Matice

$$\begin{aligned} 1x + 3y &= 4 \\ 2x + 3y &= 5 \end{aligned} \quad - \quad \underbrace{\begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}}_{A} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \underbrace{\begin{pmatrix} 4 \\ 5 \end{pmatrix}}_{b}$$

$$\boxed{x = 1}$$

$$1 + 3y = 4$$

$$3y = 3 \quad /:3$$

$$\boxed{y = 1}$$

$$3^{-1} = \frac{1}{3}$$

$$\cancel{x \cdot A = b \quad /:A}$$

$$\cancel{x = \frac{b}{A}}$$

$$3x = 9 \quad /\cdot 3^{-1}$$

$$x \cdot A = b \quad /\cdot A^{-1}$$

$$\underbrace{3^{-1} \cdot 3}_{1} \cdot x = \underbrace{3^{-1} \cdot 9}_{\frac{1}{3} \cdot 9 = 3} \rightarrow \underline{x = 3}$$

Př. 2

$$x + 2y + 3z = 4$$

$$4x - 2y - z = 1$$

$$y - z = 2$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & -3 & -1 \\ 0 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$$

$$Ax = b \quad / \cdot A^{-1}$$

$$I \leftarrow \boxed{A^{-1}A}\, x = A^{-1} b$$

$$I . x = A^{-1} b$$

$$\boxed{x = A^{-1} b}$$

$$x . A^{T} = b$$

$$x . A^{T} (A^{T})^{-1} = b (A^{T})^{-1}$$

$$x . I = b . (A^{T})^{-1}$$

$$\boxed{x = b . (A^{T})^{-1}}$$

$$(A^{-1} . A = I)$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

pouze pro čtvercové matice

Def. $\quad A^{-1} = \dfrac{1}{\det(A)} \cdot \operatorname{adj} A$

$\det(A) = 0 \rightarrow$ matice nemá inverzní matici (singulární)

$A^{m\times n} + B^{m\times n}$

$A - B$

stejná velikost

$A^{2\times 2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$\vec{B}^{2\times 3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$C = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

2×3

$B^{2\times 3} - C^{2\times 3} = \begin{pmatrix} 1-2 & 0-1 & 0-0 \\ 0-0 & 1-1 & 0-2 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

skalár = číslo

$4 \cdot A = \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix}$

matice

$\vec{v} = \begin{pmatrix} 1 & 2 \end{pmatrix}$

násobení matic

$$A^{3\times 2} \cdot B^{2\times 3} = C^{3\times 3}$$

$$C^{1\times 4} \cdot D^{5\times 2} \quad \times$$

$$E^{2\times 2} \cdot F^{2\times 2} = G^{2\times 2}$$

$$E = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$F = \begin{pmatrix} 4 & 5 \\ 6 & -1 \end{pmatrix}$$

$$E \cdot F = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 4 & 5 \\ 6 & -1 \end{pmatrix} = \begin{pmatrix} 1\cdot 4 + 2\cdot 6 & 1\cdot 5 + 2\cdot(-1) \\ 0\cdot 4 + 3\cdot 6 & 0\cdot 5 + 3\cdot(-1) \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 3 \\ 18 & -3 \end{pmatrix}$$

$$E = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$$

$$F = \begin{pmatrix} 4 & 5 \\ 6 & -1 \end{pmatrix}$$

$E \cdot F$

$$\begin{array}{cc|c|c|}
 & & 4 & 5 \\
 & & 6 & -1 \\
\hline
1 & 2 & 16 & 3 \\
\hline
0 & 3 & 18 & -3 \\
\hline
\end{array}$$

$E \cdot F \times F \cdot E$

$F \cdot E$

$$\begin{array}{cc|c|c|}
 & & 1 & 2 \\
 & & 0 & 3 \\
\hline
4 & 5 & 4 & 23 \\
\hline
6 & -1 & 6 & 9 \\
\hline
\end{array}$$

$$E \cdot F = \begin{pmatrix} 16 & 3 \\ 18 & -3 \end{pmatrix} \qquad F \cdot E = \begin{pmatrix} 4 & 23 \\ 6 & 9 \end{pmatrix}$$

Definice $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\det(A) = a \cdot d - b \cdot c$$

$$\operatorname{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$$

$$\boxed{A^{-1} = \frac{1}{ad - b \cdot c} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}}$$

$$1x + 3y = 4$$
$$2x + 3y = 5$$

$$\rightarrow \underset{1\times 2}{(x \quad y)} \cdot \underset{2\times 2}{\begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix}}^{T} = \underset{1\times 2}{(4 \quad 5)}$$

$$A^T = \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix}$$

$\boxed{\text{1. způsob}}$

$$\det(A^T) = 1.3 - 2.3 = -3$$

$$\operatorname{adj}(A^T) = \begin{pmatrix} 3 & -2 \\ -3 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det(A^T)} \cdot \operatorname{adj}(A^T)$$

$$A^{-1} = \frac{1}{-3} \cdot \begin{pmatrix} 3 & -2 \\ -3 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1 & 2/3 \\ 1 & -1/3 \end{pmatrix}$$

$$x = b \cdot (A^T)^{-1}$$

$$(x \quad y) = (4 \quad 5) \cdot \begin{pmatrix} -1 & 2/3 \\ 1 & -1/3 \end{pmatrix}$$

$$(x \quad y) = (-4+5 \quad 8/3 - 5/3)$$

$$(x \quad y) = (1 \quad 1)$$

$1x + 3y = 4$
$2x + 3y = 5$ $\rightarrow$

$$\begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$\boxed{\text{2. způsob}}$

$$\det(A) = 1.3 - 2.3 = -3$$

$$\operatorname{adj}(A) = \begin{pmatrix} 3 & -3 \\ -2 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \cdot \operatorname{adj}(A)$$

$$A^{-1} = \frac{1}{-3} \cdot \begin{pmatrix} 3 & -3 \\ -2 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1 & 1 \\ 2/3 & -1/3 \end{pmatrix}$$

$$x = A^{-1} \cdot b$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 2/3 & -1/3 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1.4 + 1.5 \\ 2/3 \cdot 4 - 1/3 \cdot 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$x+y+z=1$$
$$x-y+z=2$$

$$z=t, \quad t\in\mathbb{R}$$

$$x+y=1-t$$
$$x-y=2-t$$

$$A \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \longrightarrow A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$x_1 - 2x_2 + 3x_3 = 9$$
$$-x_1 + 3x_2 - x_3 = -6$$
$$2x_1 - 5x_2 + 5x_3 = 17$$

$$\begin{array}{ccccc} X & \cdot & A & & b \\ \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} & \cdot & \begin{pmatrix} 1 & -2 & 3 \\ -1 & 3 & -1 \\ 2 & -5 & 5 \end{pmatrix} & = & \begin{pmatrix} 9 & -6 & 17 \end{pmatrix} \end{array}$$

$$X \cdot \underset{1}{A^T} = b \quad | \cdot (A^T)^{-1}$$

$$X = b \cdot (A^T)^{-1}$$

$$x + 3y = 4$$
$$2x + 3y = 5$$

$$\overbrace{(x\ \ y)}^{x} \cdot A^T = b$$
$$x = b \cdot (A^T)^{-1}$$

1×2 $\quad$ 2×2 $\quad$ T

$$\cancel{\begin{vmatrix} x & y \end{vmatrix}} \cdot \begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} x+2y & 3x+3y \end{pmatrix} = \begin{pmatrix} 4 & 5 \end{pmatrix}$$

		1	3
	+	2	3
x	y	x+2y	3x+3y

$$\begin{pmatrix} x & y \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 5 \end{pmatrix}$$

$$x + 3y = 4$$
$$2x + 3y = 5$$

$$\begin{pmatrix} 1 & 3 \\ 2 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

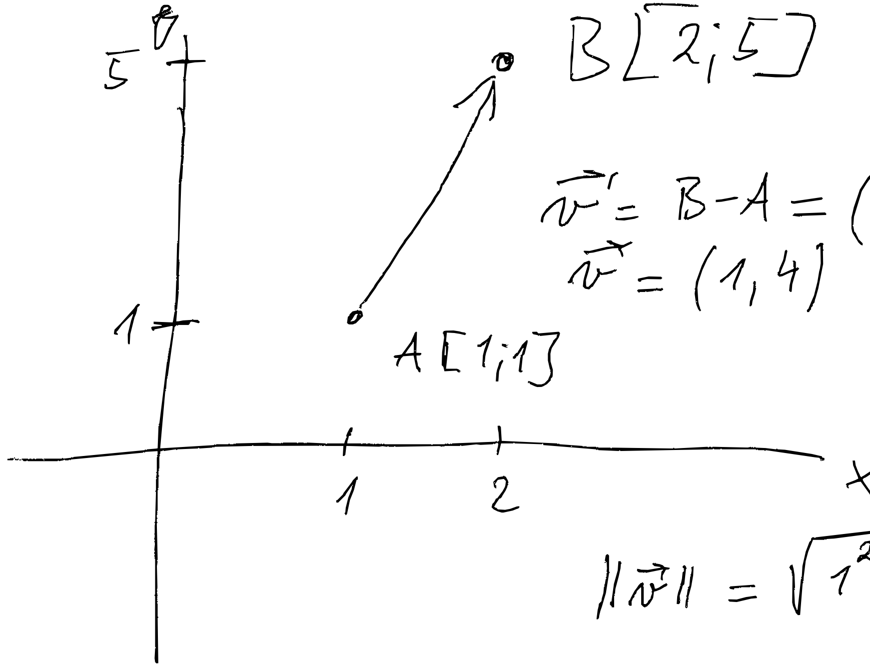
$$\begin{array}{c|c|c}
 & \begin{matrix} x \\ y \end{matrix} & \\
\hline
1 \quad 3 & x + 3y & \\
\hline
2 \quad 3 & 2x + 3y & \\
\hline
 & &
\end{array}$$

$$x + 3y = 4$$
$$2x + 3y = 5$$

$$\begin{pmatrix} 1 & -2 & 3 \\ -1 & 3 & -1 \\ 2 & -5 & 5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \\ 17 \end{pmatrix}$$

$$A \cdot x = b \quad | \cdot A^{-1}$$

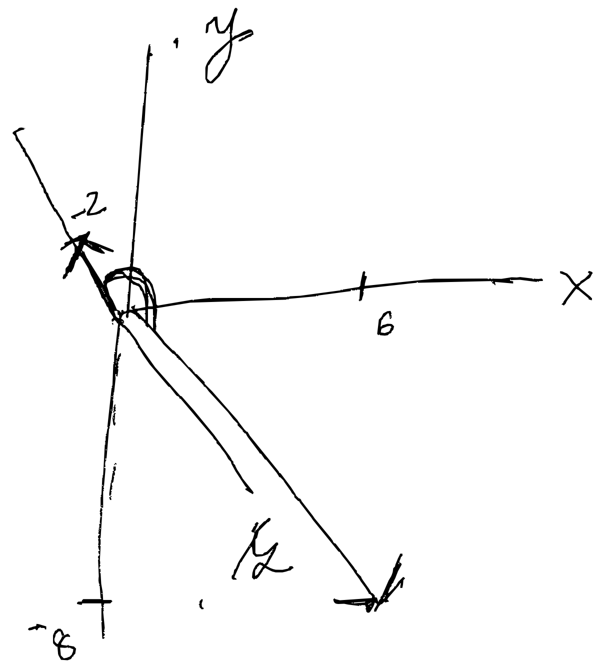
$$x = A^{-1} \cdot b$$



$\vec{v} = B - A = (2-1; 5-1)$

$\vec{v} = (1, 4)$

$\|\vec{v}\| = \sqrt{1^2 + 4^2} = \underline{\underline{\sqrt{17}}}$



$a \cdot b = \|a\| \cdot \|b\| \cdot \cos(\gamma)$

$\cos(\gamma) = \dfrac{a \cdot b}{\|a\| \cdot \|b\|} \quad / \cdot \arccos$

$\boxed{\arccos(\cos(x)) = x}$

$\boxed{\gamma = \arccos\left(\dfrac{a \cdot b}{\|a\| \cdot \|b\|}\right)}$

$\vec{a} = (2, 4, 2) \rightarrow 2 + 4 + 2 = 8$

$\vec{a}/8 = \left(\frac{1}{4}; \frac{1}{2}; \frac{1}{4}\right)$ $\quad \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = \frac{1+2+1}{4} = 1$

vlastní číslo

$$Av = \lambda v \qquad \lambda = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$Av - \lambda v = 0 \qquad A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$(A - \lambda)v = 0$$

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$$

$$A = \begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix} = 0$$

$$\det|A| = (2-\lambda)\cdot(2-\lambda) - 1\cdot 1 = 0$$

$$4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$(\lambda - 1)\cdot(\lambda - 3) = 0$$

$$\lambda_1 = 1 \qquad \lambda_2 = 3$$

$$\boxed{\lambda = 1} \qquad v_{\lambda_1} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

2×2 2×1

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$v_1 + v_2 = 0 \rightarrow \underline{v_1} = -v_2$$

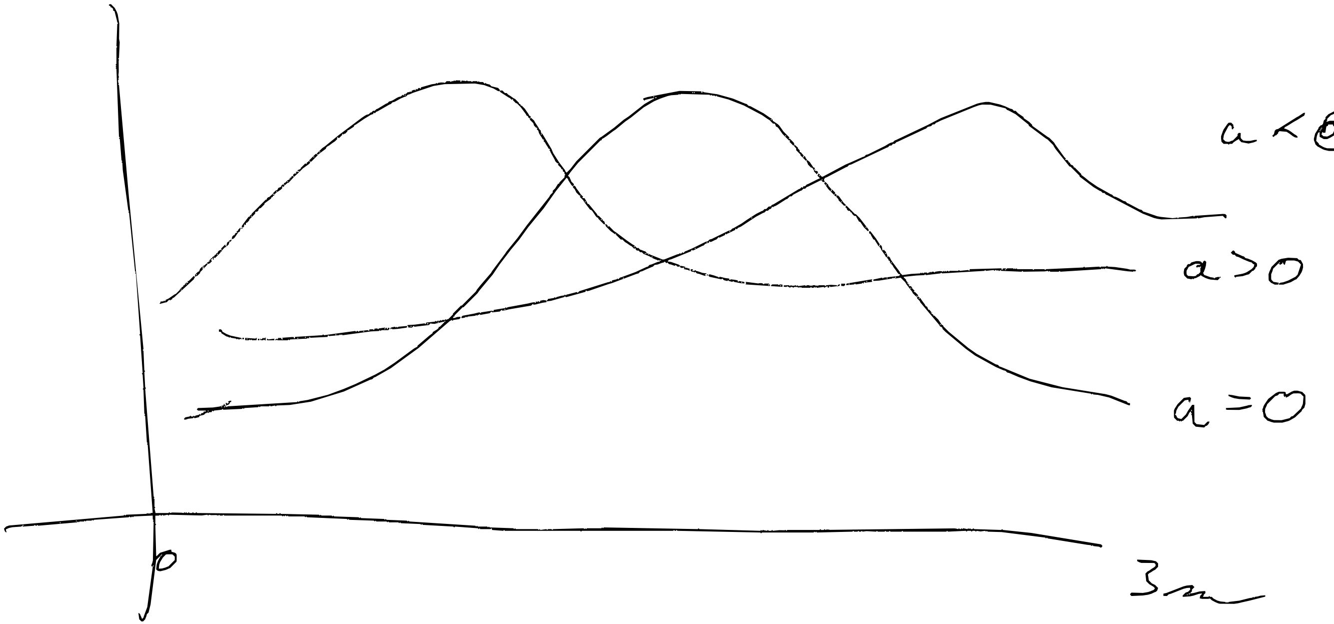
$$v_1 + v_2 = 0$$

$$\lambda = 3 \quad \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

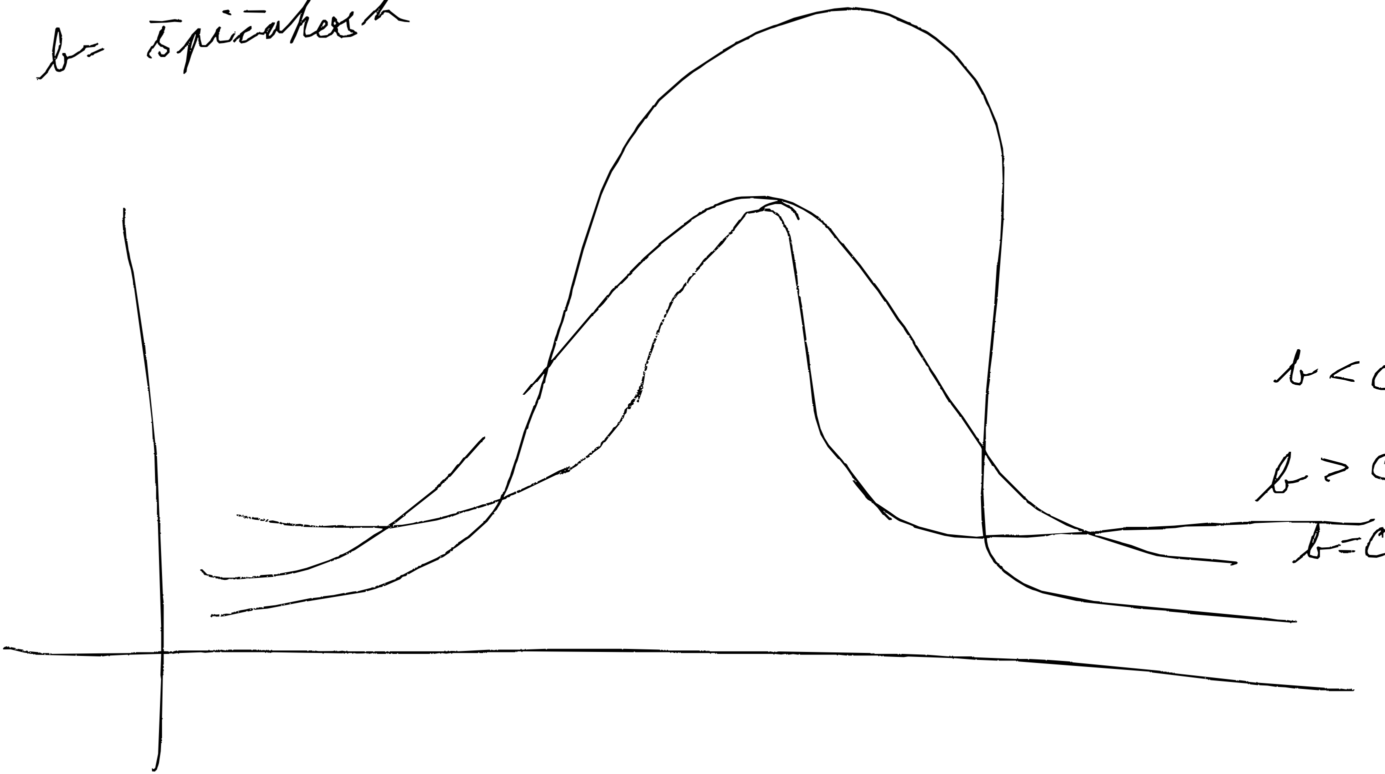
$$\lambda_2 = 3 \quad v_{\lambda_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$v_1 = v_2 \quad \begin{cases} -v_1 + v_2 = 0 \\ v_1 - v_2 = 0 \end{cases}$$

a = Tilmacht



b = Spitzigkeit



3 a méně $\{1; 2; 3\}$

$$\frac{\#\text{ úspěšných jevů}}{\#\text{ všech jevů}} = \frac{3}{6} = 50\%$$

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2 kostky $= 5$

$\{1; 4\}$

$\{2; 3\}$

$\{3; 2\}$

$\{4; 1\}$

$$P = \frac{4}{36} = \frac{1}{9} \doteq 11\%$$

$6 \cdot 6$

$$P = \frac{\#\text{úspěšných}}{\#\text{všech}}$$

$$\Omega = \{1, 2, 3, 4, 5, 6\} = \underline{100\,\%}$$

jev jistý

$$P(\emptyset) = 0$$

$$0 \leq P(A) \leq 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$1 - \frac{1}{6} = \frac{5}{6}$$

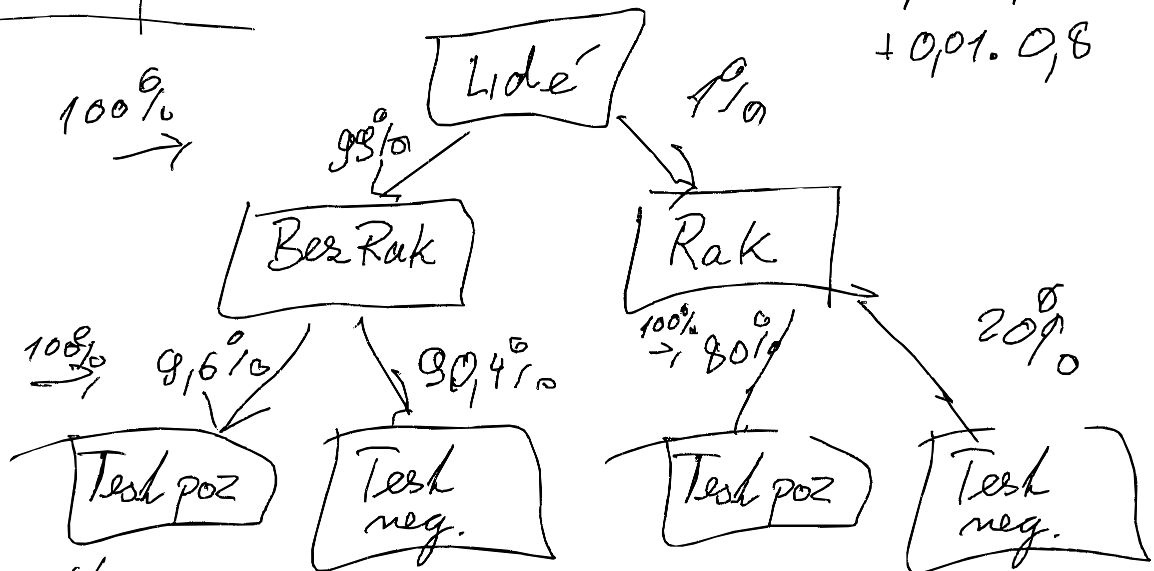
	Bez rak 99%	s rak 1%
Neg	90,4%	20%
Test Poz	9,6%	80%

$$0{,}01 \cdot 0{,}08$$

$$P(A|B) = \frac{\boxed{P(B|A) \cdot P(A)}}{P(B)}$$

$$P(B) = 0{,}99 \cdot 0{,}096 + 0{,}01 \cdot 0{,}8$$

Poz + Rak



$0{,}99 \cdot 0{,}096$

$0{,}01 \cdot 0{,}8$

výsledek testu pozitivní a při nemám rakovinu

$$\frac{0{,}99 \cdot 0{,}096}{0{,}99 \cdot 0{,}096 + 0{,}01 \cdot 0{,}8} =$$