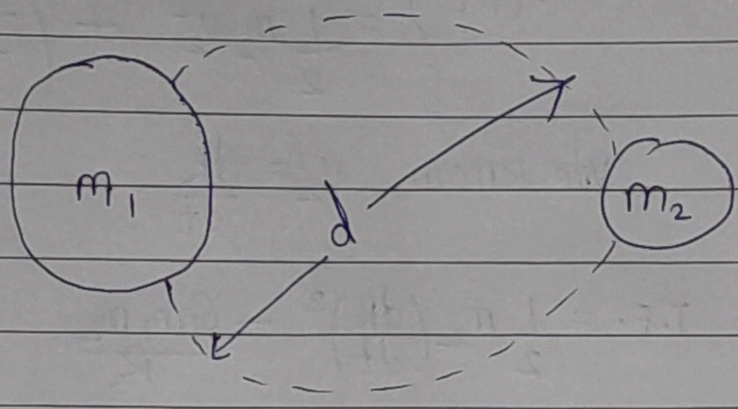
Problem # 3



In a stable orbit, gravitational force provides centripetal force (F_c)

$$\therefore \boxed{F_c = \frac{G m_1 m_2}{d^2}} \quad \text{—} (1)$$

Here, $G = 6.674 \times 10^{-11}\ N \cdot m^2/kg^2$

Centripetal force acting on m_1 is also given by:-

$$\boxed{F_c = m_1 \omega^2 d} \quad \text{—} (2)$$

ω is angular velocity of m_1 w.r.t. m_2

Equating (1) & (2),

$$\frac{G m_1 m_2}{d^2} = \frac{m_1 \omega^2 d}{\cancel{2}}$$

$$\therefore \omega^2 = \frac{\cancel{2} G m_2}{d^3}$$

$$\boxed{\omega = \sqrt{\frac{\cancel{2} G m_2}{d^3}}}$$

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Net force on m_2 (F_{net}) = $F_{gravity}$ + F_{drag}

From Newton's second law,

$$F_{net} = m_2 \cdot a_2$$

~~{a_2 = acc^n of m_2}~~

$$\therefore \quad m_2 a_2 = \frac{G m_1 m_2}{R^2} + (-\beta m_2 v_2)$$

$$\text{Here, } v_2 = \left(\frac{dR}{dt}\right)$$

As $m_1 >> m_2$, we can consider m_1 to be stationary.

Relative acc^n of m_2 w.r.t m_1 = $a_2 = \left(\frac{d^2R}{dt^2}\right)$

$$m_2\left(\frac{d^2R}{dt^2}\right) = \frac{G m_1 m_2}{R^2} - \beta m_2\left(\frac{dR}{dt}\right)$$

$$\therefore \quad m_2\left(\frac{d^2R}{dt^2}\right) + \beta m_2\left(\frac{dR}{dt}\right) = \frac{G m_1 m_2}{R^2}$$

$$\therefore \quad \boxed{\left(\frac{d^2R}{dt^2}\right) + \beta\left(\frac{dR}{dt}\right) = \frac{G m_1}{R^2}}$$

Solving this second order DE, we get R as a func^n of t.

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Total energy of system (T.E.) = K.E. + P.E.

$$= \frac{1}{2} m_2 V_2^2 + \left(\frac{-G m_1 m_2}{R}\right)$$

We know, $V_2 = \frac{dR}{dt}$

$$\therefore \quad T.E. = \frac{1}{2} m_2 \left(\frac{dR}{dt}\right)^2 - \frac{G m_1 m_2}{R}$$

Total energy as a funcⁿ of time =

$$\boxed{E(t) = \frac{1}{2} m_2 \left(\frac{dR}{dt}\right)^2 - \frac{G m_1 m_2}{R}}$$

Problem 4

Centripetal acc^n of star at radius R_o is given by

$$a_c = \frac{V_o^2}{R_o}$$

(V_o is orbital velocity)

Oort's constants A, B describe rotation of the Milky Way's disc, where A is related to local rotation of the galaxy and B is related to shear in rotation curve.

$$a_c = A \cdot r_2 + B$$

~~let a~~

Diff. in centripetal acc^{n}s for star (a_c) and sun (a') is due to the dark matter contribution

$$a' = a_c - (A \cdot r_1 + B)$$

Dark matter contribution (a_r) to the acc^n at location of star $= \frac{V_o^2}{r_2} - Ar_2 - B$

Acc^n due to dark matter at location of sun is related to a' as:-

$$a' = a + A(r_2 - r_1)$$

$$a = a' - A(r_2 - r_1)$$

$$a' = \frac{V_o^2}{\cancel{r_2} R_2} - Ar_2 - B + A(r_2 - r_1)$$

$$a' = \frac{V_o^2}{R_2} + Ar_1 - B$$

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V_0 can be written as:-

$$V_0 = \sqrt{Ar_1 + B}$$

$$a' = \frac{A \cdot r_1 + B}{r_2} + Ar_1 - B$$

$$a' = \frac{Ar_1 + B + Ar_1 r_2 - Br_2}{r_2}$$

dark matter density (ρ_{DM}) is related to a' and G by:-

$$\rho_{DM} = \frac{a'}{4\pi G}$$

Substitute a' into expression for ρ_{DM}.

$$\rho_{DM} = \frac{Ar_1 + B + Ar_1 r_2 - Br_2}{4\pi G r_2}$$

further simplifications lead to given expression:-

$$k = \frac{2(A-B)^2 - \frac{da}{dr}}{4\pi G}$$

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Physical significance of (A−B) term

A represents contribution of local rotation to the stellar acceleration
It signifies the rotation rate due to differential rotation of stars in the galactic disk.

B is associated with the shear in the rotation curve, describing how the rotation rate changes with distance from the galactic center.

Higher value of (A−B) contributes to a higher dark matter density. The term plays a crucial role in understanding galactic rotation curve, which describes how the orbital velocities of stars and gas change with radial distance from galactic center.
In the dark matter density equation, (A−B) is squared in the part of numerator.
Its square signifies importance of understanding the interplay between local rotation and shear in determining the dark matter density in the vicinity of the Sun.
When $(A-B) > 0$, it implies that contribution of local rotation is greater than shear.
A larger (A−B) indicates that the observed acceleration is more likely attributed to the presence of dark matter.
If (A−B) is small or −ve, it implies that

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rotational and shear together might explain a significant portion of observed accn without requiring a substantial contribution from dark matter.

∴ Hence, in summary, the (A−B) term serves as a measure of the relative importance of local rotation and shear in the galaxy's dynamics.