to nearly satisfy $f''(x_0) = f''(x_n) = 0$. An alternative to the natural boundary condition that does not require knowledge of the derivative of f is the *not-a-knot* condition, (see [Deb2], pp. 55–56). This condition requires that $S'''(x)$ be continuous at x_1 and at x_{n-1}.

EXERCISE SET 3.5

1. Determine the natural cubic spline S that interpolates the data $f(0) = 0$, $f(1) = 1$, and $f(2) = 2$.
2. Determine the clamped cubic spline s that interpolates the data $f(0) = 0$, $f(1) = 1$, $f(2) = 2$ and satisfies $s'(0) = s'(2) = 1$.
3. Construct the natural cubic spline for the following data.

a.

x	$f(x)$
8.3	17.56492
8.6	18.50515

b.

x	$f(x)$
0.8	0.22363362
1.0	0.65809197

c.

x	$f(x)$
−0.5	−0.0247500
−0.25	0.3349375
0	1.1010000

d.

x	$f(x)$
0.1	−0.62049958
0.2	−0.28398668
0.3	0.00660095
0.4	0.24842440

4. Construct the natural cubic spline for the following data.

a.

x	$f(x)$
0	1.00000
0.5	2.71828

b.

x	$f(x)$
−0.25	1.33203
0.25	0.800781

c.

x	$f(x)$
0.1	−0.29004996
0.2	−0.56079734
0.3	−0.81401972

d.

x	$f(x)$
−1	0.86199480
−0.5	0.95802009
0	1.0986123
0.5	1.2943767

5. The data in Exercise 3 were generated using the following functions. Use the cubic splines constructed in Exercise 3 for the given value of x to approximate $f(x)$ and $f'(x)$, and calculate the actual error.
   a. $f(x) = x \ln x$; approximate $f(8.4)$ and $f'(8.4)$.
   b. $f(x) = \sin(e^x - 2)$; approximate $f(0.9)$ and $f'(0.9)$.
   c. $f(x) = x^3 + 4.001x^2 + 4.002x + 1.101$; approximate $f(-\frac{1}{3})$ and $f'(-\frac{1}{3})$.
   d. $f(x) = x \cos x - 2x^2 + 3x - 1$; approximate $f(0.25)$ and $f'(0.25)$.
6. The data in Exercise 4 were generated using the following functions. Use the cubic splines constructed in Exercise 4 for the given value of x to approximate $f(x)$ and $f'(x)$, and calculate the actual error.
   a. $f(x) = e^{2x}$; approximate $f(0.43)$ and $f'(0.43)$.
   b. $f(x) = x^4 - x^3 + x^2 - x + 1$; approximate $f(0)$ and $f'(0)$.
   c. $f(x) = x^2 \cos x - 3x$; approximate $f(0.18)$ and $f'(0.18)$.
   d. $f(x) = \ln(e^x + 2)$; approximate $f(0.25)$ and $f'(0.25)$.
7. Construct the clamped cubic spline using the data of Exercise 3 and the fact that
   a. $f'(8.3) = 3.116256$ and $f'(8.6) = 3.151762$
   b. $f'(0.8) = 2.1691753$ and $f'(1.0) = 2.0466965$
   c. $f'(-0.5) = 0.7510000$ and $f'(0) = 4.0020000$
   d. $f'(0.1) = 3.58502082$ and $f'(0.4) = 2.16529366$
8. Construct the clamped cubic spline using the data of Exercise 4 and the fact that
   a. $f'(0) = 2$ and $f'(0.5) = 5.43656$
   b. $f'(-0.25) = 0.437500$ and $f'(0.25) = -0.625000$