

Review of Collaborative Filtering in Recommender System

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Abstract

Keyword

SVD, SVD++, Recommender System, Collaborative Filtering

1 Background

Developed in 1990s, the recommender system (RS) has been applied in e-commerce, music apps, job portal, social networking and more [2]. The recommender system collects information on users' behaviors explicitly (users' ratings on items) and implicitly (users' mouse movements, attention on one page, etc.) and makes prediction on users' preferences. For example, Netflix applies a five-star ratings to help users find their favorite movies and maintain their subscriptions [4], and Amazon selects products based on the predicted users' favors to gain profits.

In October 2006, Netflix launched a competition for which they rewarded the team that can beat Netflix's Cinematch system by at least 10% Root Mean Squared Error [4]. This competition aroused great attention in collaborative filtering field as the dataset covered 100 million ratings for 500,000 anonymous customers on 17,000 movies, which was greater than previous public dataset in the orders of magnitude [11]. The final grand prize was given to "Bellkora's Pragmatic Chaos" team in 2009 [4] [10].

In this competition, matrix factorization approaches

raised people's attention as the top teams in this competition frequently applying it to beat Bayes or probabilistic approaches [9]. Singular Value Decomposition as the classical latent semantic indexing approach in information retrieval was found as a great fit for spanning customers data with low dimensions and often adapted to various variants as regularized SVD, SVD++, iterative SVD, and more [4] [7] [6] [9].

2 Rationale behind Collaborative Filtering Recommender System

2.1 User-item Interaction

Matrix factorization models represent the user-item interaction in a latent joint space [11]. Each item i is associated with a vector $q_i \in \mathbb{R}^f$ and user u is associated with a vector $p_u \in \mathbb{R}^f$. The rationale behind such representation is: item i will possess some unknown factors in a measure, positive or negative, stored in entries of q_i , and user will have different favors toward these factors, and such interest is represented as entries in p_u . Therefore, we can take inner product to get an approximate of the user u 's

rating on item i as [11]

$$\hat{r}_{ui} = q_i^T p_u \quad (1)$$

Conventional SVD suffers from an incomplete (or even sparse) rating matrix. Researchers often resort to imputation to fill in the matrix, or through regularized least squared model as [11].

$$\min_{q,p} \sum_{(u,i) \in \kappa} (r_{ui} - q_i^T p_u)^2 + \lambda(\|q_i\|^2 + \|p_u\|^2) \quad (2)$$

in which κ is the set of known ratings. The regularizing term $\|q_i\|^2 + \|p_u\|^2$ controls the magnitude of q_i and p_u to avoid overfitting.

2.2 Bias

Early recommender system exhibit a systematic bias for users giving generally higher ratings than others or items receive higher ratings than others [11]. Therefore, as Koren, et al. pointed out, the entire user-item interaction cannot be simply explained by $q_i^T p_u$, and a bias term exists. A first-order approximation of bias is as follows [11]

$$b_{ui} = \mu + b_i + b_u \quad (3)$$

in which b_{ui} is the bias and μ is the overall average rating of training data, and b_u, b_i are user u and item i 's average ratings compared to the overall average rating [11]. We can transform our estimated rating as

$$\hat{r}_{ui} = b_{ui} + q_i^T p_u \quad (4)$$

and the objective becomes

$$\min_{q,p} \sum_{(u,i) \in \kappa} (r_{ui} - q_i^T p_u - b_{ui})^2 + \lambda(\|q_i\|^2 + \|p_u\|^2 + b_u^2 + b_i^2) \quad (5)$$

3 SVD-Related Methods

3.1 Standard SVD-CF Model

SVD, as a common dimension reduction tool, originally used in information retrieval to identify latent semantic factor (as Latent Semantic Indexing), can represent the user-item

interactions in a low dimensional space [11] [15]. SVD approximates a matrix $A \in \mathbb{R}^{m \times n}$ in a form as

$$A = U \Sigma V^T \quad (6)$$

Suppose the rank of A is r , there are three main properties in SVD[15] [4]:

- The initial r singular values of diagonal matrix Σ holds that $\forall i \in [1, r] \sigma_i > 0$ and $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r$.
- The first r columns of orthogonal matrix U are eigenvectors of $A A^T$ and span the column space of A .
- The first r columns of orthogonal matrix V are eigenvectors of $A^T A$ and span the row space of A .

People usually takes the best rank- k approximation of matrix A as

$$A_k = U_k \Sigma_k V_k^T \quad (7)$$

with $U_k \in \mathbb{R}^{m \times k}$, $\Sigma_k \in \mathbb{R}^{k \times k}$, $V_k \in \mathbb{R}^{k \times n}$.

We will take a rating matrix R with users u_1, u_2, \dots, u_i in the row and items i_1, i_2, \dots, i_j in the column, and the entry R_{ij} is the estimated interest of user u_i on item i_j . [11] [8] [15].

Notice that matrix R is often sparse and require some preprocessing. A common preprocessing approach is to take the row averages to replace all of the missing values in the matrix R and normalize R by subtracting row averages [15] [8].

We then compute $U_k \Sigma_k V_k^T$ as equation 7, and form two matrix products: $U_k \sqrt{\Sigma_k^T}$ and $\sqrt{\Sigma_k} V_k^T$.

There are some variants regarding the computation of predicted rating [15] [14]. For example, Manolis et al. [14] provided a formula for SVD-CF prediction rating for user u_i on item i_j as:

$$P_{ij} = \bar{r}_i + U_k \sqrt{\Sigma_k^T[i]} \sqrt{\Sigma_k} V_k^T(j) \quad (8)$$

Another variant is based on PCA, with the previous steps identical except that we need to replace and subtract column average \bar{c}_j for preprocessing R . In addition, prediction score for user u_i on item i_j will be

$$P_{ij} = \bar{c}_j + U_k \sqrt{\Sigma_k^T[i]} \sqrt{\Sigma_k} V_k^T(j) \quad (9)$$

Some researchers incorporate the reduced matrix $R_{\text{red}} = U_k \Sigma_k V_k^T$ into the calculation of similarity measure to predict the new user or item's score based on the existing clusters. These approaches are usually categorized to user-based collaborative filtering or item-based collaborative filtering depending on the subject of similarity measure.

Pearson correlation and adjusted cosine similarity are common similarity measures in literature [13] [15] [12] [1] [3]:

- Pearson correlation

\mathcal{I}_{uv} means common item rated by both user u and user v . \bar{r}_u means average rating of user u on the shared item set.

$$r = \frac{\sum_{i \in \mathcal{I}_{uv}} (r_{ui} - \bar{r}_u)(r_{vi} - \bar{r}_v)}{\sqrt{\sum_{i \in \mathcal{I}_{uv}} (r_{ui} - \bar{r}_u)^2 (r_{vi} - \bar{r}_v)^2}} \quad (10)$$

- Adjusted cosine similarity

\bar{r}_u means average rating of user u on the entire item set.

$$r = \frac{\sum_i (r_{ui} - \bar{r}_u)(r_{vi} - \bar{r}_v)}{\sqrt{\sum_i (r_{ui} - \bar{r}_u)^2 (r_{vi} - \bar{r}_v)^2}} \quad (11)$$

Pearson-correlation looks really similar to adjusted cosine similarity except that adjusted cosine similarity is calculated over the entire set of rated vectors while person correlation is calculated over co-rated vector [5]. For the missing value case, a typical approach is to set it as zero [12].

After we calculate similarity measure, we will

3.2 SVD++ Model

The traditional SVD method only considers users' explicit ratings but do not take implicit data into account. Although implicit data (mouse move, reading time, etc.) might not be publicly available, there is one kind of implicit data always available: whether a user rates on an item (rate vs. not rate).

3.3 Regularized SVD Method

4 Experiment

similarity

5 Conclusion

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