Lab Assignment-9

MTH 308 and & MTH 308B: Numerical Analysis and Scientific Computing-I

January-April 2024, IIT Kanpur

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1. **(Trapezoidal/Trapozoidal)**: Write a C/Matlab program on Trapozoidal rule to find approximate integral of a function in an interval.
Hint: (You may use the following algorithm)
INPUT : End points a, b of the interval of the interval $[a, b]$, integer N or h (for number of partition of the interval $[a, b]$.
OUTPUT: Approximation of $\int_a^b f(x)\,dx$ is T_h.
Step-1: Set $h = (b-a)/N$; or $N = (b-a)/h$; (of course choose h so that N is an integer)
$$T_h = (f(a) + f(b))/2;$$
Step-2: For $i = 1, 2, \dots, N-1$, set $T_h = T_h + f(a + ih)$.
Step-3: $T_h = h \cdot T_h$
Step-4: OUTPUT (T_h).
Step-5: STOP.

2. **(Simpson's 1/3)**: Write a C/Matlab program on Simpson's rule to find approximate integral of a function in an interval.
Hint: (You may use the following algorithm)
INPUT : End points a, b of the interval of the interval $[a, b]$, an integer N or h (for number of partition of the interval $[a, b]$).
OUTPUT: Approximation of $\int_a^b f(x)\,dx$ is S_h.
Step-1: Set $h = (b-a)/2N$; or $N = (b-a)/2h$; (of course choose h so that N is an integer)
$$T_h = (f(a) + f(b));$$
Step-2: For $i = 1, 2, \dots, N-1$, set $T_h = T_h + 2 \cdot f(a + 2ih)$.
Step-3: For $i = 1, 2, \dots, N$, set $T_h = T_h + 4 \cdot f(a + (2i-1)h)$.
Step-4: $T_h = hT_h/3$
Step-5: OUTPUT (T_h).
Step-6: STOP.
Find the approximate integration of the following with your coding:

$$(i)\ \int_{0.5}^{1} x^4\,dx, \quad (ii)\ \int_1^{1.5} x^2 \ln x\,dx, \quad (iii)\ \int_1^{1.6} \frac{2x}{x^2-4}\,dx, \quad (iv)\ \int_0^{\frac{\pi}{4}} x \sin x\,dx$$

$$(v)\ \int_0^{0.5} \frac{2}{x-4}\,dx, \quad (vi)\ \int_0^1 x^2 e^{-x}\,dx \quad (vii)\ \int_0^{0.35} \frac{2}{x^2-4}\,dx, \quad (viii)\ \int_0^{\frac{\pi}{4}} e^{3x} \sin 2x\,dx.$$

3. **(Euler's method)**: Write a C/Matlab program on Euler method to solve and ODE.
Hint: (You may use the following algorithm)
INPUT : End points a, b of the interval of the interval $[a, b]$, integer N or h (for number of partition of the interval $[a, b]$) and initial condition y_0.
OUTPUT: Approximation z to y at the $t = a + Nh = b$
Step-1: Set $h = (b - a)/N$; or $N = (b - a)/h$; (of course choose h so that N is an integer)

$$t(1) = a;$$

$$w(1) = y_0;$$

Step-2: For $i = 2, 3, \ldots, N + 1$, set $w(i) = w(i - 1) + hf(t(i - 1), w(i - 1))$,

$$t(i) = a + (i - 1)h.$$

Step-3: OUTPUT (t, w).

Step-4: Plot the Graph of T-vs-W.

Step-5: STOP.
Solve the following ODEs with your coding:

(a) i. $y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 0$, with $h = 0.2$
ii. $y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1$, with $h = 0.5$
iii. $y' = 1 + \frac{y}{t}, \quad 1 \le t \le 2, \quad y(1) = 2$, with $h = 0.25$
iv. $y' = \cos 2t + \sin 3t, \quad 0 \le t \le 1, \quad y(0) = 1$, with $h = 0.2$.

(b) i. $y' = e^{t-y}, \quad 0 \le t \le 1, \quad y(0) = 1$, with $h = 0.1$
ii. $y' = \frac{1+t}{1+y}, \quad 1 \le t \le 2, \quad y(1) = 2$, with $h = 0.1$
iii. $y' = -y + ty^{\frac{1}{2}}, \quad 2 \le t \le 3, \quad y(2) = 2$, with $h = 0.25$
iv. $y' = t^{-2}(\sin 2t - 2ty), \quad 1 \le t \le 2, \quad y(1) = 2$, with $h = 0.2$.

Note: Problem 3 is not included in endsem lab exam, and can be tried after Monday's(15.04.24) lecture.

End.