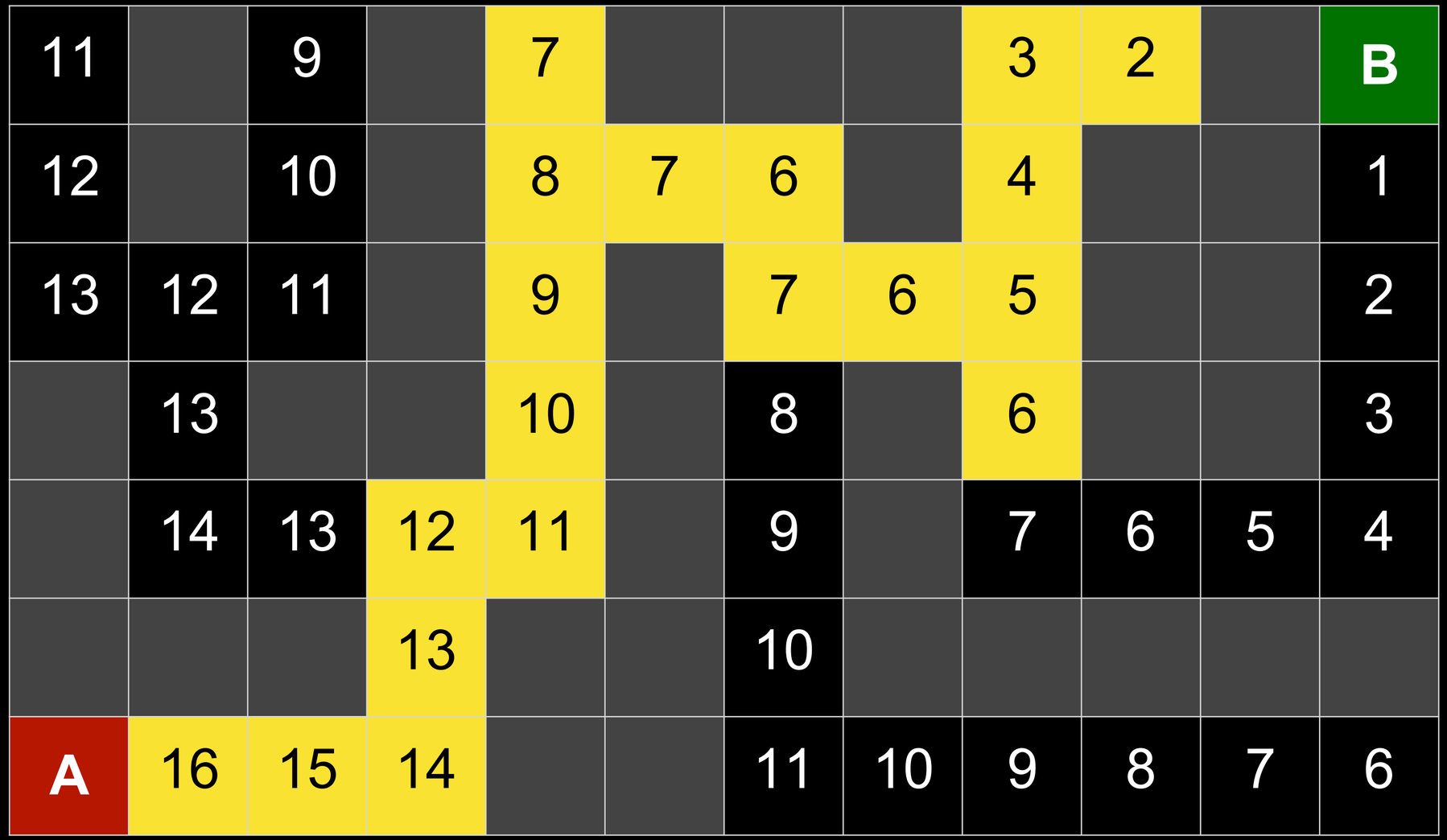
Greedy Best-First Search



Greedy Best-First Search



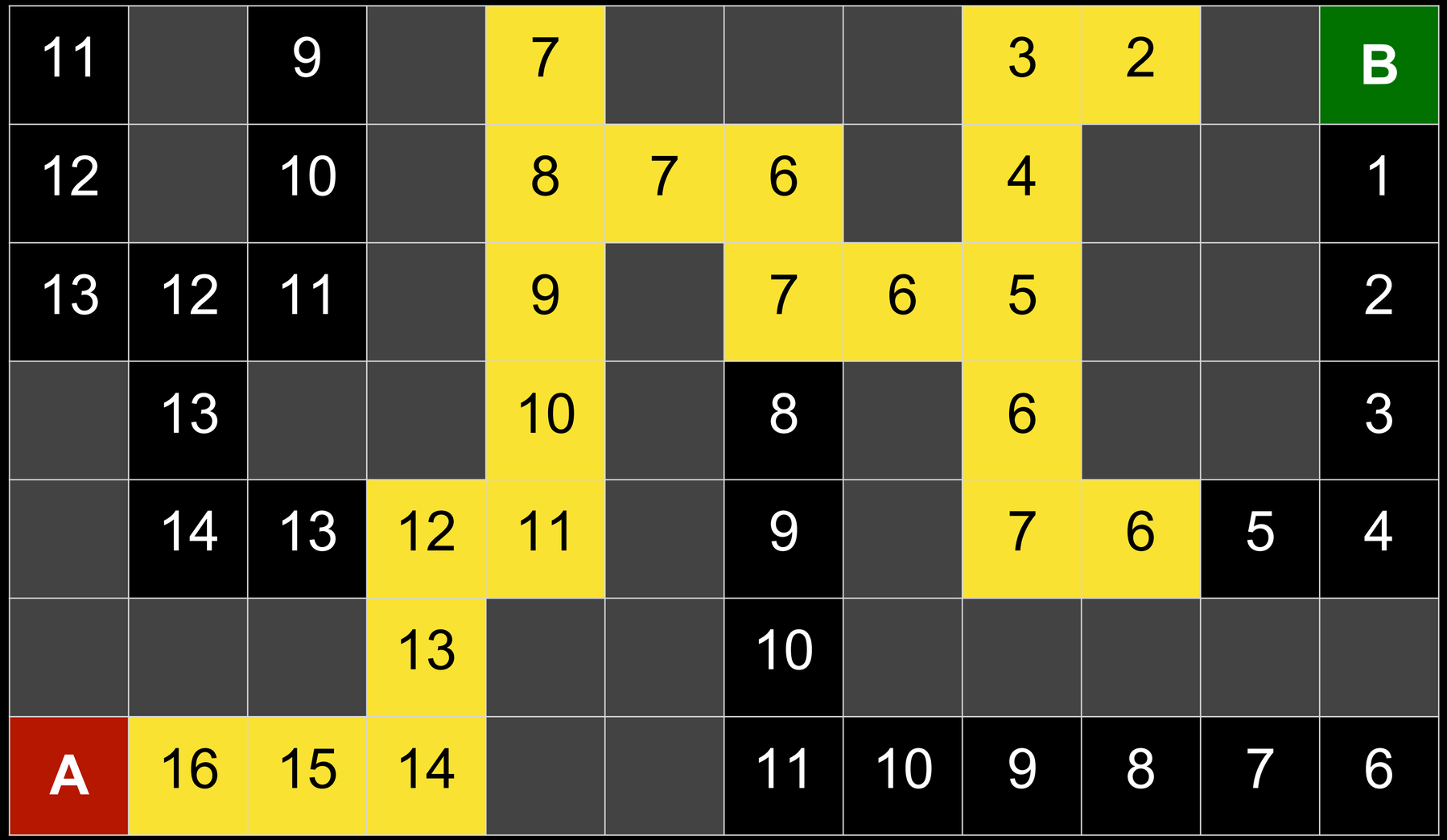
Greedy Best-First Search



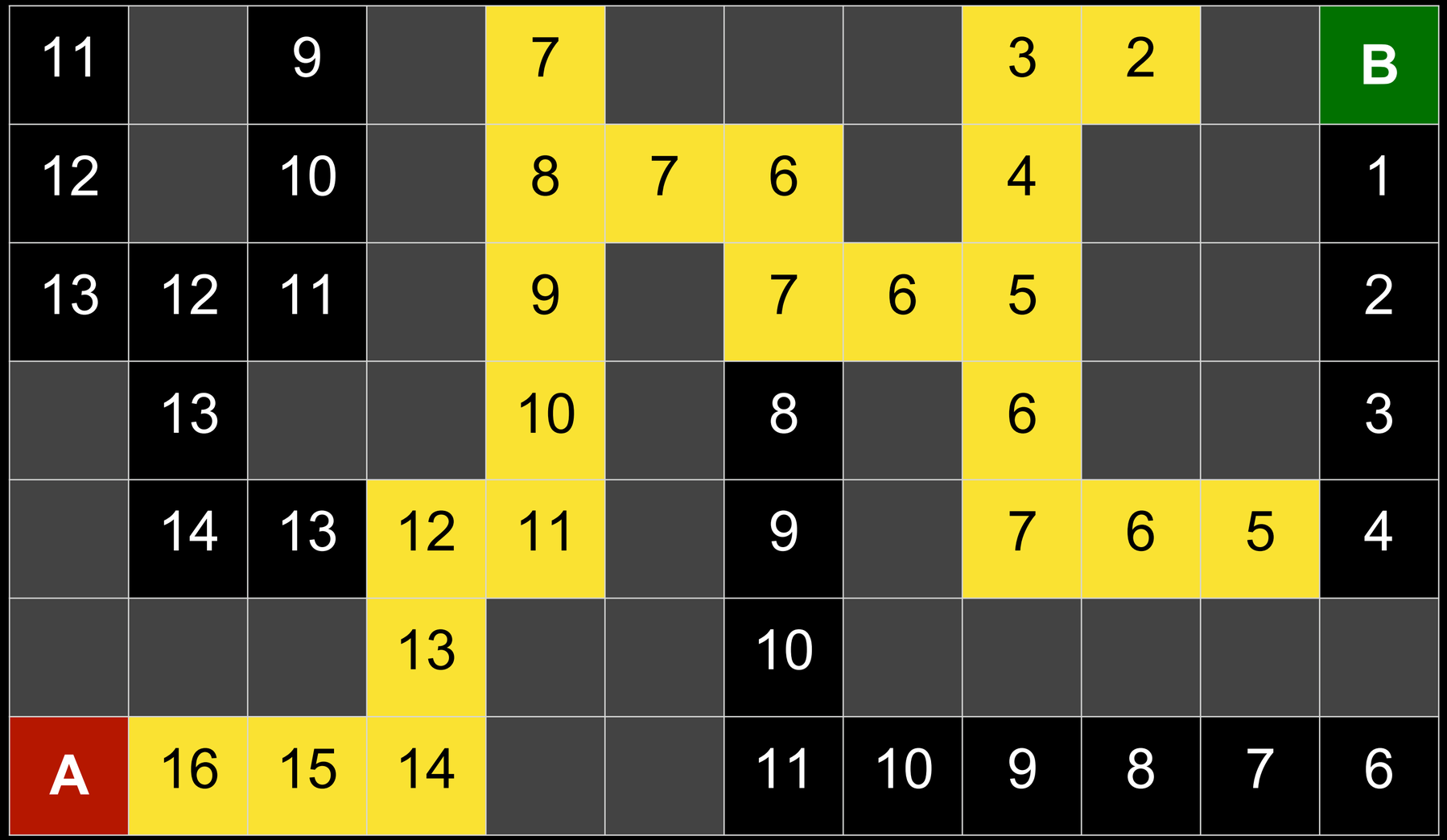
Greedy Best-First Search

11		9		7				3	2		**B**
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search



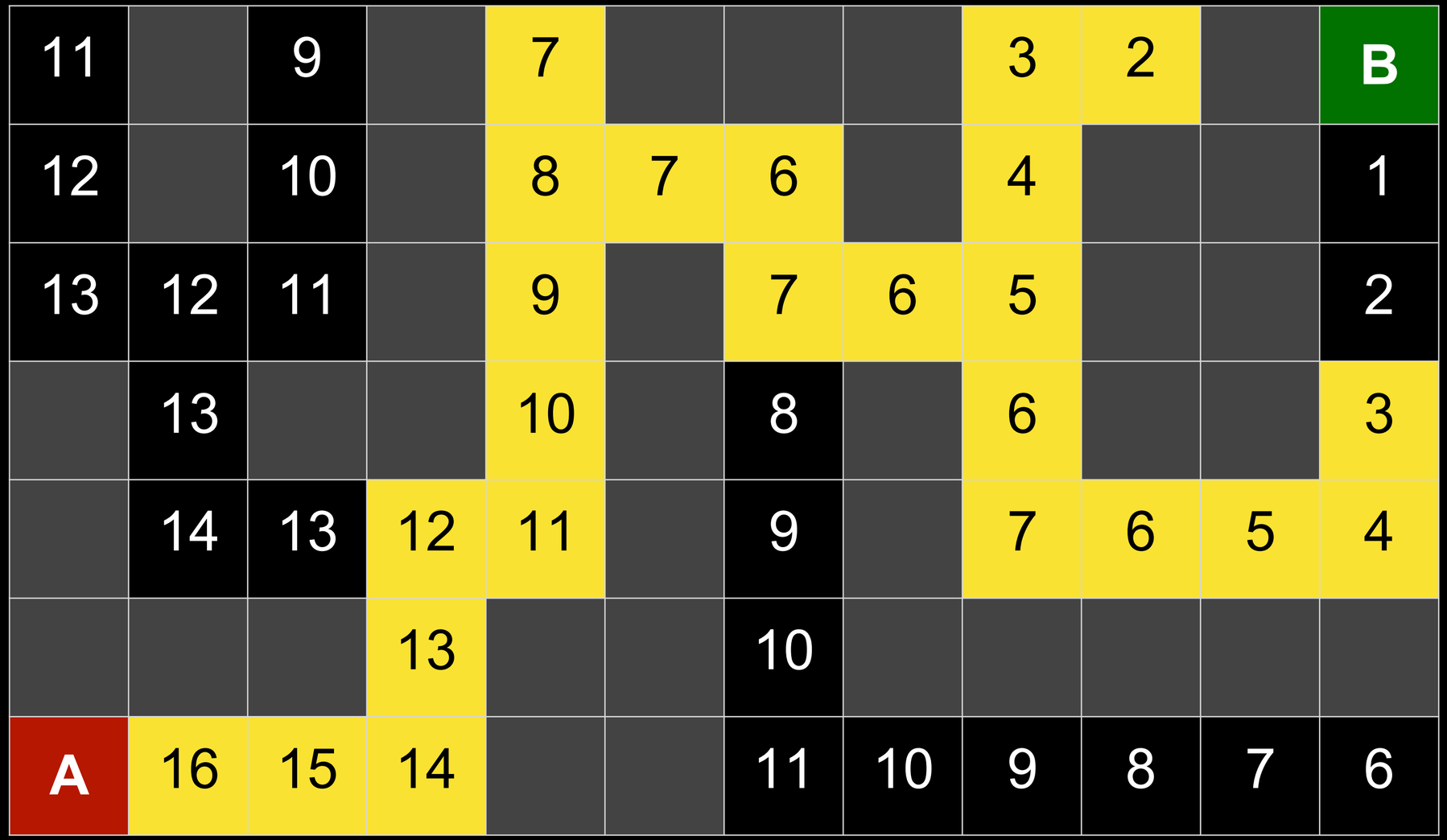
Greedy Best-First Search



Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

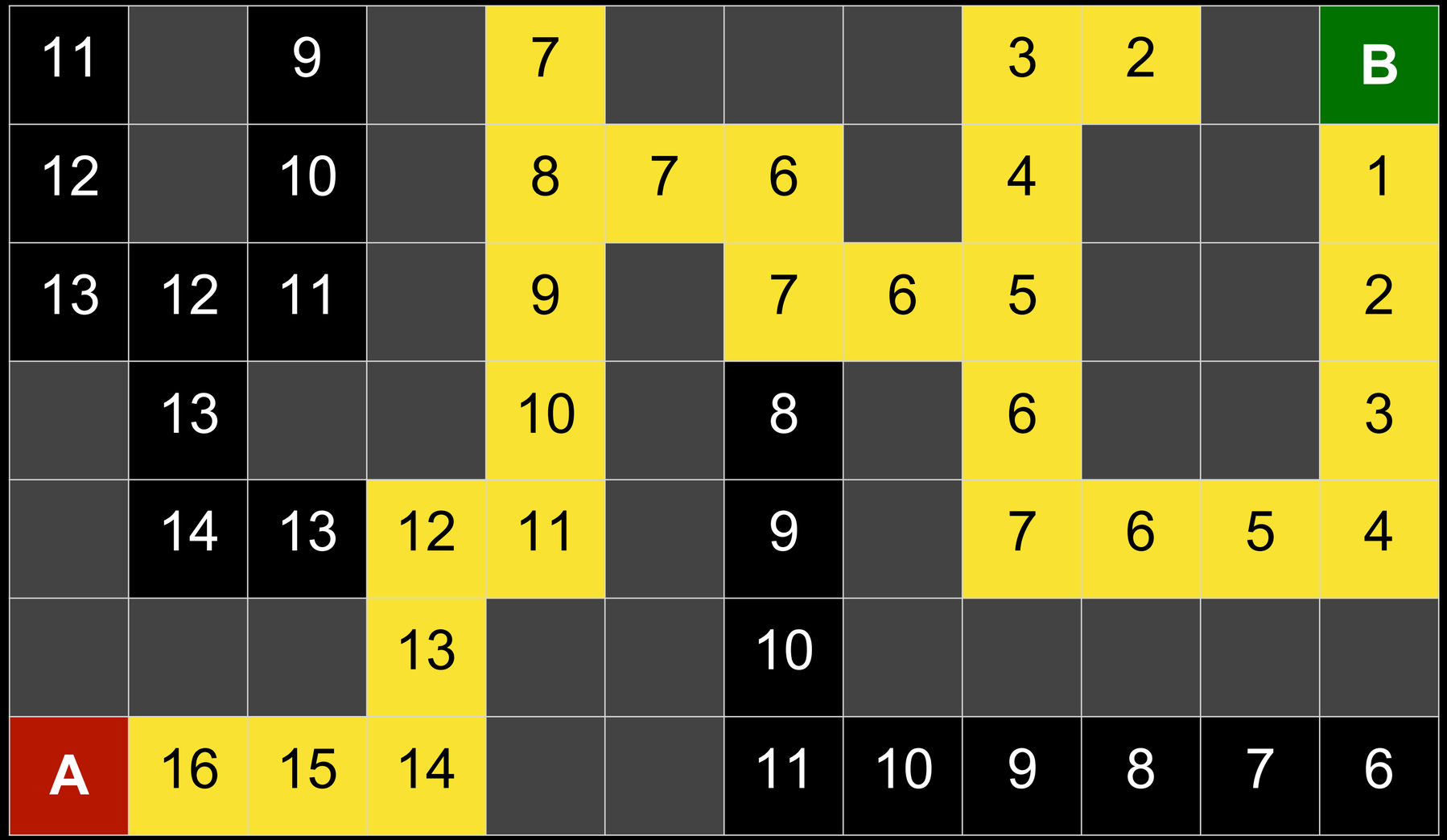
Greedy Best-First Search



Greedy Best-First Search

11		9		7				3	2		B
12		10		8	7	6		4			1
13	12	11		9		7	6	5			2
	13			10		8		6			3
	14	13	12	11		9		7	6	5	4
			13			10					
A	16	15	14			11	10	9	8	7	6

Greedy Best-First Search



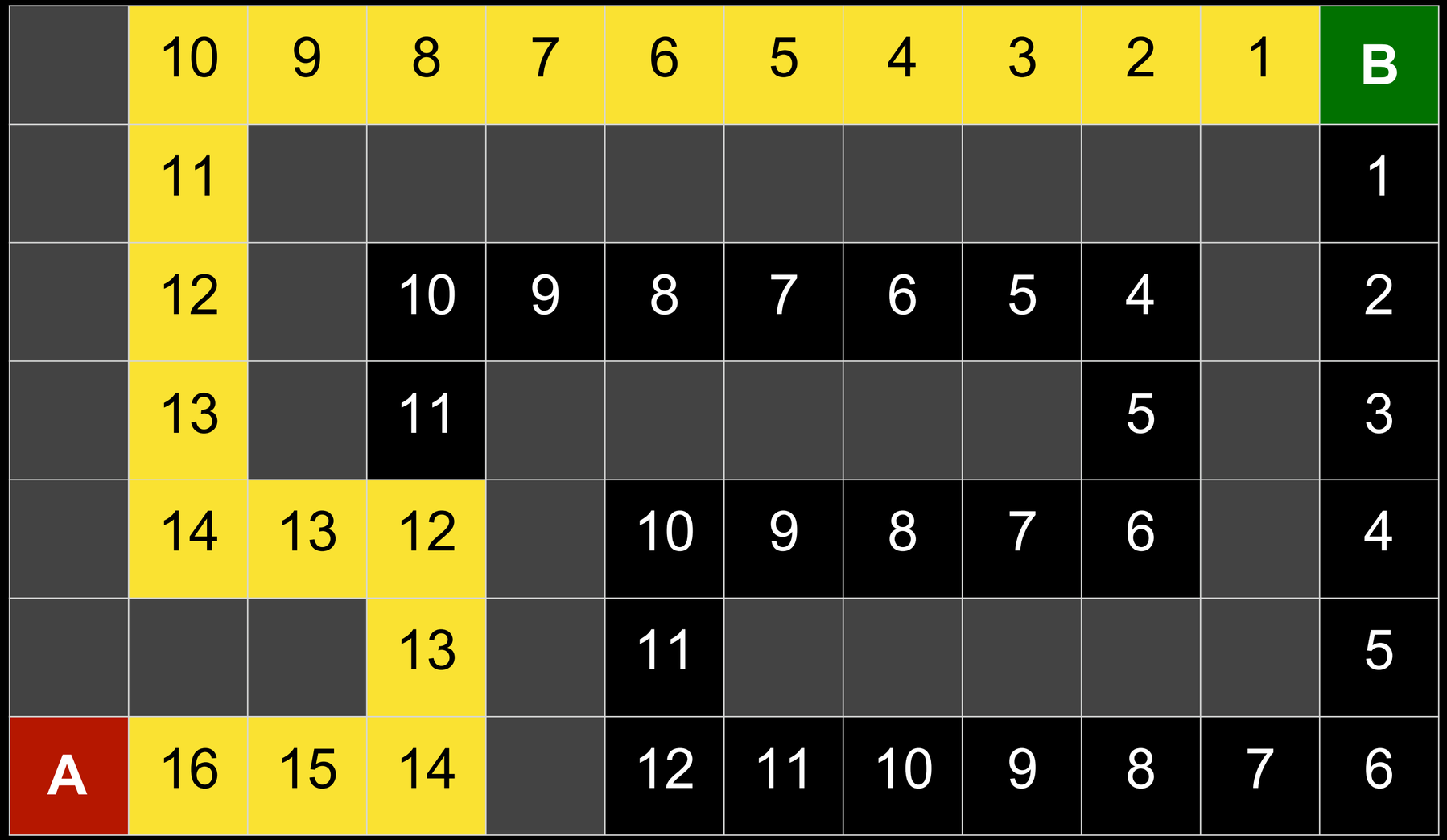
Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Greedy Best-First Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

Greedy Best-First Search



A* search

search algorithm that expands node with lowest value of $g(n) + h(n)$

$g(n)$ = cost to reach node
$h(n)$ = estimated cost to goal

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	16	15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	2+15	14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		10	9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	6+13	5+12		10	9	8	7	6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	**B**
	11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	10	9	8	7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	9	8	7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	8	7	6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	6	5	4	3	2	1	B
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	19+2	1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* Search

	11+10	12+9	13+8	14+7	15+6	16+5	17+4	18+3	19+2	20+1	**B**
	10+11										1
	9+12		7+10	8+9	9+8	10+7	11+6	12+5	13+4		2
	8+13		6+11						14+5		3
	7+14	6+13	5+12		10	9	8	7	15+6		4
			4+13		11						5
A	1+16	2+15	3+14		12	11	10	9	8	7	6

A* search

optimal if

- $h(n)$ is admissible (never overestimates the true cost), and
- $h(n)$ is consistent (for every node n and successor n' with step cost c, $h(n) \leq h(n') + c$)

Thank You!