

1. Elementary algebra

P1.1 Simplify $\frac{x^{n+2}}{x^{n-2}} \div \frac{x^{n-2+4}}{x^{n-2}} = x^4$

P1.2 $x^{-1} \times 8 = 2$
 $x^{-1} \times 2^3 = 2$
 $x^{-1} = \frac{2}{2^3} = \frac{1}{4}$
 $x = 4$

P1.3 Calculate the missing value. If $a = 5$ & $b = 10$ then $(a^b)^0 =$
 $= (5^{10})^0 = 1$

P1.4 Calculate $\frac{\sqrt{4x}}{\sqrt{x}} = \frac{2\sqrt{x}}{\sqrt{x}} = 2$

P1.5 $x^2 + (x+1)^2 = (x+2)^2$
 $x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$
 $x^2 - 2x - 3 = 0$
 $(x+1)(x-3) = 0$
 $x_1 = -1$
 $x_2 = 3$

P1.6 Sol. for Inequality: $2^x > 1024$
 $2^x > 2^{10}$
 $x > 10$

2. Functions of one variable

P2.1 Relationship between temps measured in Cels & F. is linear.

Celsius $0^\circ\text{C} \xrightarrow{+100} 100^\circ\text{C}$

Fahrenheit $32^\circ\text{F} \xrightarrow{+180} 212^\circ\text{C}$

$$y = 32 + 1.8x$$

$$\begin{aligned} 32 + 1.8x &= x \\ 32 &= -0.8x \\ x &= -40 \end{aligned}$$

P2.2 $f(x) = 5x + 4$

$y = ?$ if $f(3) = y$

$f(3) = 5 \times 3 + 4 = 19$

P2.3 Invest 10 HUF for 90 days

$i = 2\%$

How much money do you receive 90 days later?

$$10 \times 1.02^{90} = 59.43$$

or $10 \times e^{90 \times 0.02} = 60.49$

P2.3 $x^2 - 4x + 3 = 0$
 $(x-1)(x-3) = 0$

$x_1 = 1$

$x_2 = 3$

3. Calculus \rightarrow sum

$$P_{3.1} \sum_{i=1}^{\infty} \frac{12}{6^i} = ?$$

$$a_n = 12 \times \frac{1}{6^i}$$

$$a = 12 \quad b = \frac{1}{6}$$

$$\sum_{i=1}^{\infty} \frac{12}{6^i} = \frac{12 \times \frac{1}{6}}{1 - \frac{1}{6}} = \frac{\frac{2}{1}}{\frac{5}{6}} = \frac{6 \times 2}{5} = \frac{12}{5}$$

$P_{3.3} \rightarrow$ slope $f(x) = x^5 - 8$ at $x = -3$

$$f'(x) = 5x^4$$

$$f(-3) = 5(-3)^4 = 81 \times 5 = 405$$

$P_{3.5} \rightarrow$ sec deriv $\frac{d^2}{dx^2} 4x^4 + 4x^2 = ?$

$$d_1 = 16x^3 + 8x$$

$$d_2 = 48x^2 + 8$$

$P_{3.7} f(x) = 3x^2 - 5x + 2 \rightarrow d_1 = 6x - 5$

$$\text{stationary point} = \frac{5}{6} \quad d_2 \geq 6 \quad \begin{matrix} 6x - 5 = 0 \\ 6x = 5 \\ x = \frac{5}{6} \end{matrix}$$

$$d_2 = 6 - \text{local min}$$

	$-\infty$	$5/6$	$5/6$	$+\infty$
$f(x)$	\nearrow	local min	\nearrow	
$f'(x)$	$-$	0	$+$	
$f''(x)$	$+$	$+$	$+$	

$P_{3.9} f(x, y) = \ln(x - y)$

for what comb. x & y is defined?

$$f(x, y) = \ln(x - y) \rightarrow (x - y) > 0$$

\rightarrow limit (2) Baribaulton Davl

$$P_{3.2} \lim_{x \rightarrow 1} \frac{6^{1-x}}{x-1} = \frac{6^{1-x} \times 6^x}{x \times 6^x} = \frac{6}{x \times 6^x} = \frac{6}{1 \times 6^1} = 1$$

$P_{3.4} \rightarrow$ deriv $\frac{d}{dx} \frac{x^3 + 2x - 1}{x - 2} =$

$$= \frac{(3x^2 + 2)(x - 2) - (x^3 + 2x - 1) \cdot 1}{(x - 2)^2} =$$

$$= \frac{3x^2 - 6x^2 - 2x - 4 - x^3 - 2x + 1}{x^2 - 4x + 4} =$$

$$= \frac{-x^3 - 3x^2 - 4x - 3}{x^2 - 4x + 4}$$

$P_{3.6} \rightarrow$ der $\frac{d}{dx} \frac{\ln x}{e^x} = \frac{x \times e^x - \ln x \cdot e^x}{(e^x)^2} =$

$$= \frac{x - \ln x}{e^x}$$

$P_{3.8} f(x, y) = x^2 + y^3$

$$f(2, 3) = ?$$

$$f(x, y) = x^2 + y^3$$

$$f(2, 3) = 2^2 + 3^3 = 4 + 27 = 31$$

\rightarrow deriv $P_{3.10} \frac{\partial}{\partial x} x^5 + xy^3 = 5x^4 + y^3$

\rightarrow local max or min

$P_{3.11} f(x, y) = x^2 y^2 + 10$

$$f'_x = 2xy^2 \quad f'_y = x^2 2y$$

$$f''_{xx} = 2y^2 \quad f''_{yy} = x^2 \times 2$$

local min for

P_{3,12} $\max x^2 y^2$
 $x + y = 10$

$$\left. \begin{array}{l} 2xy^2 - N = 0 \\ 2yx^2 - N = 0 \end{array} \right\} \begin{array}{l} 2xy^2 = 2yx^2 \\ x = y \end{array}$$

$$\left. \begin{array}{l} x + y - 10 = 0 \\ x = y \end{array} \right\} \begin{array}{l} x + y = 10 \\ x = y = 5 \end{array}$$

4. Linear algebra

Pr 4.1

$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 2 \times 1 + 6 \times 2 & 1 \times 2 + 8 \times 6 & 7 \times 2 + 6 \times 2 \\ 5 \times 1 + 2 \times 2 & 1 \times 5 + 8 \times 1 & 7 \times 5 + 1 \times 2 \\ 1 \times 1 + 9 \times 2 & 1 \times 1 + 8 \times 8 & 7 \times 1 + 2 \times 9 \end{bmatrix} = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$$

Pr 4.2

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$B \times A = \begin{bmatrix} 1 \times 2 + 9 \times 4 + 1 \times 1 & 1 \times 2 + 9 \times 6 + 1 \times 3 \\ 2 \times 2 + 1 \times 4 + 2 \times 1 & 2 \times 2 + 1 \times 6 + 2 \times 3 \end{bmatrix} \quad BA = \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$$

Pr 4.3 Transpose

$$\begin{bmatrix} 7,1 & 8,1 & 4,7 \\ 2 & 7,8 & 1,1 \\ 4 & 4,44 & 0 \end{bmatrix}^T = \begin{bmatrix} 7,1 & 2 & 4 \\ 8,1 & 7,8 & 4,44 \\ 4,7 & 1,1 & 0 \end{bmatrix}$$

Pr 4.4 determinant

$$\begin{bmatrix} 1 & 8 \\ 2 & 8 \end{bmatrix} = 1 \times 8 - 8 \times 2 = -10$$

5. Probability theory

Pr 5.1

d_1	1	2	3	4	5	6
d_2	1	1	2	5	4	1

Pr 5.2 Drug test

Pr 5.3 Drug test

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We can take data from 5.2 pr.

	Drug		
Drug user	$1 \times 99\%$	$1 \times 1\%$	$\approx 0,99\% \quad 0,01\%$
Not a drug user	$99 \times 5\%$	$99 \times 99,5\%$	$\approx 0,495\% \quad 98,50\%$

$$\frac{0,99\%}{0,99\% + 0,495\%} \approx \frac{1}{3} \approx 66,7\%$$