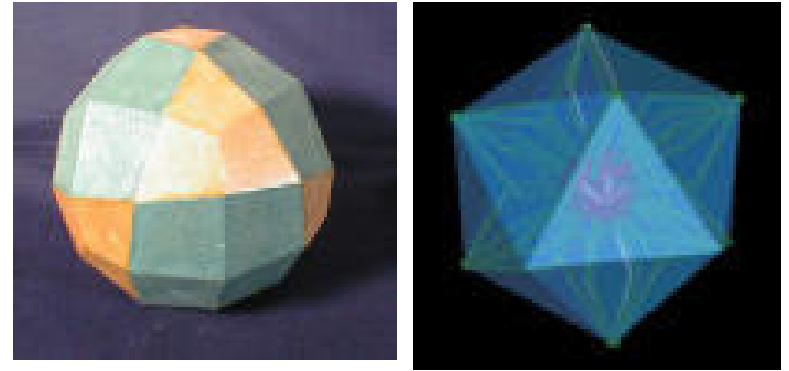


# Solids

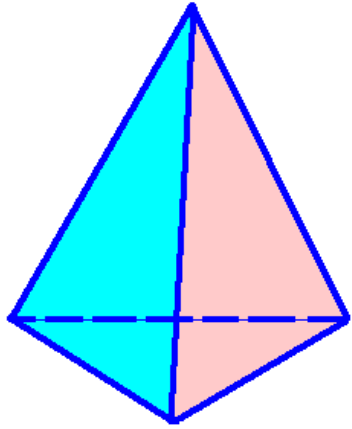
A 3-D object having length, breadth and thickness and bounded by surfaces which may be either plane or curved, or combination of the two.

- Classified under two main headings
  - Polyhedron
  - Solids of revolution

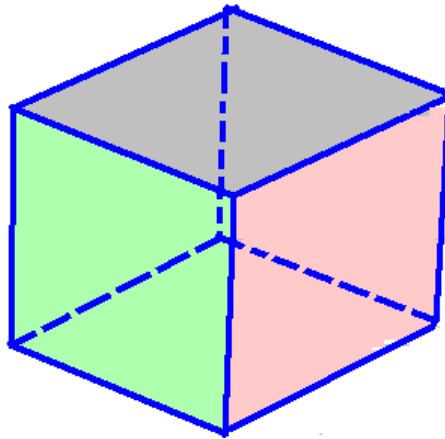


- **Regular polyhedron** – solid bounded only by plane surfaces (faces). Its faces are formed by regular polygons of same size and all dihedral angles are equal to one another.
- **Other polyhedra** – when faces of a polyhedron are not formed by equal identical faces, they may be classified into prisms and pyramids.

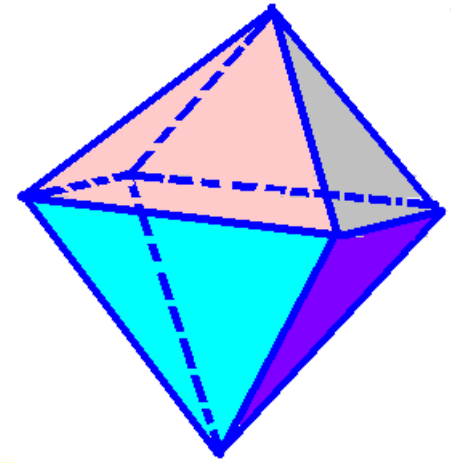
# Five regular polyhedra



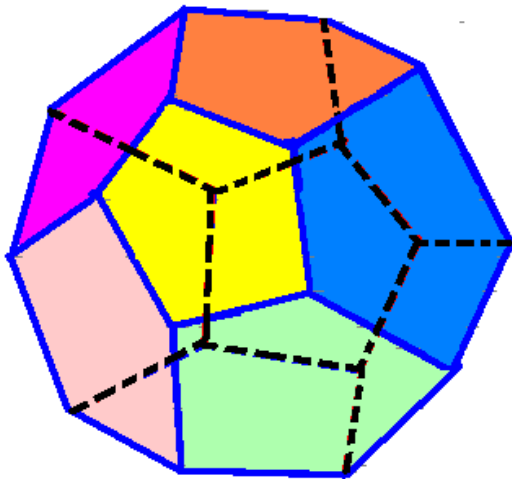
**Tetrahedron** — four equal equilateral triangular faces



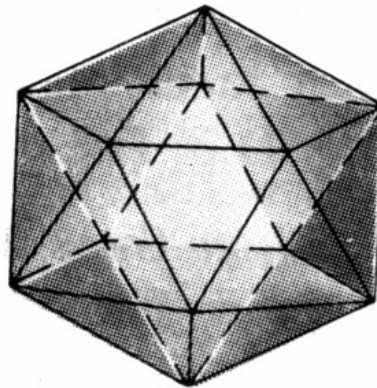
**Cube/hexahedron** — six equal square faces



**Octahedron**— eight equal equilateral triangular faces



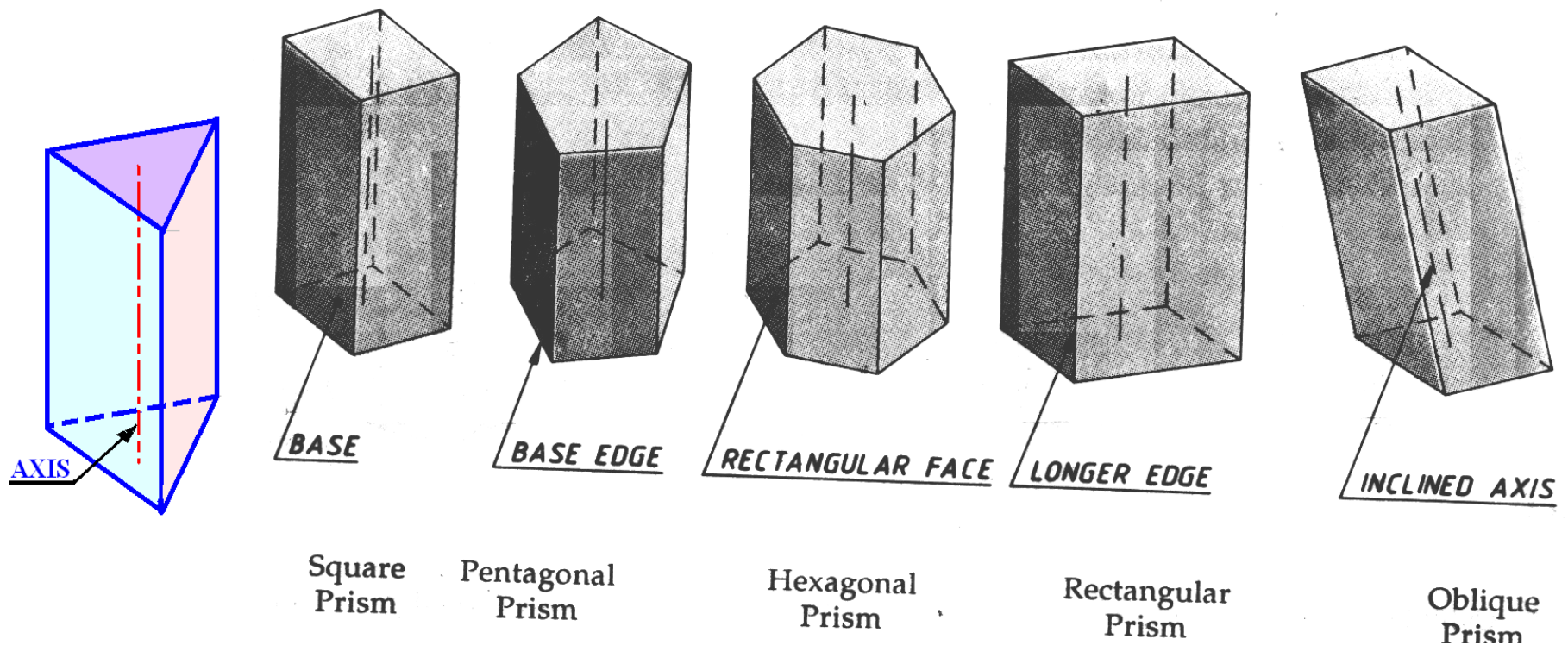
**Dodecahedron** — twelve equal regular pentagonal faces



**Icosahedron**— twenty equal equilateral triangular faces

**Prism** – a polyhedron formed by two equal parallel regular polygon, end faces connected by side faces which are either rectangles or parallelograms.

## Different types of prisms



**Pyramids** – a polyhedron formed by a **plane surface as its base** and a **number of triangles as its side faces**, all meeting at a point, called **vertex** or **apex**.

**Axis** – the imaginary line connecting the apex and the center of the base.

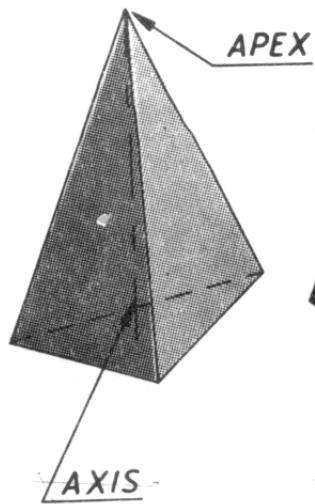
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**Inclined/slant faces** – inclined triangular side faces.

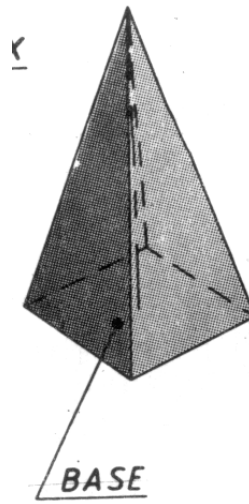
**Inclined/slant/longer edges** – the edges which connect the apex and the base corners.

**Right pyramid** – when the axis of the pyramid is perpendicular to its base.

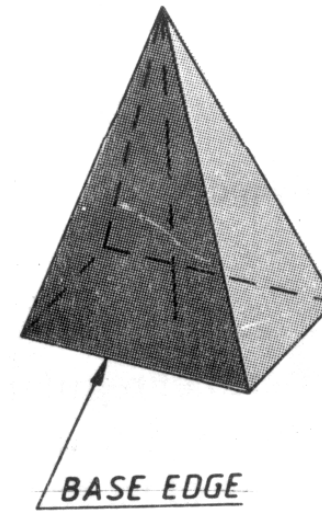
**Oblique pyramid** – when the axis of the pyramid is inclined to its base.



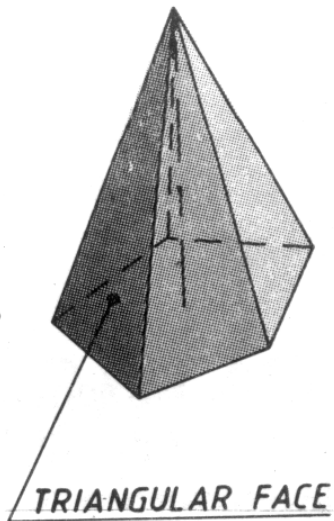
**Triangular pyramid**



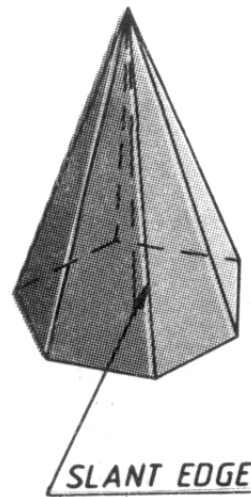
**Square pyramid**



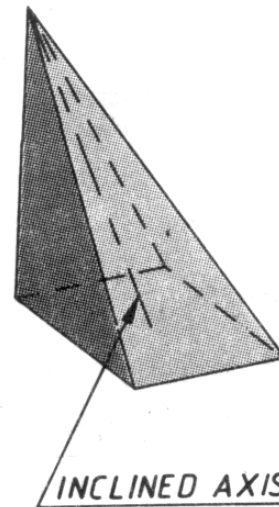
**Rectangular pyramid**



**Pentagonal pyramid**



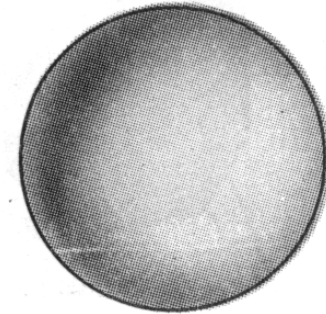
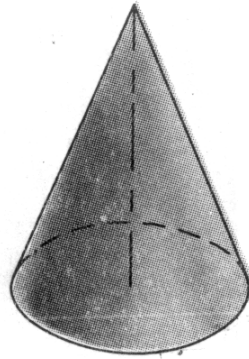
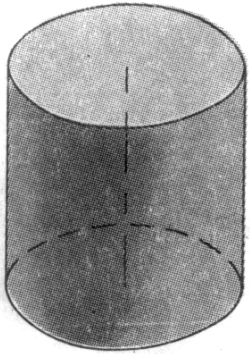
**Hexagonal pyramid**



**Oblique pyramid**



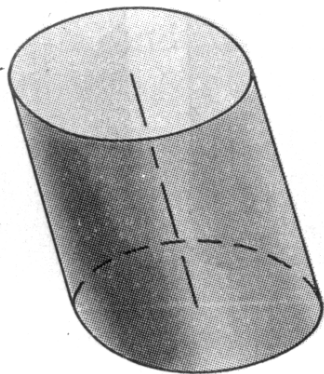
**Solids of revolution** – when some of the plane figures are revolved about one of their sides – solids of revolution is generated.



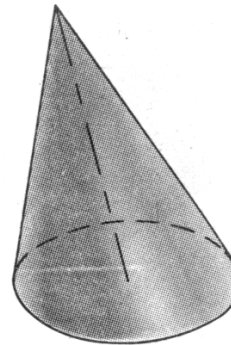
**Cylinder** – when a rectangle is revolved about one of its sides, the other parallel side generates a cylinder.

**Cone** – when a right triangle is revolved about one of its sides, the hypotenuse of the right triangle generates a cone.

**Sphere** – when a semi-circle is revolved about one of its diameters, a sphere is generated..

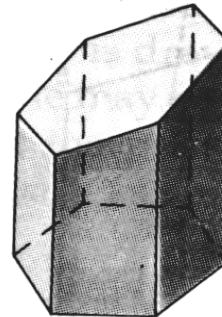
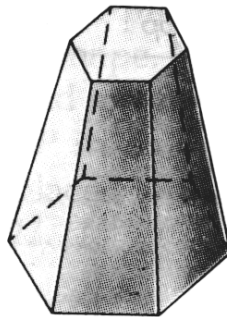
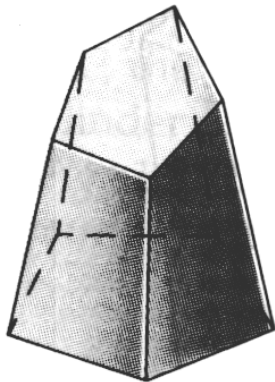
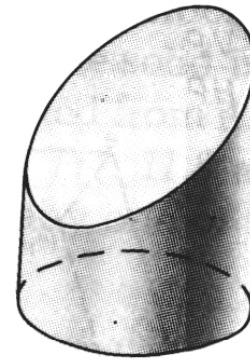
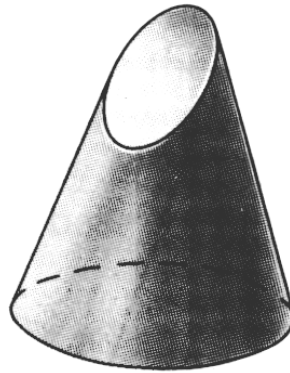
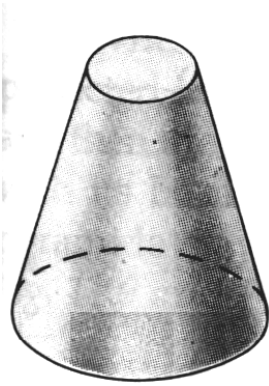


**Oblique cylinder** – when a parallelogram is revolved about one of its sides, the other parallel side generates a cylinder.



**Oblique cone**

**Truncated and frustums of solids** – when prisms, pyramids, cylinders are cut by cutting planes, the lower portion of the solids (without their top portions) are called, either truncated or frustum of these solids.



**Visibility** – when drawing the orthographic views of an object, it will be required to show some of the **hidden details as invisible** and are shown on the orthographic views by **dashed lines**

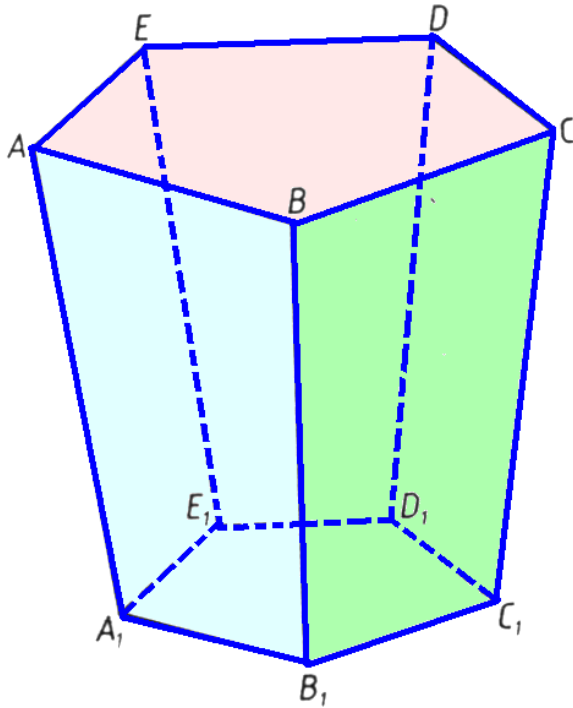
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### **Rules of visibility**

**All outlines of every view are visible – the outlines of all the views are shown by full lines.**



**In the top view**, the highest portions of the object are visible.



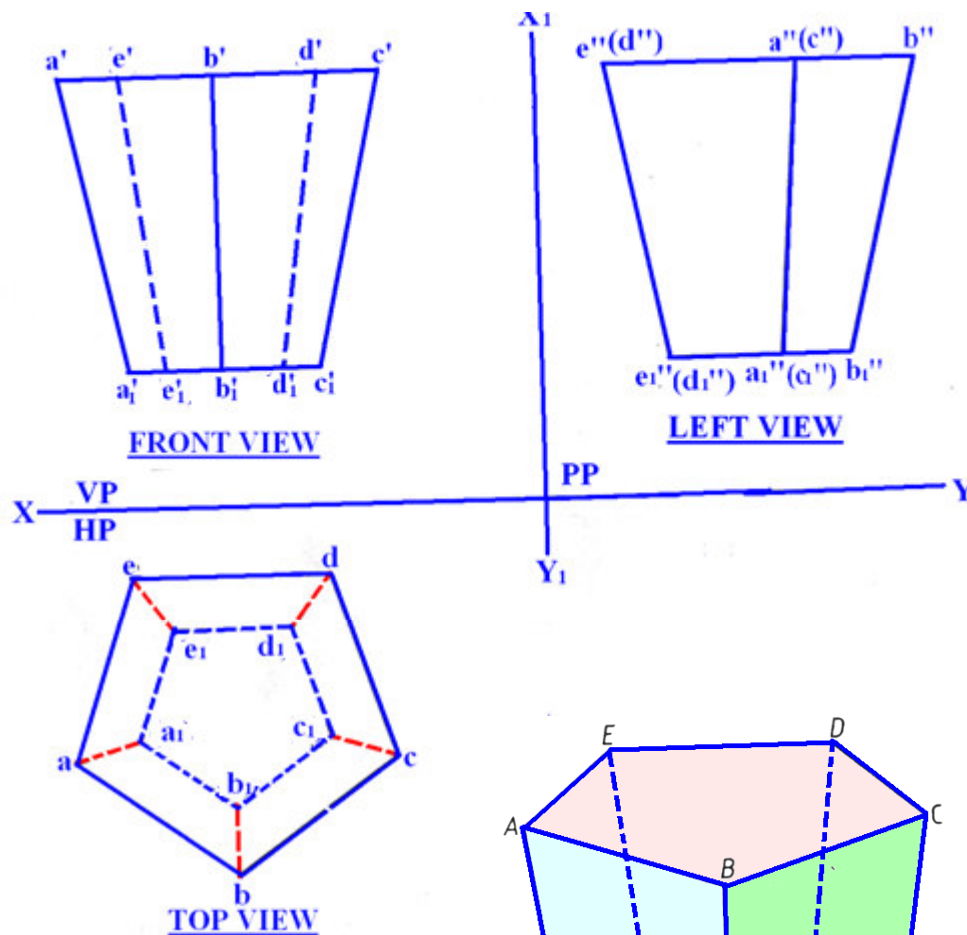
Frustum of a pentagonal pyramid – the top face ABCDE is the highest, it is completely visible in the top view.

In the top view, edges **ab, bc, cd, de** and **ea** are shown as **full lines**. The bottom pentagonal faces **A<sub>1</sub>B<sub>1</sub>C<sub>1</sub>D<sub>1</sub>E<sub>1</sub>** is smaller than the top face, hence **invisible**.

The slant edges **AA<sub>1</sub>, BB<sub>1</sub>, CC<sub>1</sub>, DD<sub>1</sub>** and **EE<sub>1</sub>** are invisible in the top view, hence they are shown as **lines of dashes**.

**The line connecting a visible point and an invisible point is shown as an invisible line of dashes unless they are outlines.**

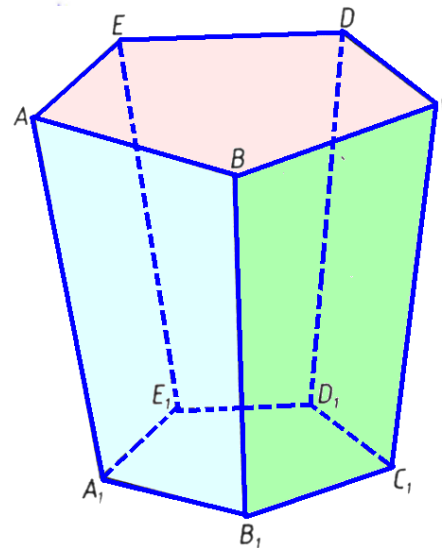
**In the front view** - the front faces of the object are visible.



In the front view – the faces  **$ABB_1A_1$**  and  **$BCC_1B_1$**  are the front faces, hence are visible.

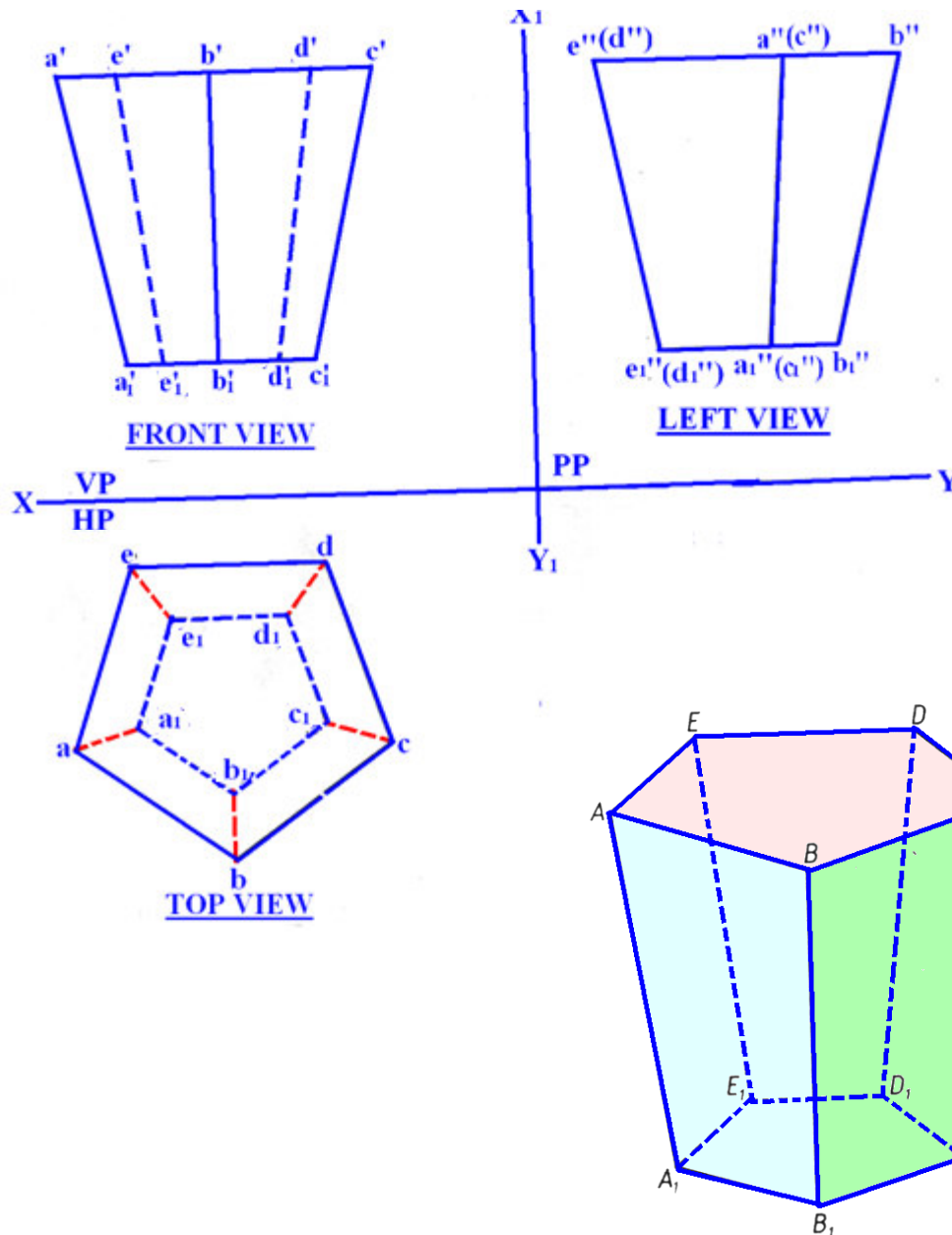
In the front view, the corners  **$a$ ,  $b$ ,  $c$**  and  **$a_1$ ,  $b_1$ ,  $c_1$**  are visible to the observer. Hence in the front view, the lines  **$a'a_1'$ ,  $b'b_1'$**  and  **$c'c_1'$**  are shown as full lines.

The corners  **$d$ ,  $e$ ,  $d_1$**  and  **$e_1$**  are invisible in the front view. The lines,  **$e'e_1'$ ,  $d'd_1'$**  are invisible, hence shown as dashed lines. The top rear edges  **$a'e'$ ,  $e'd'$**  and  **$d'c'$**  coincide with the top front visible edges  **$a'b'$**  and  **$b'c'$** .



Object

**In the side view** - the face lying on that side are visible.



As shown in the left side view, the corners **e, a, b** and **e<sub>1</sub>, a<sub>1</sub>, b<sub>1</sub>** lie on left side and are visible in the left view.

Hence the lines, **e''e<sub>1</sub>''**, **a''a<sub>1</sub>''** and **b''b<sub>1</sub>''** are shown as full lines. The edges **d''d<sub>1</sub>''**, **c''c<sub>1</sub>''** coincide with the visible edges **e''e<sub>1</sub>''** and **a''a<sub>1</sub>''** respectively.

# Projections of solids placed in different positions

The solids may be placed on HP in various positions

(1) The way the axis of the solid is held with respect to HP or VP or both -

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- **Perpendicular to HP or VP**
- **Parallel to either HP or VP and inclined to the other**
- **Inclined to both HP and VP**

# **Axis of the solid perpendicular to HP**

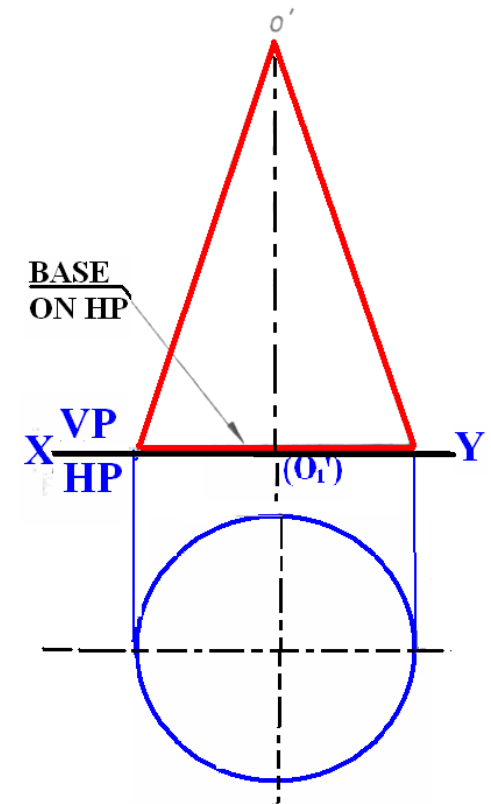
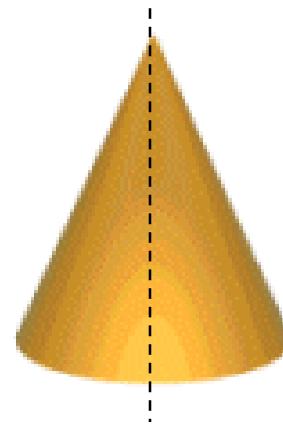
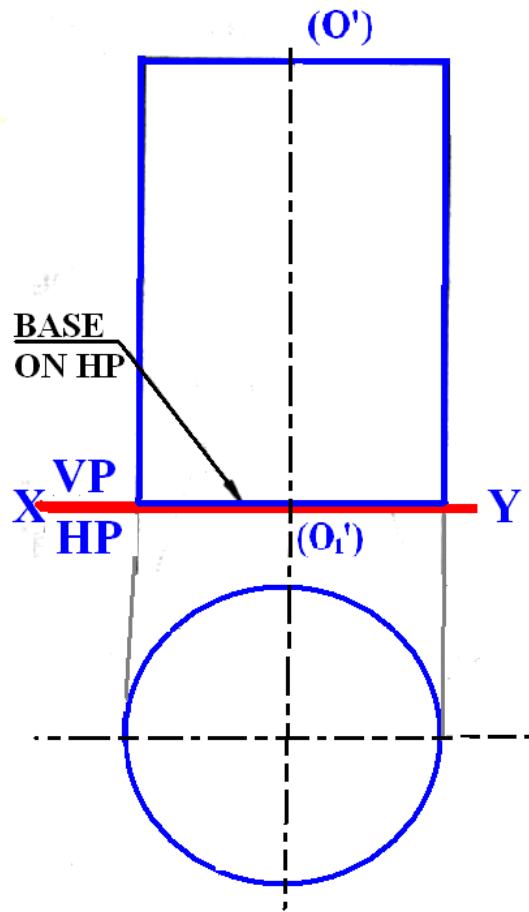
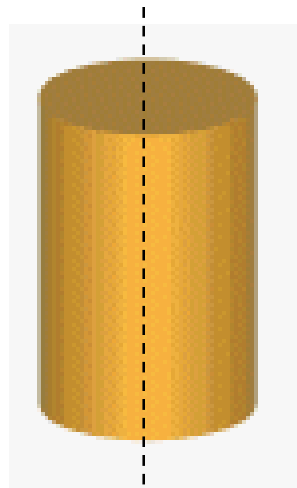
A solid when placed on HP with its axis perpendicular to it, then it will have its base on HP. This is the simplest position in which a solid can be placed.

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**When the solid is placed with the base on HP position, in the top view, the base will be projected in its true shape.**

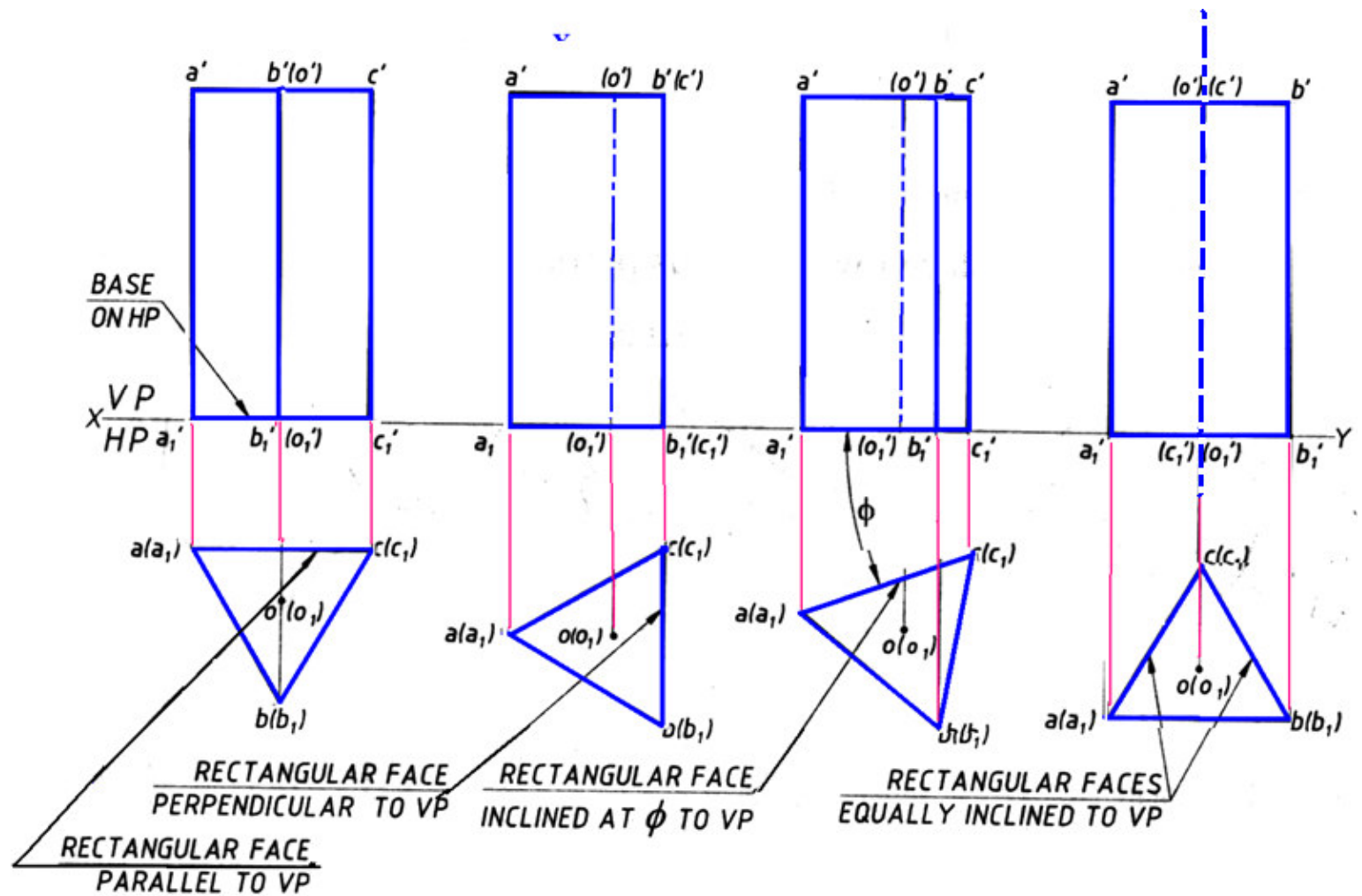
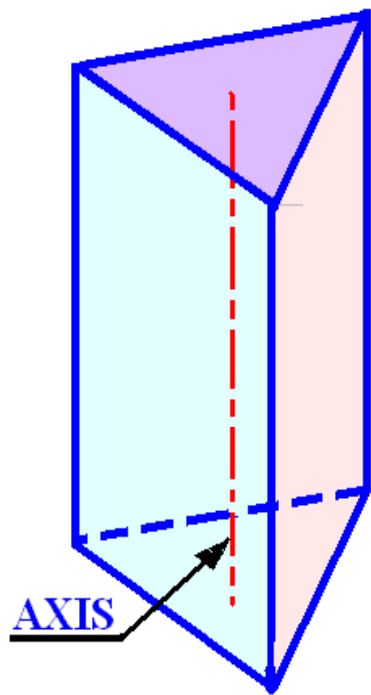
Hence, when the base of the solid is on HP, the top view is drawn first and then the front view and the side views are projected from it.

Only one position in which a **cylinder** or a **cone** may be placed with its base on HP.

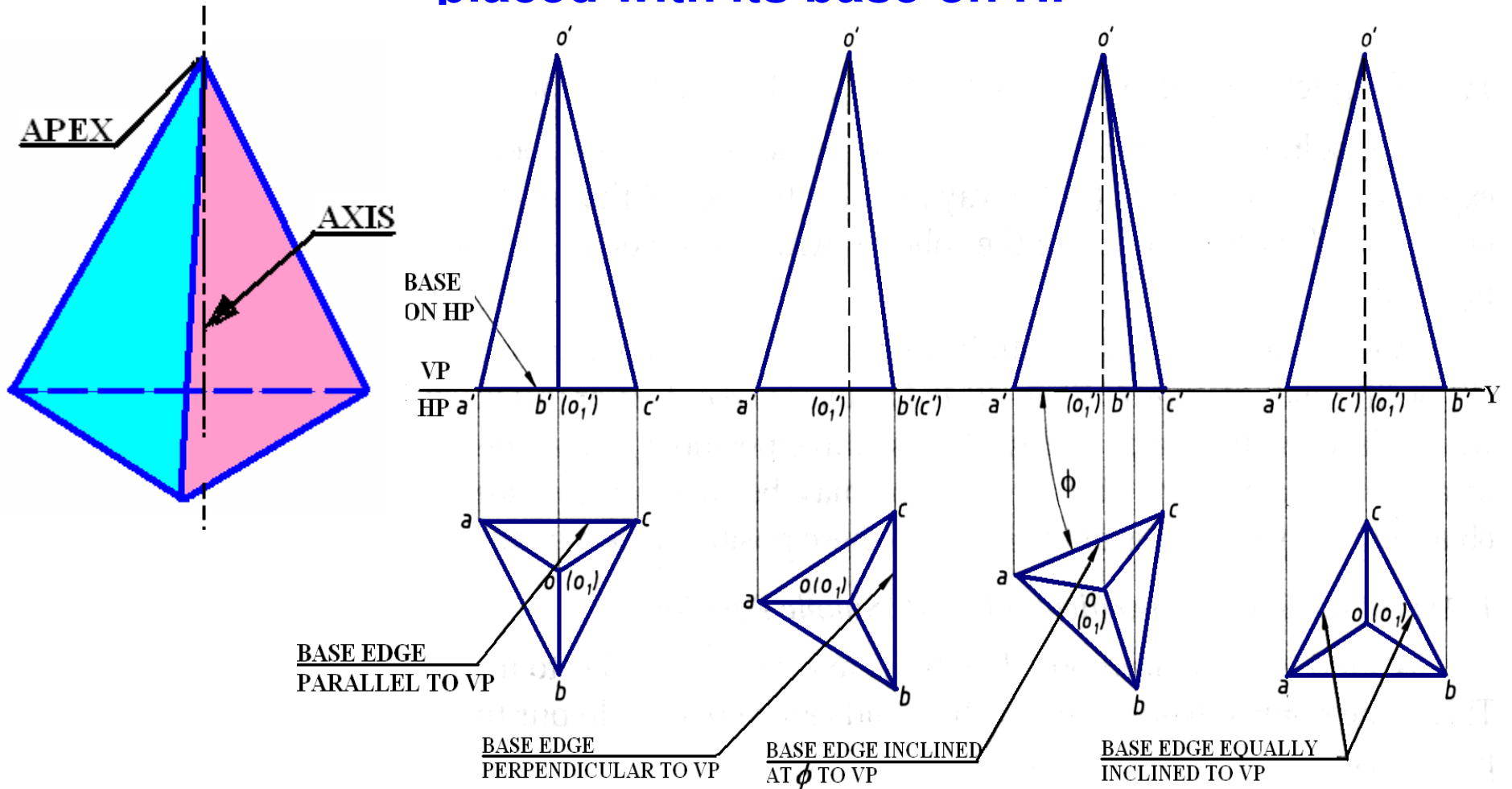




# Four positions of a prism placed with its base on HP.



# Four positions of a triangular pyramid placed with its base on HP



## **Axis of the solid perpendicular to VP**

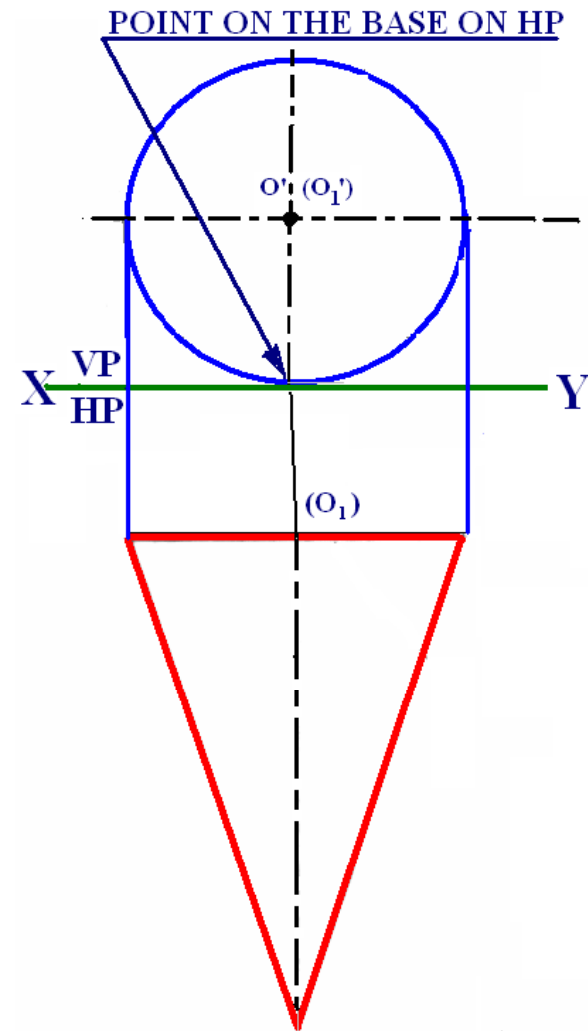
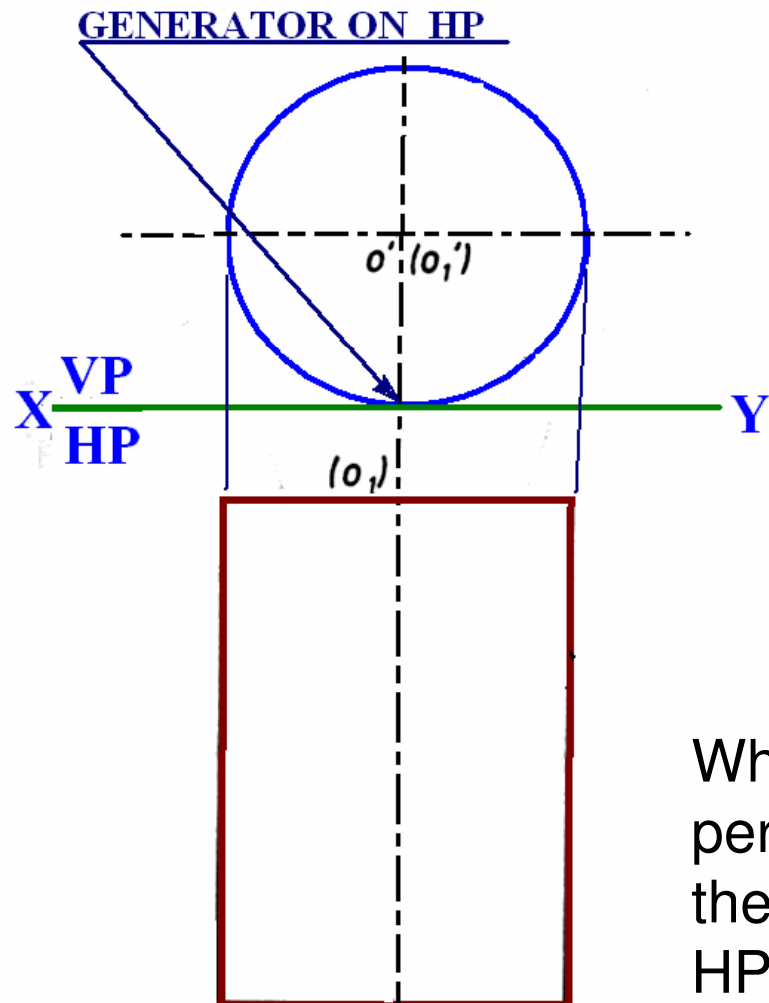
When a solid is placed with its axis perpendicular to VP, the base of the solid will always be perpendicular to HP and parallel to VP.

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**Hence in the front view, base will be projected in true shape**

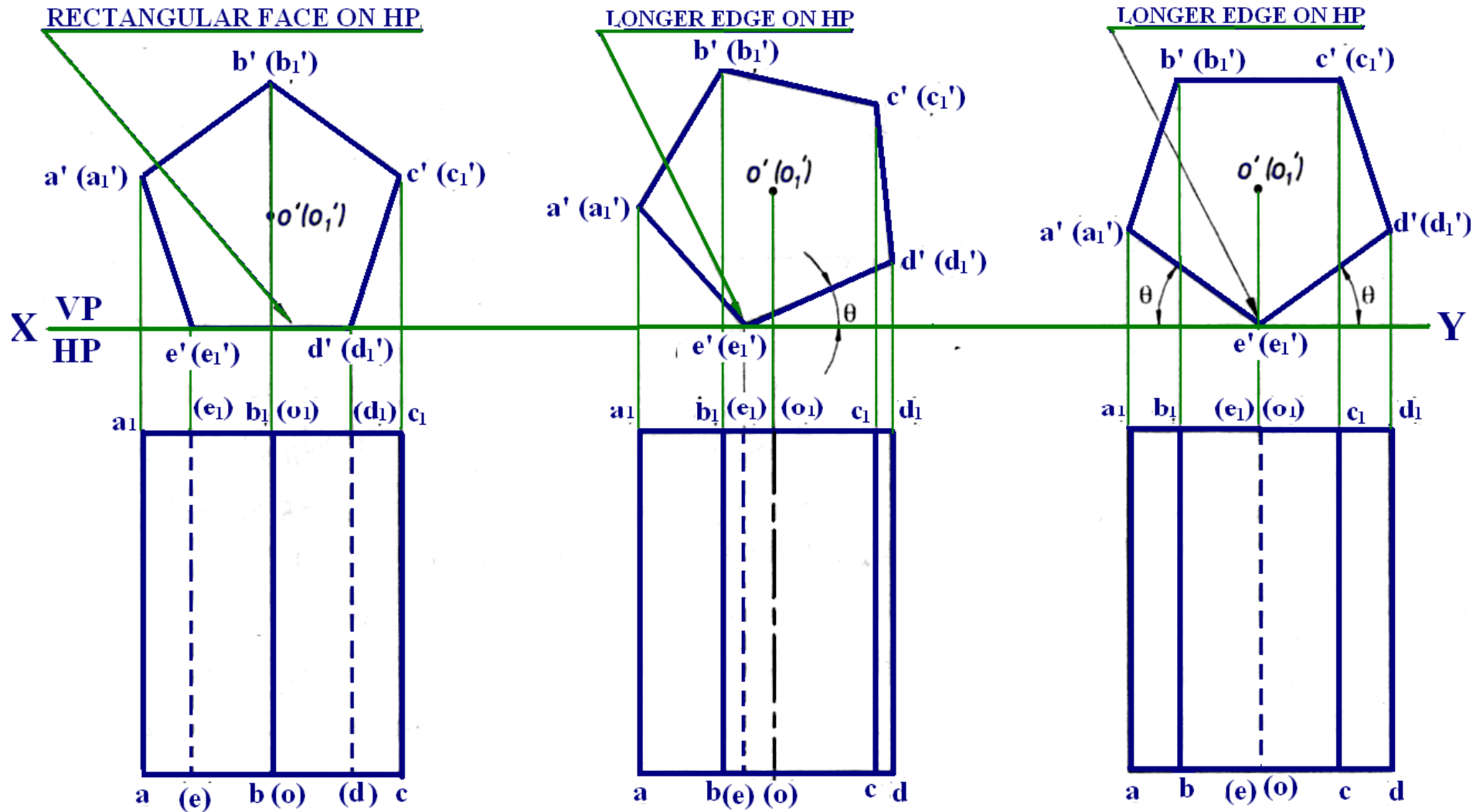
Therefore, when the axis of the solid is perpendicular to VP, the front view is drawn first and then the top and side views are drawn from it.

When a cylinder rests on HP with its axis perpendicular to VP, one of its generators will be on HP.

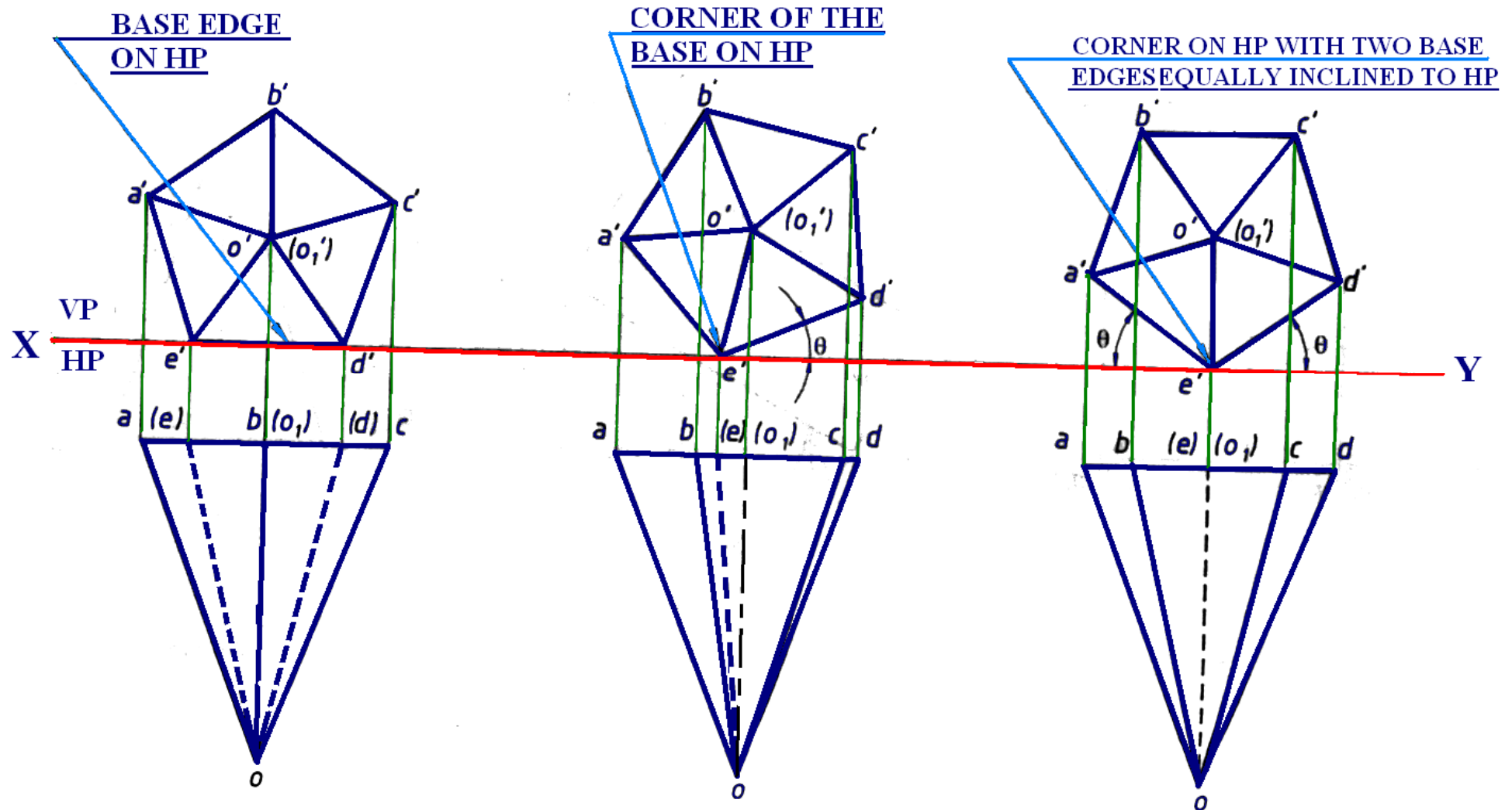


When a cone rests on HP with its axis perpendicular to VP, one of the points on the circumference of the base will be on HP.

**Prism** placed with their axis perpendicular to VP in three different positions.



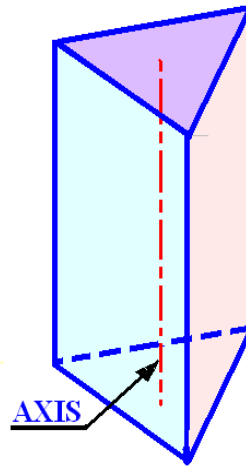
**Pyramid** placed with their axis perpendicular to VP in three different positions.



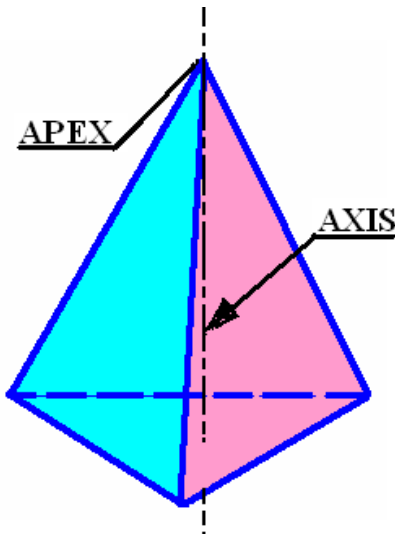


## Axis of the solid inclined to HP and parallel to VP

When a solid is **placed on HP** with its **axis inclined** to HP, the **elemental portion of the solid that lies on HP** depends upon the **type of the solid**.



When a **prism** is placed on HP with its **axis inclined to it**, then it will lie either on one of its **base edges** or **on one of its corners** on HP.

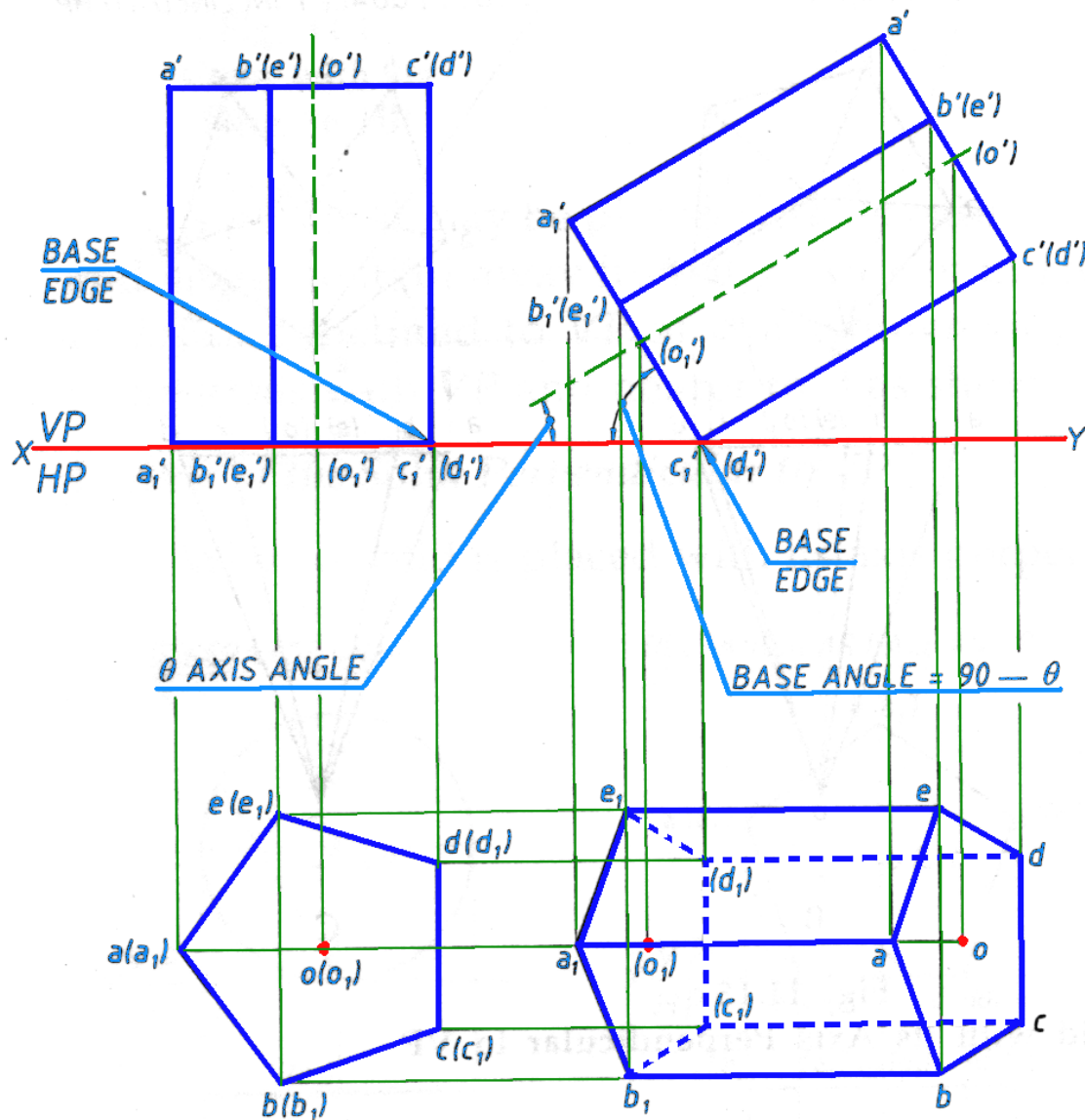


When a **pyramid** is placed on HP with its **axis inclined to HP**, then we will have one of its **base edges** on HP or one of its **base corners** on HP or **one of its slant edges** on HP or one of its **triangular faces** on HP or **an apex** on HP.

## **Case 1. When the solid lies with an edge of base on HP**

**If the solid is required to be placed with an edge of the base on HP, then initially the solid has to be placed with its base on HP such that an edge of the base is perpendicular to VP, i.e., to XY line in top view preferably to lie on the right side.**

# When the solid lies with an edge of base on HP



When a **pentagonal prism** has to be placed with **an edge of base on HP** such that the **base or axis is inclined to HP**, then initially, the **prism is placed with its base on HP with an edge of the base perpendicular to VP** and the lying on the right side.

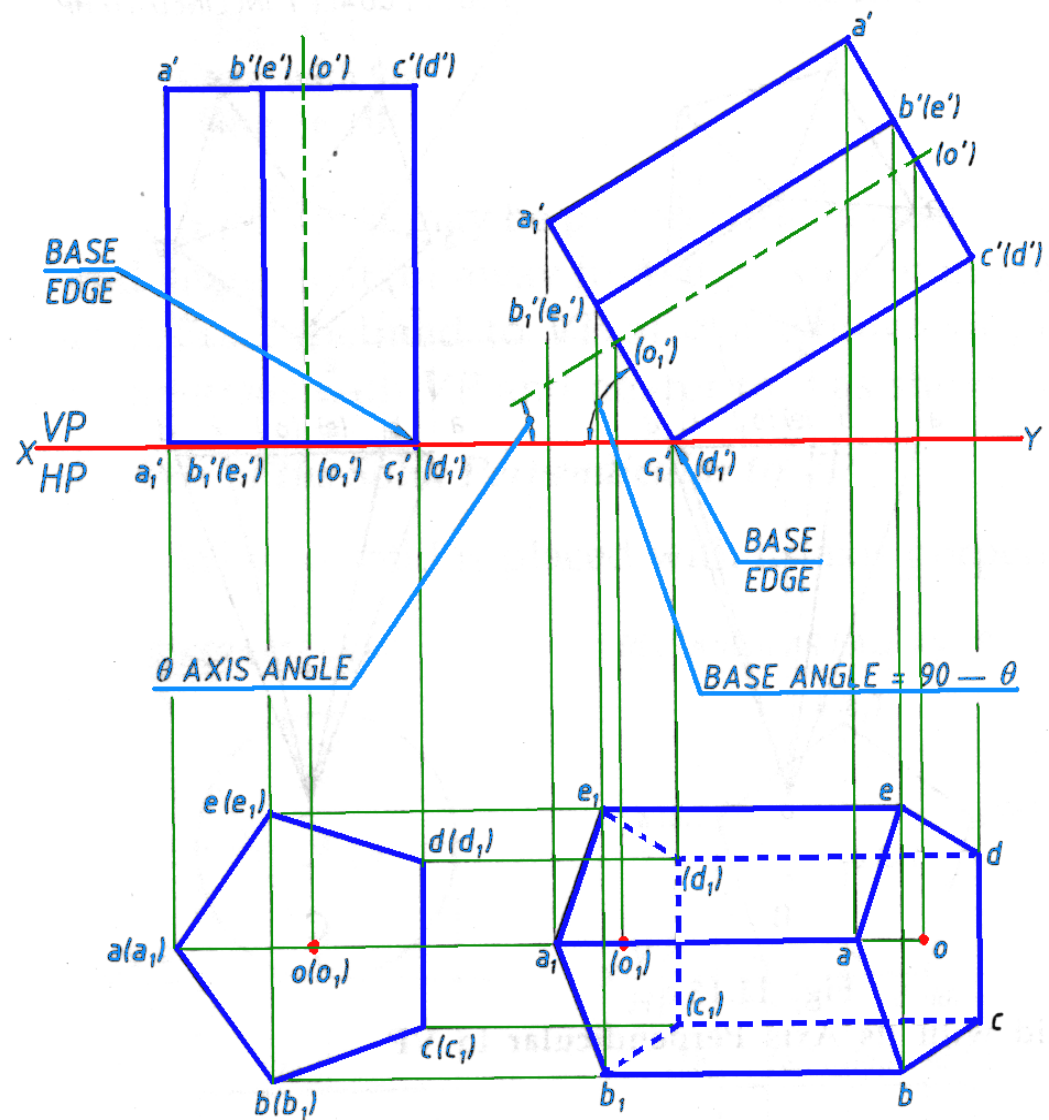
In this position, the first set of top and front views are drawn with the base edges  **$(c_1)(d_1)$**  perpendicular to  **$XY$**  line in the top view. In the front view, this edge  **$c_1'(d_1')$**  appears as a point.

Since the prism has to lie with an edge of the base on **HP**, the front view of the prism is tilted on the edge **c<sub>1</sub>'(d<sub>1</sub>)'** such that the axis is inclined at  $\theta$  to **HP**.

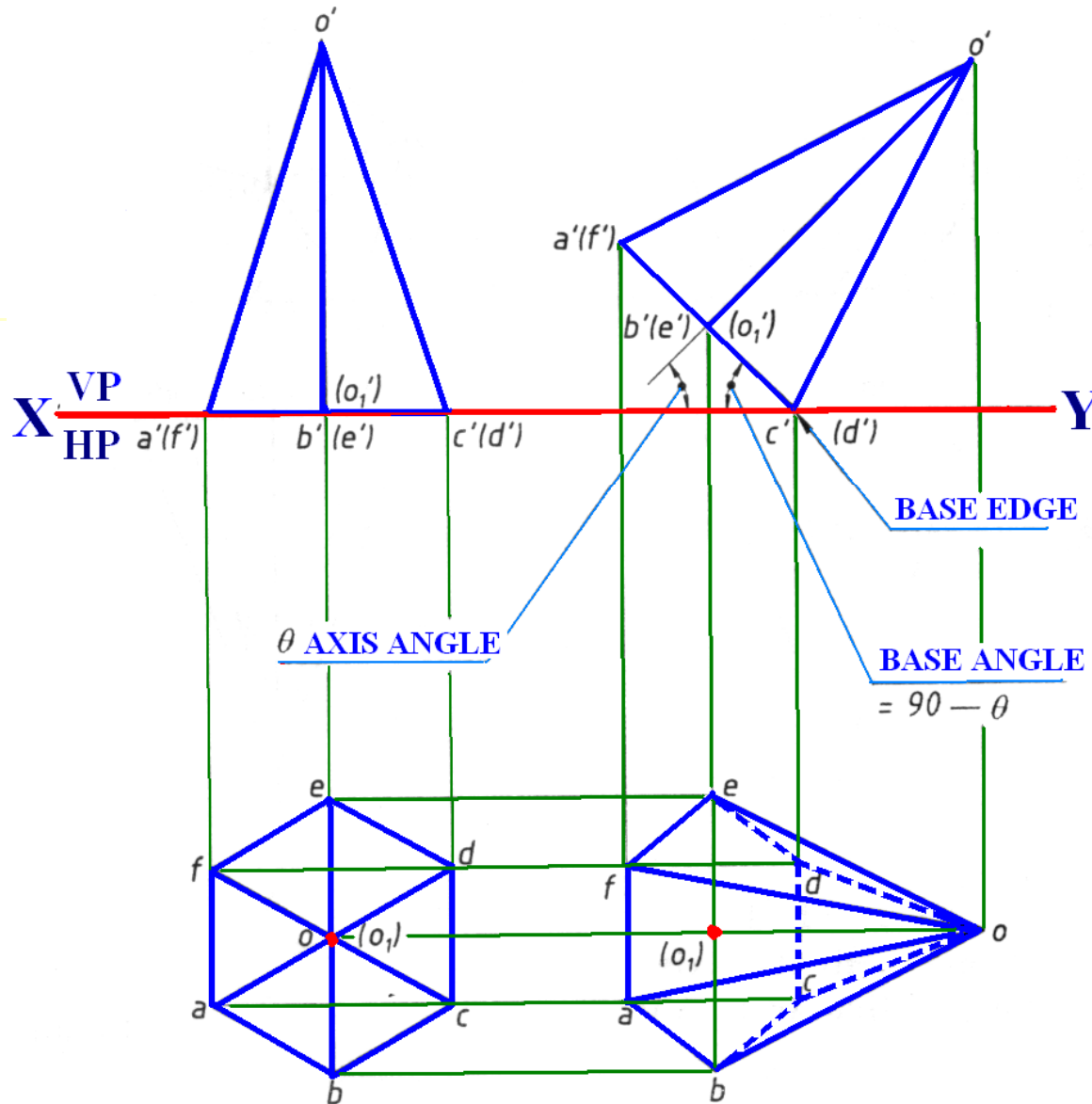
Redraw the first front view in the tilted position.

Whenever the inclination of axis  $\theta$  with **HP** is given, first the base is drawn at **(90-  $\theta$ )** in the front view, otherwise improper selection of the position of the axis may result in the base edge **c<sub>1</sub>'(d<sub>1</sub>)'** lying above or below the **XY** line.

The second top view is projected by drawing the vertical projectors from the corners of the second front view and the horizontal projectors from the first top view.



Top and the front views of a hexagonal pyramid when it lies on HP on one of its base edges with its axis or the base inclined to HP.



## **Case.2 : When the solid lies on one of its corners of the base on HP**

When a solid lies on one of its corners of the base on HP, then the two edges of the base containing the corner on which it lies make either **equal inclinations** or **different inclination** with HP.

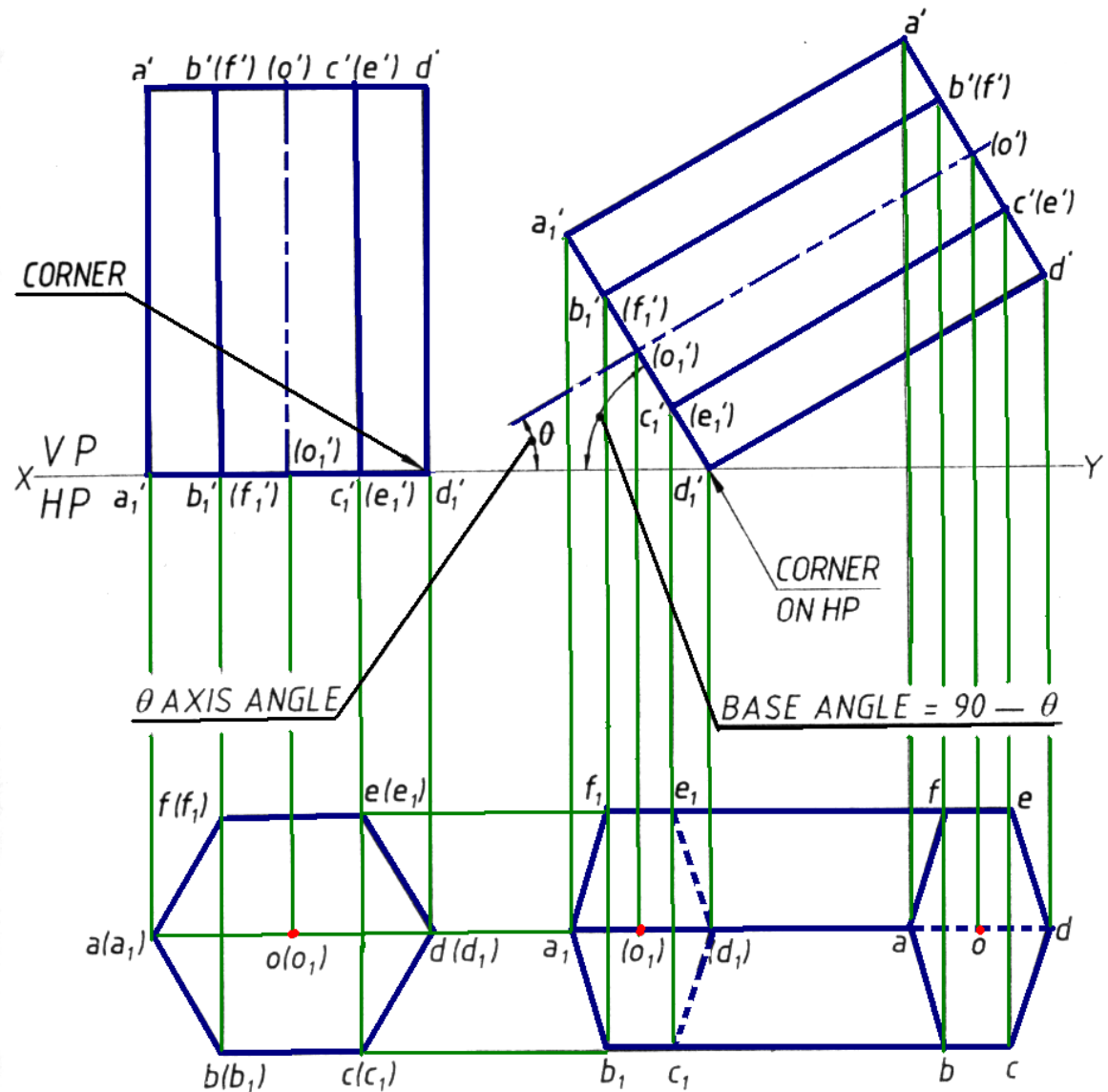
**Corner of the base on HP with two base edges containing the corner on which it rests make equal inclinations with HP**

Initially the solid should be placed with its base on HP such that **an imaginary line connecting the center of the base and one of its corners is parallel to VP**, i.e. to XY line in the top view, and preferably to lie on the right side.



For example, when a **hexagonal prism** has to be placed **with a corner of the base on HP** such that the **base or the axis is inclined to HP**, then initially the the prism is placed with its base on HP such that **an imaginary line connecting the center of the base and a corner is parallel to VP** and it **lies on the right side**.

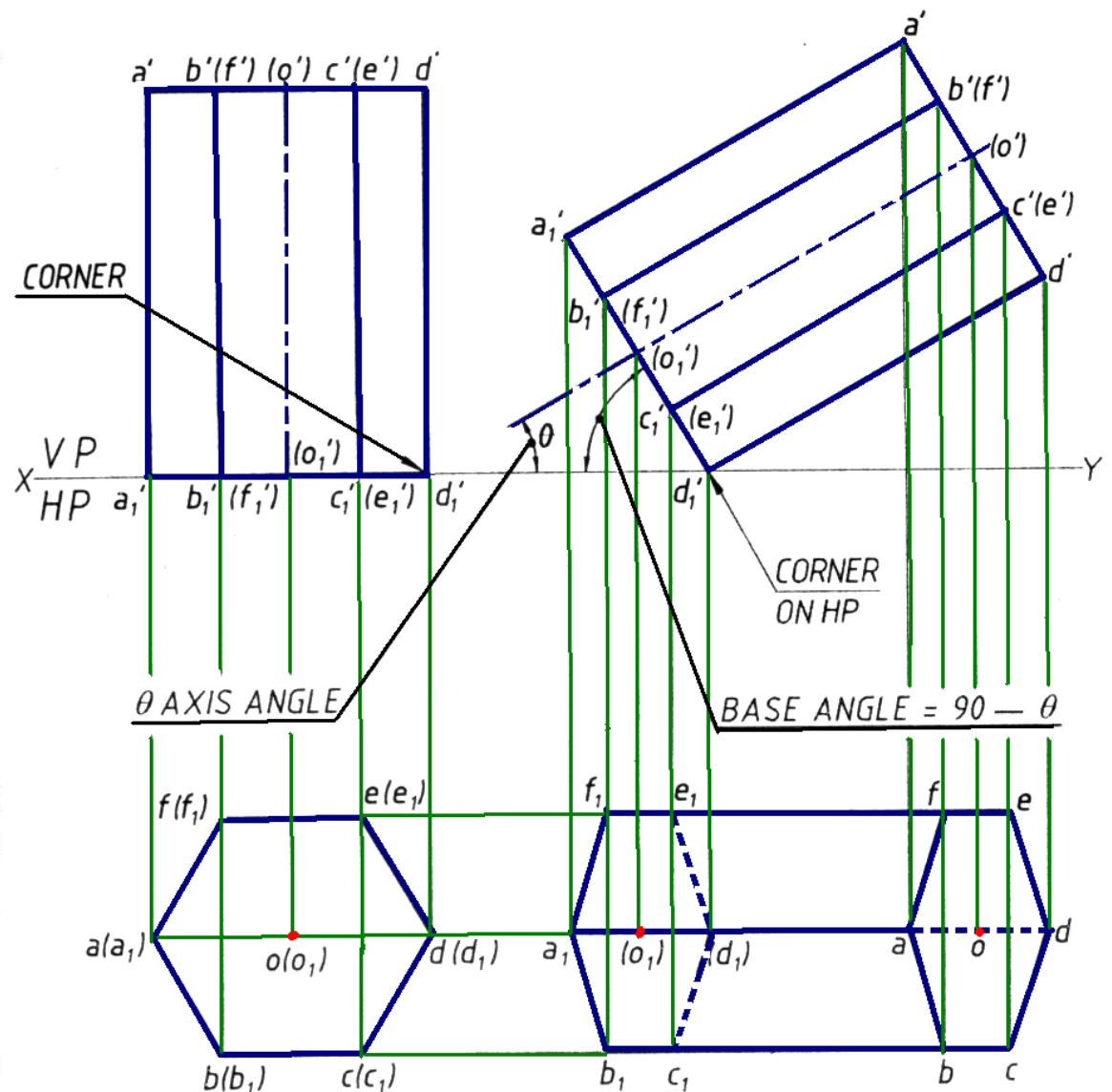
In this position, the first set of top and front views are drawn – the line  $(o_1)(d_1)$  is parallel to the **XY** line in the top view.



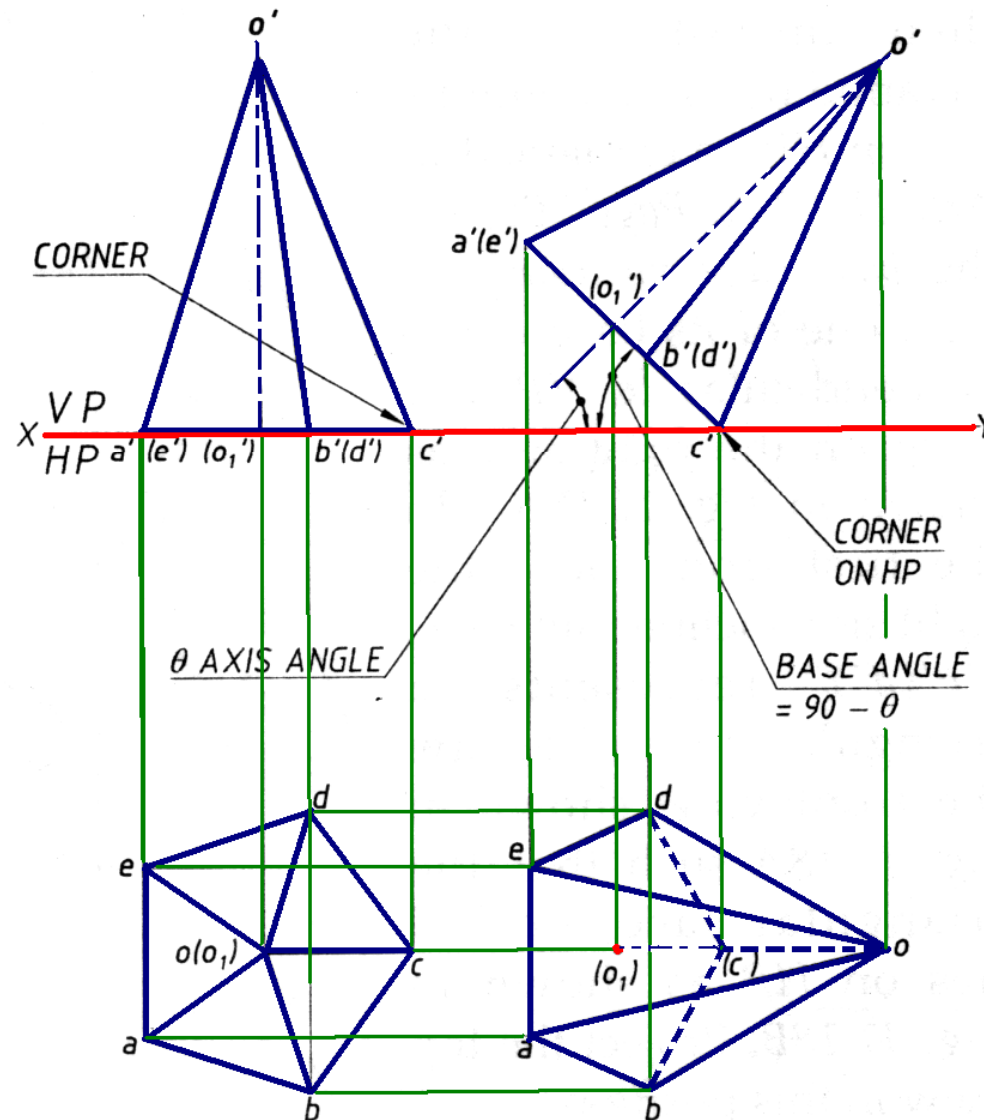
Since the prism **has to lie on one of its corners of the base on HP**, the front view of the prism is tilted on the corner  $d_1'$  such that the axis is inclined at  $\theta$  to HP.

Redraw the front view in the tilted position. The base edge is drawn at  $(90 - \theta)$  in the front view.

The second top view is projected by drawing the vertical projectors from the corners of the second front view and horizontal projectors from the first top view.



**Case.2 for Pyramid:** The top and front views of the pyramid when it rests on HP on one of its base corners such that the two base edges containing the corner on which it rests make equal inclination



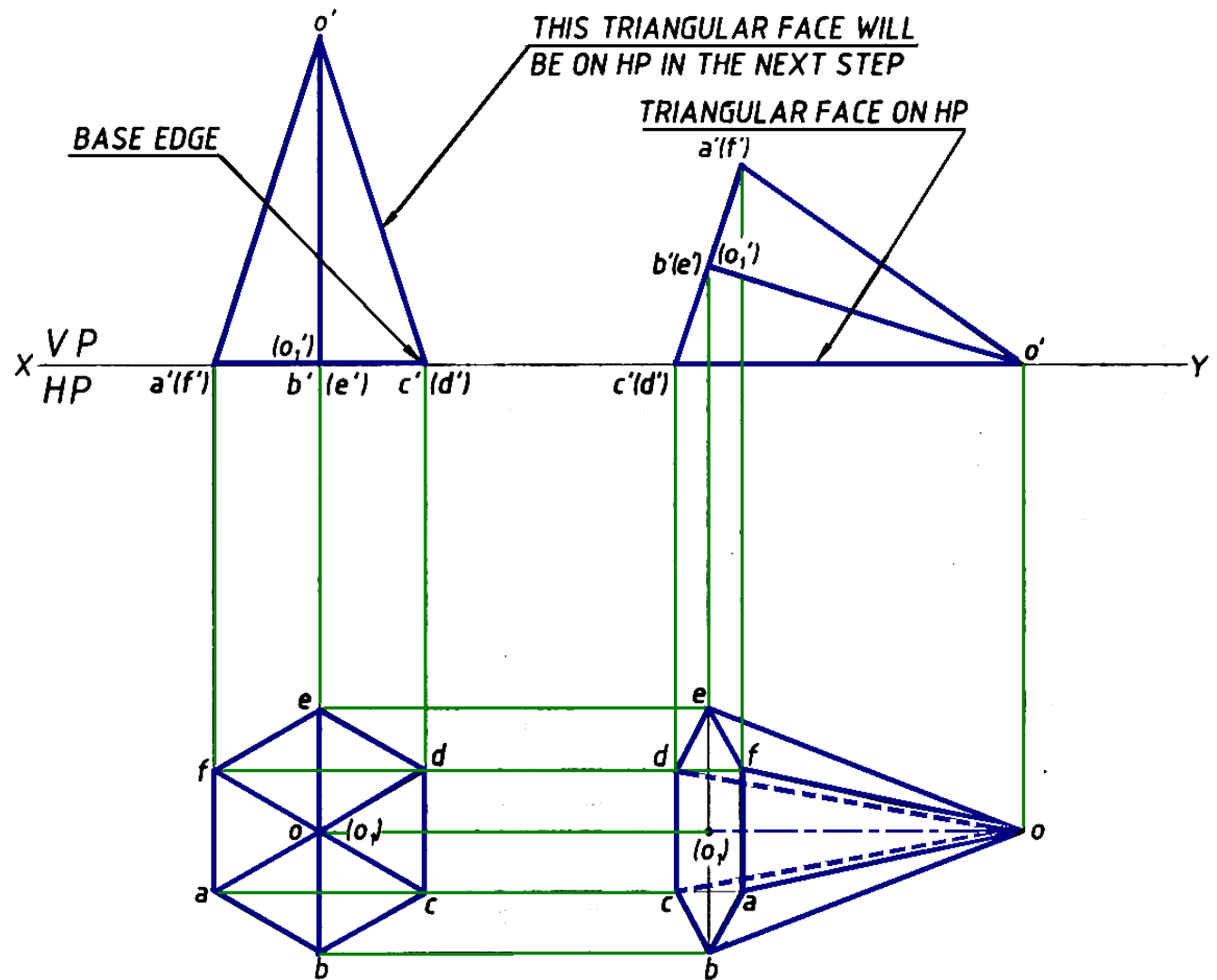
### Case-3 When a pyramid lies on one of its triangular faces on HP

If a pyramid has to be placed on one of its triangular faces on HP, then initially let the pyramid be placed with its base on HP.

In the first front view, the right side inclined line, i.e.,  $o'c'(d')$  represents a triangular face.

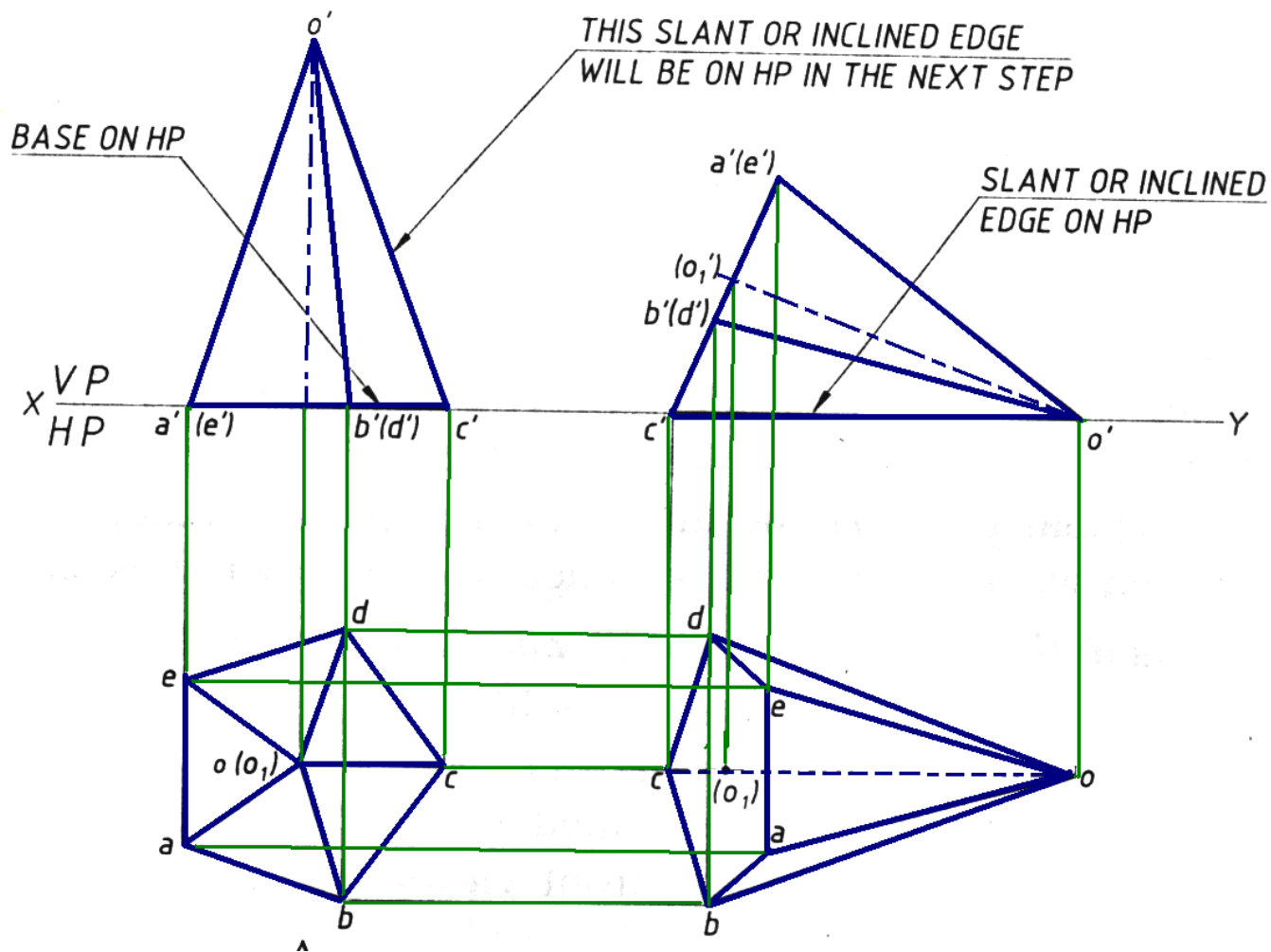
Redraw the front view such that the triangular face  $o'c'(d')$  lies on HP.

Project the top view in this position.



## CASE-4: When a pyramid lies on one of its slant edges on HP

When a pyramid lies with one of its slant edges on HP, then two triangular faces containing the slant edge on which it rests make either **equal inclinations** or **different inclinations** with HP.



## Problem5.

- A cone of base 60 mm diameter and the axis 80 mm long lies on HP with its axis inclined at  $45^\circ$  and  $30^\circ$  to HP and VP, respectively. Draw the top and front views of the cone.

