Lab Assignment-3

MTH 308 and & MTH 308B: Numerical Analysis and Scientific Computing-I

January-April 2024, IIT Kanpur

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1. Write a C program on Gaussina Elimination (on $n \times n$ system) with backward substitution.
   Hint: (You can use the following algorithm)
   INPUT : Number of unknowns and equations n, the augmented matrix $A = [a_{ij}]$ for $1 \leq i \leq n$ and $1 \leq j \leq n+1$.
   OUTPUT: The solution vector $x = [x_j]$ or message that the system has no unique solution.
   Step-1: For $i = 1, 2, \ldots, n-1$ do Steps 2 to 4.

   Step-2: Let p be the smallest integer with $i \leq p \leq n$ and $a_{pi} \neq 0$.
   If no integer p can be found, then OUTPUT ('no unique solution exists');
   STOP

   Step-3: If $p \neq i$ then perform $R_i \leftrightarrow R_p$

   Step-4: For $k = i+1, \ldots, n$ do Steps 5 and 6.

   Step-5: Set $m_{ki} = \frac{a_{ki}}{a_{ii}}$
   Step-6: Perform $(R_k - m_{ki} R_i) \rightarrow R_k$

   Step-7: If $a_{nn} = 0$, then OUTPUT ('no unique solution exists');
   STOP.
   Step-8: Set $x_n = \frac{a_{n,n+1}}{a_{nn}}$
   Step-9: For $i = n-1, \ldots, 1$, set $x_i = (a_{i,n+1} - \sum_{j=i+1}^{n} a_{ij} x_j)/a_{ii}$.
   Step-10: OUTPUT $x = [x_1, x_2, \ldots, x_n]$
   STOP.

2. Write a C program on LU decomposition of a square matrix A.
   Hint: (You can use the following algorithm)
   INPUT: The dimension n and the entries $a_{ij},\ 1 \leq i, j \leq n$, of A, the diagonal

$$l_{11} = l_{22} = \cdots = l_{nn} = 1$$

   of L or the diagonal $u_{11} = u_{22} = \cdots = u_{nn} = 1$ of U.
   OUTPUT: The entries l_{ij}, $1 \leq j \leq i$, $1 \leq i \leq n$ of L and the entries, u_{ij}, $i \leq j \leq n$, $1 \leq i \leq n$ of U.
   Step-1: Select $l_{11}$and u_{11} satisfying $l_{11} u_{11} = a_{11}$.
   If $l_{11} u_{11} = 0$, then OUTPUT ('Factorization impossible');
   STOP.
   Step-2: For $j = 2, \ldots, n$ set $u_{1j} = a_{1j}/l_{11}$; or $l_{j1} = a_{j1}/u_{11}$;
   Step-3: For $i = 2, \ldots, n-1$ do Steps 4 and 5.

   Step-4: Select $l_{ii}$and u_{ii} satisfying $l_{ii} u_{ii} = a_{ii} - \sum_{k=1}^{i-1} l_{ik} u_{ki}$.
   If $l_{11} u_{11} = 0$ then OUTPUT ('Factorization impossible');
   STOP.

Step-5: For $j = i+1, \ldots, n$, set $u_{ij} = \frac{1}{l_{ii}} \left[a_{ij} - \sum_{k=1}^{i-1} l_{ik} u_{kj} \right]$,
$l_{ji} = \frac{1}{u_{ii}} \left[a_{ji} - \sum_{k=1}^{i-1} l_{jk} u_{ki} \right]$.

Step-6: Select l_{nn} and u_{nn} satisfying $l_{nn} u_{nn} = a_{nn} - \sum_{k=1}^{n-1} l_{nk} u_{kn}$.
Step-7:
OUTPUT (l_{ij} for $j = 1, \ldots, i$ and $i = 1, \ldots, n$); and
OUTPUT (u_{ij} for $j = i, \ldots, n$ and $i = 1, \ldots, n$);
STOP.

End.