

AI1110 Assignment 1

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EE22BTECH11208

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Question: 10.13.2.12 An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be atleast 4 successes.

Solution:

Given that probability of failure = x

The probability of success = $2x$

$$\therefore x + 2x = 1 \quad (1)$$

$$\implies 3x = 1 \quad (2)$$

$$\implies x = \frac{1}{3} \quad (3)$$

$$\therefore 2x = \frac{2}{3} \quad (4)$$

Let X be the random variable that represents the number of trials.

Hence, by binomial distribution, we have

$$P(X = x) = {}^nC_x p^{n-x} q^x$$

\therefore Probability of having at least 4 successes

$$= P(X \geq 4) \quad (5)$$

$$= P(X = 4) + P(X = 5) + P(X = 6) \quad (6)$$

$$= {}^6C_4 \frac{1}{3}^2 \frac{2}{3}^4 + {}^6C_5 \frac{1}{3} \frac{2}{3}^5 + {}^6C_6 \frac{2}{3}^6 \quad (7)$$

$$= \frac{15 \times 2^4}{3^6} + \frac{6 \times 2^5}{3^6} + \frac{2^6}{3^6} \quad (8)$$

$$= \frac{31 \times 2^6}{3^6} \quad (9)$$

$$= \frac{31}{9} \left(\frac{2}{3}\right)^4 \quad (10)$$

$$= 0.68 \quad (11)$$